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THE DISCRETE TIME BETA

by Roger J. Bowden*

Abstract

Empirical studies of the CAPM commonly utilise returns measured over discrete time intervals. In such a sampled data frame, the correct choice of the risk free rate is far from clear- it is commonly taken as the yield on a discount bill or bond, whose time to maturity coincides with the sampling interval. It is also unclear whether use of discrete data preserves the independence of the CAPM relationship from idiosyncratic variance elements. This paper relates the continuous beta to what one would observe from discrete data, assuming continuous underlying no- arbitrage trading processes. It is shown that the risk free rate can be eliminated, as a nuisance variable, and the CAPM cast in terms of two benchmarks, namely the market rate of return and the yield on a default free discount bond; the latter more or less corresponds to the 'risk free rate' as it appears in the usual empirical study. In such terms, the bond beta contaminates the measured stock beta, powering up the latter, so that high beta stocks appear even higher, and low beta stocks even lower. It is shown that the bond beta is equal to one half times the proportion of the overall market portfolio devoted to risky assets. Thus one can expect the powering up - and hence the measured stock betas - to vary over the business cycle. Moreover, idiosyncratic variance elements appear as an intercept term in the CAPM relationship, though this effect vanishes as the sampling interval becomes small.

Key words: *Beta, CAPM, bond yields, continuous trading, cost of capital, empirical theory, equivalent martingale, no-arbitrage trading process, risk free rate, sampled data.*

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1 Introduction.

The literature on the CAPM beta is both huge and indecisive. It remains unclear why a theory of such robustness in alternative assumptions should perform so badly in its empirical fitting. In recent times, inquests from authors such as Fama and French (1996) under the 'beta is dead' catch phrase have attracted much attention, as well more long standing reservations implicit in the Roll critique (Roll, 1977), and a variety of other authors on anomalies, the question of time stability, time series versus cross section, and other issues. However while much of the literature has been preoccupied with the actual techniques of estimation or testing, little of it- apart from the Roll contribution- has actually considered the matter of the underlying empirical theory, as distinct from the econometrics. In other words, one starts with a relationship derived in a particular theoretical world, and the problem then arises as to what one can expect to happen when the translation is made to the world represented by the available data. The present paper addresses two related issues that arise within such a framework. The first considers the empirical meaning of the risk free rate. The second is concerned with the choice of time interval over which the returns are to be measured. Both are ultimately concerned with what one would expect to happen when the CAPM model is actually fitted.

Inevitably, the available data, for the time series phase of the study, spans some finite time interval: an hour, day, a month, a year, and so on. Where all are available, as is now pretty much the case with sophisticated commercial databases, which should be used? Most researchers would not feel comfortable in estimating a stock's beta on the basis of hourly or even daily trading data, which may very well be dominated by extraneous "noise", in the terminology of Fischer Black(1986). Moreover if the proposed use is for cost of capital studies, the time interval to be employed is also relevant. On the other hand, if a longer period of time is to be considered, measurement of the risk free rate becomes a problem. On the basis of monthly data, for example, one could elect to use the yield on 30 day T-bills as the risk free rate. However this may not coincide with the true risk free rate as it is actually being used to measure the opportunity cost of risk by investors. One might ask, for instance, why investor decision horizons should coincide so beautifully with the precise monthly boundaries presumed by the study. Where they do not, the measured risk free rate is far from risk free. To some extent, this difficulty has been overcome in the more recent literature by the use of the yield on a default free zero coupon instrument whose term coincides with the measurement period, to form a stochastic discount factor. In the 'law of one price' methodology of Hansen and Jagannathan (1991), the betas are assessed in terms of the covariance between the stock's return and a stochastic discount factor, whose expectation is the price of the zero coupon bond. The origins of this approach lie in the modern theory of no arbitrage financial general equilibrium, where the discount factor represents discounted state prices for risk. In this respect the development extends Breeden's (1979) idea of the consumption beta as determined by the marginal utility of consumption in various states of the world. While this is conceptually much more satisfactory, it does not relate the risk premium for the stocks directly to their betas with respect to a recognisable rate of return on the market, or even with that on the bond itself. Thus the stochastic discount factor approach does not address the question of just what one might expect to happen where studies cast in terms of the more traditional framework of expected rates of return of the stocks with the market.

