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AN IMPROVED STATE SPACE REPRESENTATION FOR CYCLICAL TIME SERIES

John Haywood and Granville Tunnicliffe Wilson

An improved state space representation for cyclical time series

John Haywood[*] and Granville Tunnicliffe Wilson[†]

Abstract

We consider a commonly used two state model for a cyclical time series and show how a particular specification of its observation vector allows maximum generality of the range of model spectra.

Keywords

Seasonal time series; cyclical time series; state space representation; model flexibility.

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1 The model and parameter restrictions

Consider the following two state model for a scalar cyclical discrete time series y_t:

$$y_t = hx_t, \quad x_t = Tx_{t-1} + e_t, \tag{1}$$

where $h = (\alpha \; \beta)$,

$$x_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix}, \; T = r\begin{pmatrix} C & -S \\ S & C \end{pmatrix}, \; e_t = \begin{pmatrix} e_{1,t} \\ e_{2,t} \end{pmatrix}, \; \text{var}\,(e_t) = V = \begin{pmatrix} V_1 & W \\ W & V_2 \end{pmatrix},$$

and the series e_t is bivariate white noise, uncorrelated at all non-zero lags. We take the scalar $r \in (0, 1]$ together with $C = \cos 2\pi F$ and $S = \sin 2\pi F$, for some frequency $F \in (0, \frac{1}{2})$, to be fixed parameters. We then consider the specification of the five free parameters α, β, V_1, V_2 and W.

Such a model for cyclical behaviour is presented, using different notation, by Harvey (1989, p.39) with the particular specification of $h = (1 \; 0)$ and either $V_1 = V_2$ or $W = 0$. He later uses the collection of such terms for $F = F_j = j/12$, $j = 1, \ldots, 5$, to represent evolving monthly seasonal harmonics, taking $r = 1$. With the addition of the single harmonic term at the Nyquist frequency, that model has 11 states. Hannan (1964) proposed a harmonic model for seasonality with independent autoregressive processes for the cosine and sine coefficients. For each harmonic frequency, with $h = (1 \; 0)$ and both $V_1 = V_2$ and $W = 0$, the Hannan and Harvey models are equivalent and are related by a (time-varying) transformation. These models have been used frequently since their introduction (see, for example, Bruce and Jurke (1996), Canova and Hansen (1995), Hannan, Terrell and Tuckwell (1970), Hylleberg and Pagan (1997), Young, Pedregal and Tych (1998)).

The spectrum of model (1), or the pseudo-spectrum in the limit $r \to 1$, has a peak close to frequency F. As we note in section 2, that spectrum is of the form

$$P(f) = \frac{\frac{1}{2}(1 + \cos 2\pi f)G + \frac{1}{2}(1 - \cos 2\pi f)H}{4\left\{(uC - r\cos 2\pi f)^2 + (vS)^2\right\}} \tag{2}$$

where $u = \frac{1}{2}(1 + r^2)$ and $v = \frac{1}{2}(1 - r^2)$. The five free parameters in (1) only influence the two quantities G and H. It is therefore natural to seek some way of restricting these five parameters to just two which provide as wide a range as possible of the two parameter spectrum (2). The free parameters can be reduced to V_1, V_2 and W without loss of generality by any fixed choice of α and β such as $h = (1 \; 0)$. This is because a similarity transformation consisting of a rotation and scaling applied to (1) leaves the transition equation unchanged in form and allows h to be transformed to any non-zero vector. Further simple constraints on V_1, V_2 and W, to reduce them to just two free parameters, may however be unduly restrictive. For example taking $W = 0$ it may be shown that, when $F = 0.25$, $P(f)$ is a function of $V_1 + V_2$ so that there is only *one* effective free parameter.

In the next section we present a theorem which ensures that the spectrum (2) is as flexible as possible, whilst still meeting the constraints necessary for a non-negative model spectrum.

2 A modified observation vector

We propose the use of the free parameter constraints specified in the following

Theorem. Let the vector h be $(s\ c) = (\sin \pi F\ \ \cos \pi F)$, and set W to zero so that $V_1 \geq 0$ and $V_2 \geq 0$ are the two free parameters. Then, in the limit $r \to 1$, model (1) admits all non-negative spectra of the form (2), i.e. those with $G \geq 0$ and $H \geq 0$.

Proof. The particular structure of T gives $(I - T\mathrm{B})^{-1} = (I - T'\mathrm{B})/det(I - T\mathrm{B})$, where B is the backward shift operator. So model (1) may be expressed as

$$y_t = h(I - T\mathrm{B})^{-1}e_t = (1 - 2rC\mathrm{B} + r^2\mathrm{B}^2)^{-1}w_t$$

where $w_t = h(I - T'\mathrm{B})e_t$. The spectrum of y_t is then:

$$\frac{H(f)}{|1 - 2rCz + r^2z^2|^2} \tag{3}$$

where $z = \exp(2\pi i f)$. The denominator of (3) may be shown to be that of (2) and $H(f)$ is the spectrum of w_t, given by

$$H(f) = h(I - T'z)V(I - T\bar{z})h'. \tag{4}$$

This has the form of the numerator (2) with the values of G and H obtained by setting $z = 1$ and $z = -1$ respectively in (4). Take $z = 1$ and now exploit the choice of $h = (s\ c)$ which results in $hT' = r(-s\ c)$, a rotation of h through the angle F from $\frac{1}{2}\pi - \frac{1}{2}F$ to $\frac{1}{2}\pi + \frac{1}{2}F$. Consequently $h(I - T') = \begin{pmatrix} s(1+r) & c(1-r) \end{pmatrix}$ and

$$G = s^2(1+r)^2V_1 + c^2(1-r)^2V_2 + 2cs(1-r^2)W.$$

Similarly, by taking $z = -1$,

$$H = s^2(1-r)^2V_1 + c^2(1+r)^2V_2 + 2cs(1-r^2)W.$$

As $r \to 1$, $G \to 4s^2V_1$ and $H \to 4c^2V_2$ so that the range of values $V_1 \geq 0$ and $V_2 \geq 0$ is equivalent to $G \geq 0$ and $H \geq 0$. Note that as $r \to 1$ the covariance W does not enter the spectrum so that there is no loss of generality caused by the constraint $W = 0$.

■

Provided r is close to 1 the constraint $W = 0$ may also be applied with little practical limitation on the range of feasible spectra. By contrast the same constraint with the choice of $h = (1\ 0)$ leads, as $r \to 1$, to $G = 4s^2(s^2V_1 + c^2V_2)$ and $H = 4c^2(c^2V_1 + s^2V_2)$. Values of $G = 0$ or $H = 0$ are not possible with $V_1 + V_2 > 0$, and when $F = 0.25$ equality of s and c leads to $G = H$ whatever the values of V_1 and V_2.

Applications of this result to time series modeling, forecasting and seasonal adjustment are presented by Haywood and Tunnicliffe Wilson (1999).

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