In the present contribution, we take the approach that if the true definition and empirical identification of the risk free rate (or a zero beta security) is a problem, then we can bypass it, essentially by treating the risk free rate as a nuisance variable that can be eliminated. In this approach, the two benchmarks of the conventional theory, namely the risk free rate and the market return, are replaced by the bond rate and the market respectively. In many ways this is a natural thing to do for cost of capital studies, for it highlights the margin of the stock in question not only with respect to the market as a whole, but also relative to the rate of return quoted in the fixed interest market over the chosen observation interval.

Once this is done, then we can also relate the observed discrete time betas to the underlying parameters of continuous time trading processes. Supposing the latter to constitute basic parameters, it emerges that the discrete betas are simple transformations of the continuous time betas, characterised by the following:

(a) The discrete betas are contaminated by the beta of the bond with respect to the market, which is neither zero nor unity. Specifically, the stock betas are powered up by the bond beta. High stock betas become even higher, and low stock betas become even lower. One could call this the 'over-estimation effect'. We show that the bond beta is equal to one half the proportion of the overall market portfolio devoted to risky assets. It can therefore be expected to vary over time according to cyclical attitudes towards risk, or less formally as to whether times are good or bad, in terms of perceived investment opportunities and/or the marginal utility of wealth. In turn, the powering up effect on stock betas will inherit this variation, so one would expect a cyclical effect on measured stock betas. The powering up effect is independent of the sampling interval employed.

(b) The discrete beta is contaminated by idiosyncratic variance elements. Stocks that are more intrinsically variable therefore appear to have higher betas, no matter that they are not so penalised in the underlying continuous time trading. This effect is dependent on the time interval - the shorter the chosen interval, the smaller the effect.

In short, discrete observation betas are more variable in cross section than are the underlying continuous time betas; and there may be additive contributions to the CAPM relationship itself from an idiosyncratic risk term. With regard to time series, the betas are also more variable across different phases of the business cycle. The latter aspect could be thought of as providing theoretical motivation for empirical studies that attempt to allow for the possibility that beta does in fact change; for examples see Fisher and Kamin (1985) or Ferson and Harvey (1991).

The scheme of the paper is as follows. Section II establishes the underlying continuous time trading model, which in itself is quite standard. Within such a framework, we derive the instantaneous beta equation for any chosen risky security. The corresponding equation is also obtained for default free zero coupon bonds, relative to the market fund of risky assets; it is the same for zero coupon bonds of all maturities. The two sets are combined into a single beta relationship that eliminates the risk free rate between the two beta equations, one for the stock and one for the bond. The yield on the bond now replaces the risk free rate. In section III this relationship is integrated up over the unit interval to obtain the discrete beta. Powering properties are exposited in terms of the mapping between the continuous and discrete betas. Some concluding comments briefly explore the implications for cost of capital assessments.

II Continuous time: The risk premium process and the global portfolio.

2.1 *The risk premium process and the reproducing portfolio.*

For simplicity we review here only the case of the complete market. The formal algebra is the same for the incomplete case but the risk premium process is no longer necessarily unique. Even with complete markets, the analysis that follows can be generalised further, but in order to dispense with mathematical details, we cover only the most basic case. Suppose that there is a set of N securities that span an underlying or driving set of stochastic influences, the latter modelled in continuous time by some N dimensional Ito process. The security returns are then themselves Ito processes - or more precisely so are the values P_i of the corresponding accumulation funds, which represent the progressive rewards in terms of dividends, capital gains and other reward elements. Security returns in the chosen spanning set are specified to have means $\boldsymbol{\mu}$ and covariance matrix Σ_* conditional on information $\mathcal{F}_s$ up to time s. In the present section time s will be taken as continuous - later we shall reserve the index t for discrete time. The mean and covariance matrix are defined in the Ito sense as the instantaneous drift and coefficient of the Brownian motion terms. They need not be constant over time. In what follows, the time unit will be taken as the interval (s, s+ds). There is also an asset whose instantaneous return over the interval is the risk free rate ρ . The latter will in general be stochastic over time (the 'spot rate', in the terminology of interest rate theory).

Given the above specifications, there exists a risk premium process ξ that evolves according to the Ito equation

$$(1) \qquad d\xi = -\frac{1}{2}\xi\,\theta'\theta\,ds - \xi\,\theta'\,d\boldsymbol{B},$$

where $\theta = \Sigma_*^{-\frac{1}{2}}(\boldsymbol{\mu} - \rho\mathbf{1})$, with $\mathbf{1}$ as the unit vector, and $\mathbf{B}$ denotes the underlying Brownian motion process. For brevity, we shall sometimes refer to the above as an equation in $d\xi/\xi = \xi'$, say, with drift ($-\frac{1}{2}\theta'\theta ds$) and volatility coefficient ($-\theta$). In complete market theory, the risk premium process is used as a stochastic discount factor to value future payoffs. Duffie (1992) is an excellent discussion in the context of state price theory; a compendious recent reference is Magill and Quinzii (1996).

Define a vector $\mathbf{x} = \Sigma_*^{-\frac{1}{2}}\theta$, and consider a portfolio M of nominal value \$1 with amounts $\mathbf{x}$ in the risky assets and $1 - \mathbf{1}'\mathbf{x}$ in the safe asset. The returns on this portfolio are such that the drift or mean term is $\mu_m = \theta'\theta + \rho$ and the instantaneous variance is $\sigma^2_m = \theta'\theta$ (for readability we omit the time increment ds in the present context). The quantity $\lambda_m = \frac{\mu_m - \rho}{\sigma_m} = (\theta'\theta)^{\frac{1}{2}}$ is often referred to as the market price of risk for the M portfolio. If we restrict attention to the risky asset part of the portfolio, this will have portfolio weights given by $\mathbf{x} / \mathbf{1}'\mathbf{x}$, and will have instantaneous variance $\sigma^2_* = \sigma^2_m / (\mathbf{1}'\mathbf{x})^2 = \theta'\theta / (\mathbf{1}'\mathbf{x})^2$. The price of risk for this portfolio $\lambda_* = \frac{\mu_* - \rho}{\sigma_*}$ is the same as for portfolio M, i.e. $\lambda_* = \lambda_m$. The interpretation of the return on the nominated portfolio of risky assets is that there exists a market accumulation fund (the reproducing portfolio) of risky assets whose price P_* obeys the Ito process

(2) $$dP_*/P_* = (\rho + \lambda_* \sigma_*)\, ds + \sigma_*\, dB_* ,$$

where the Brownian motion increment dB_* is defined in term of the constituent security increments $d\mathbf{B}$. This particular form motivates the conventional description of λ_* as the market price of risk, referring to the pricing penalty attached to the portfolio standard deviation σ_*. The market return is interpreted as the proportional change of P_*, the price of the accumulation fund generated by the reproducing portfolio of risky assets. For typographical convenience, we shall often write $dP_*/P_* = p_*$, ie use the lower case for the instantaneous proportional change. Similarly, for the individual securities the notation $p_i = dP_i/P_i$ will be used. Thus $E\, p_i$ will mean $(EP_{i,s+ds} - P_{i,s})/ P_{is}$, with a similar meaning for $\mathrm{Cov}\,(p_i, p_j)$.

To get the instantaneous beta relationships, define the conditional regression coefficients

$$\beta_i = \mathrm{Cov}\,(p_i, p_*)/ \sigma^2_*\, ds ,$$

so that collectively,

$$\beta = \frac{1}{\sigma_*^2} \frac{1}{1'x} \Sigma_*^{½} \theta$$

$$= \frac{1}{\sigma_*} \frac{1}{\sigma_* 1'x} (\mu - \rho 1)$$

$$= \frac{1}{\sigma_*} \frac{1}{\lambda_*} (\mu - \rho 1),$$

In view of the definition of λ_* it follows that for security i, the drift terms are related by $(\mu_i - \rho) = \beta_i(\mu_* - \rho)$. It follows that for any security in the economy, whether or not in the chosen spanning set, the drift terms are related by:

(3) $$E\, p_i - \rho ds = \beta_i (Ep_* - \rho ds) ,$$

which is the instantaneous CAPM.

2.2 Relationship with zero coupon rates.

The price at instant s of a zero coupon bond of maturity T > s is given by

(4) $$P_s = \left(\frac{1}{\xi_s}\right) E\left(\xi_T \, e^{-\int_s^T \rho_u du} \right),$$

where the expectation is taken conditional on information available at instant s. Define $P^n_s = \xi_s P_s$. Applying the iterated expectation to expression (4) over the infinitesimal interval from s to s+ds , we get

$$P^n_s = e^{-\rho_s ds} \underset{s}{E}(P^n_{s+ds}) ,$$

where the expectational subscript emphasises conditionality on information available at instant s. Thus P^n is the price of a risk adjusted bond, that returns an instantaneously non stochastic rate ρ_s. One has effectively transferred to a risk neutral world, the "equivalent martingale theory", in the terminology of Harrison and Kreps (1979), in which the discounted prices of securities follow martingale processes under a transformed (EM) probability measure.

Indeed, under the EM measure, the zero coupon bond follows a price process

$$dP/P = \rho ds + \theta_b' d\mathbf{B}^m ,$$

where B^m is a martingale process, and the volatility coefficients θ_b may depend on the maturity T of the bond. The relationship between the increments under B^m and B is: $d\mathbf{B}^m = d\mathbf{B} + \theta ds$. Hence under the original or natural probability measure,

(5) $$dP/P = (\rho + \theta_b'\theta)ds + \theta_b' d\mathbf{B} .$$

Now since $dP_n/P_n = p_n = p + \xi'$, in proportional change terms, and $p_n = \rho ds$, it must be that $\theta_b'\theta = \frac{1}{2}\theta'\theta$. This is an important constraint on the allowable θ_b ; it essentially means that they must lie on a hyperplane in N dimensional space that is oriented orthogonally to the vector θ .

However, our precise concern here is with the bond beta relative to the market portfolio of risky assets. Thus comparing equation (5) with (1), $\mathrm{Cov}(p, \xi) = -\frac{1}{2}\theta'\theta$, from which the beta of the bond return upon the market portfolio return p_{*} is given by:

$$\beta_b = \tfrac{1}{2}\lambda_{*}/\sigma_{*} = \tfrac{1}{2}\,\mathbf{1}'\mathbf{x} \,. \tag{6}$$

The bond beta is thus equal to half the proportion of the overall market portfolio M allocated to risky assets. Note that this means that the bond beta will be between zero and unity.

The corresponding CAPM relationship on the market portfolio of risky assets would be

$$(Ep - \rho)ds = \beta_b(E p_{*} - \rho ds) \,. \tag{7}$$

It can be verified from equation (5) that this pricing relationship is indeed consistent with the mean term for the bond price process. Thus the CAPM beta pricing relationship holds for the zero coupon bond as well as for stocks. It can also be verified that it holds for the risk adjusted bond considered above, whose price is P_n ; in this case the beta is zero.

Note finally that the instantaneous expected return is independent of the maturity of the bond. However this does not mean that the instantaneous variances of all bonds are the same. The significance of the risk premium process is that the associated volatility coefficients given collectively by $\frac{1}{2}\theta$ constitute a lower bound for the variance of bond returns. To see this, note from expression (5) that the variance of an arbitrary bond is given by $\theta_b'\theta_b$. Minimising this subject to $\theta_b'\theta = \frac{1}{2}\theta'\theta$ results in the choice $\theta_b = \frac{1}{2}\theta$. In some circumstances[1] , the minimum variance may actually be associated with a bond of specific maturity T_m, so that there is some maturity for which the variance of zero coupon bonds is minimal. This may ,of course, change over time, as these properties relate to the instantaneous variance, given $\mathcal{F}_s$. One might think that the minimal variance bonds are those of very short maturities, but this may not necessarily be so.

2.3 *Changing the instantaneous benchmark rate*

Equations (3) and (7) may be grouped together as

(8a) $$Ep_i - \rho ds = \beta_i (Ep_{*} - \rho ds) \qquad \text{(security i)}$$

(8b) $$Ep - \rho ds = \beta_b (Ep_{*} - \rho ds) \qquad \text{(bond)},$$

where $\beta_b = \frac{1}{2} (\lambda_{*}/\sigma_{*}) = \frac{1}{2}\, \mathbf{1'x}$. The two equations (8a,b) can be used to eliminate the instantaneous risk free rate. We end up with the beta type relationship:

(9) $$Ep_i = Ep + b_i (Ep_{*} - Ep) ,$$

where:

$$b_i = \frac{\beta_i - \beta_b}{1 - \beta_b} .$$

Equation (9) is a form of the continuous time CAPM where the benchmark is the instantaneous expected rate of return on the bond, rather then the risk free rate. It is the starting point for the discrete time treatment of the next section.

For each security i, construct an asset or benchmark fund of proportions b_i in the market fund and $(1-b_i)$ in the bond. The return on the benchmark fund would be

(10) $$f_i = p + b_i (p_{*} - p) ,$$

and the beta of p_i on f_i would be unity. So one could write

$$p_i - Ep_i = f_i - Ef_i + d\eta_i ,$$

where η_i is an idiosyncratic process whose differentials $d\eta_i$ are statistically independent of the benchmark returns f_i . As $Ep_i = Ef_i$, it follows that

$$(11) \qquad p_i = f_i + d\eta_i \ ,$$

so that the return on security i decomposes into the systematic part f_i and the unsystematic or idiosyncratic part $d\eta_i$ as in the standard theory, with additive variances.

III Discrete time

3.1 *The discrete CAPM*

Let t denote real time and consider a unit time interval from instant t to instant t+1 ; a month, a year, and so forth. In the case of the zero coupon bond, set T = 1, i.e. it will have unit time to maturity. We assume the instantaneous beta b_i is constant over the discrete interval, and also that Var ($d\eta_{is}$) = σ^2_{is} ds does not depend upon f_i for $u \leq s$, and can be treated as an exogenous process so far as the systematic effects are concerned. Also define

$$\sigma^2_{\eta it} = \int_t^{t+1} \sigma^2_{\eta is}\, ds \ .$$

This is the instantaneous idiosyncratic variance over the chosen interval.

The above assumptions as to the behaviour of the beta and the idiosyncratic variance over the unit period call for some comment. They are made to derive tractable results in what follows, but they do have a certain intuitive logic to them. Essentially, we suppose that change can occur, but only on the macro or inter- period scale, rather than the micro or intra- period scale. If the betas are changing quite rapidly over the time scale of the unit interval, it is difficult to justify the logic of an approach that attempts to derive the discrete cost of capital in such terms. It is more appealing to allow the betas to change but only slowly; or alternatively, with sudden bumps and grinds punctuated by periods of relative stability. Likewise, one can allow the idiosyncratic variance to change, but only over the macro scale.

Equation (11) above decomposes the price process P_{is} into two additive processes the systematic part F_{is} and the unsystematic part η_{is} . The latter has zero drift and increments that are independent of the F_{is} process. Hence integrating (10) over the unit period and taking expectations conditional on information available at the beginning of the period, i.e. instant t, we obtain

$$E_t P_{i,t+1} = P_{it}\, e^{\frac{1}{2}\int_t^{t+1} \sigma_{\eta,u}\, ds} E_t\, e^{\int_t^{t+1} \frac{dF_{iu}}{F_{iu}}} \quad ,$$

By definition, $F_{it} = P_{it}$. We end up with

$$(12) \qquad E_t P_{i,t+1} = e^{\frac{1}{2}\sigma^2_{\eta it}} E_t F_{i,t+1} \quad ,$$

Also, integrating equation (10) over the unit period we get

$$(13) \qquad \frac{F_{i,t+1}}{F_{it}} = \left(\frac{P_{t+1}}{P_t}\right)^{1-b_i} \left(\frac{P_{*,t+1}}{P_{*t}}\right)^{b_i} \quad .$$

Define the holding period yields over the period as r_{it} for security i ; r_{bt} for the bond; and r_{*t} for the market portfolio. Equation (13) implies that

$$(1 + r^f_{it}) = (1 + r_{bt})^{1-b_i}(1 + r_{*t})^{b_i} \quad .$$

Taking logs of both sides and expanding log(1+x) in Taylor series about zero, we obtain

$$(14) \qquad r^f_{it} = r_{bt} + b_i\,(r_{*t} - r_{bt}) + \delta_{it} \quad ,$$

where:

$$\delta_{it} = r^{f\,2}_{it} - [\, r^2_{bt} + b_i(r^2_{*t} - r^2_{bt})] \quad .$$

The error δ_{it} in the linear approximation (14) amounts to a difference between two squared interest rates, themselves small unless b is unrealistically large in absolute value. Hence it is ignored in what follows. Taking expectations of both sides of (14), and incorporating expression (12), we end up with the approximation

$$(15) \qquad E\, r_{it} \approx \tfrac{1}{2}\sigma^2_{\eta it} + r_{bt} + b_i\,(E\, r_{*t} - r_{bt}) \quad ,$$

where in each case the expectation s are with respect to information available at the start of the period . Equation (15) will be taken as the discrete version of the CAPM relationship. In what follows we analyse the ways in which this can be expected to differ from the conventional empirical version.

3.2 *Temporal and cross sectional stability*

Relative to the conventional CAPM, the discrete CAPM contains within itself more possibilities for variation both across securities and over time, and in the conjunction of the two aspects.

Cross sectional variation.

This refers to the term in the idiosyncratic variance, which now plays the role of an intercept in the CAPM expectational equation, or at least the equation as it would appear from the vantage point of an investigator operating with discrete data. Much depends on the purpose of the empirical exercise. If discrete discount rates are to be used in a capital budgeting or similar exercise, then the variance term adds to the cost of capital. From this point of view its appearance in the discrete CAPM relationship re-introduces the idiosyncratic risk to add to the systematic risk, considerably diminishing the appeal of the CAPM based discounting philosophy. On this view, the cost of capital depends in some measure on the total risk of the security. If, on the other hand, the object is simply to utilise the discrete data to estimate a beta, then the variance intercept is a nuisance parameter. It may or may not be difficult to get rid of. If, for example, the $\sigma^2_{\eta it}$ is itself time varying, then things may indeed be problematical, and the stress would then be on using finer time intervals so that the variance contribution would be smaller.

Beta powering.

As $b_i = (\beta_i - \beta_b) / (1 - \beta_b)$, the magnitude of the discrete beta will depend on the beta of the unit period zero coupon bond, as well as the underlying continuous time beta for the actual security i. Relative to the underlying continuous beta (β_i), the net effect is to amplify the variation in the original betas about the point $\beta = 1$. Moreover the amplification may itself depend on the variation of the bond beta through time. Earlier (see equation (6) above), the zero coupon beta was interpreted in terms of the proportion of the overall reproducing portfolio that was devoted to risky assets. One would expect that in volatile times, or when times are bad, a lower proportion would be allocated in this way. Recall, for instance, that for constant relative risk aversion utility functions, the parameter itself is related to the proportion of assets invested in risky investments, and this is a matter of the curvature of the utility function- marginal utility is increasing more rapidly when wealth is lower .

Figure 1 will help to fix ideas. It represents the mapping from the underlying continuous beta to the discrete one. Observe that regardless of the magnitude of the bond beta, the mapping has a fixed point at $\beta_i = 1$; the resulting b_i is also unity. We have assumed that the bond beta is less than unity, as one would expect from its interpretation. The two lines drawn represent the graphs of the discrete beta against the continuous, for higher (steeper) and lower (flatter) bond betas. The steeper of the two can therefore be interpreted as what happens in less volatile, or good times, and the flatter as corresponding to bad times.

As figure 1 shows, there is always magnification of the continuous beta, apart from the pivot at unity. High beta stocks become higher, while low beta stocks become lower. But the interesting thing is the cyclical effect. The analysis suggests that high natural beta stocks become even higher when times are good, while low natural beta securities become even lower. And when times are bad, low beta stocks become higher, while high beta stocks become lower. In other words there is a degree of resonance between the natural betas and the environment that suits them best. This represents another possible rationale for the Blume(1975) effect, wherein it is observed that numerically higher betas tend to regress towards unity over suceeding time intervals of measurement. The original explanation ran in terms of statistical sampling error or variation over succeeding time periods. But it might be that high beta stocks regress in succeeding time periods to lower, simply because bad times tend to follow good over the business cycle.

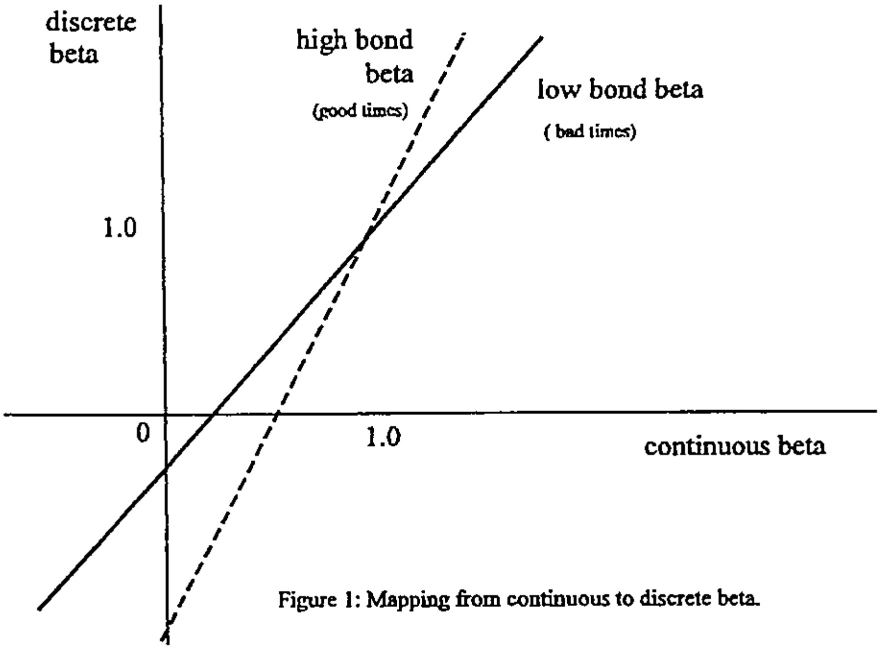

Figure 1: Mapping from continuous to discrete beta.

3.3 *Concluding remarks: beta and the cost of capital*

The purpose of computing beta is often to compute the company's cost of capital for purposes of capital budgeting or the valuation of the firm's cash flows for other reasons. In such terms, failure to identify the true continuous bets may not be a matter of much importance; the adjustment for the bond beta is, after all, the same for any company. A study of equation (15) does indicate that the time interval used to measure returns should be small, in order to avoid serious contamination by the idiosyncratic variance, at the risk of counterbalancing contamination by market noise, referred to earlier.

Possibly a more serious difficulty might arise in assessing the cost of capital for long dated projects, which might be expected to span a succession of bad times and good times over their life. Our analysis suggests that the bond beta, and hence the discrete time beta, will vary over the stages of the business cycle. Thus in employing the resulting beta in the cost of capital, one will have to face up to the likelihood of intertemporal variation in the future. Should the cost of capital to be applied to the future cash flows represent some sort of average; or should one attempt to build likely future economic conditions into the a term structure of future discount rates? In the case of shorter dated projects such problems are likely to be less serious: the cash flows might be expected to fall into one or the other category, good times or bad, and using observations of recent vintage to compute the beta would probably give a reasonable guide. However, more work remains to be done on the implications for long dated projects of intertemporal variation in the cost of capital, induced by systematic changes in beta, as well as changes in the market risk premium as a whole.

While the results concerning the over estimation effect are in a sense more graphic, the contamination by idiosyncratic variance elements itself raise some problems of practice. If the sampling interval is chosen too long, the distortions may be unacceptable in magnitude. But if too short, then market noise or measurement errors may obscure the identification of the betas. There may be some sort of limit to the precision with which we could ever hope to measure beta, Finance's very own version of the Heisenberg uncertainy principle! In short, the 'beta is dead' literature may be telling us that beta is hard to pin down as to temporal stability, and obscured even where it is stable. This would be consistent with the sort of story emerging from the present contribution.

FOOTNOTES

[1]Suppose that $\theta_1, \theta_2, \ldots \theta_N$ is a basis for $\mathbb{R}_N$ so that any vector θ_b can be represented in the form $\alpha_1\theta_1 + \alpha_2\theta_2 + \ldots \alpha_N\theta_N$. If the scalars α_i are functions of maturity T, then there may be $T = T_m > 0$ such that $\alpha_1(T_m)\theta_1 + \alpha_2(T_m)\theta_2 + \ldots \alpha_N(T_m)\theta_N = ½\theta$. If so then the zero coupon bond of maturity T_m would be the minimum variance bond.

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