

The GSBGM Working Paper Series

WORKING PAPER 2/98

**CAPM and Empirical Embedding:
When is 'near enough', good enough?**

Roger J. Bowden

© 1998 Roger J Bowden.

All rights reserved. Apart from any fair dealing for the purpose of private study, research, criticism or review, as permitted under the Copyright Act, no part of this document may be reproduced by any means, electronic, mechanical electrostatic photocopying or otherwise, or stored in a retrieval system without the prior permission of the authors.

ISSN 1173-4523

ISBN 0-475-11530-9

The GSBGM Working Paper Series 2/98 February 1998.

Roger J Bowden 'CAPM and Empirical Embedding: When is 'near enough', good enough?'

Please address inquiries concerning this paper direct to the author*

Other queries to:

Angela Dolan

Manager Administration Services

The Graduate School of Business and Government Management

The Victoria University of Wellington

PO Box 600

Wellington

New Zealand

Telephone:

64-4-471 5366

Facsimile:

64-4-496 5454

E-mail:

angela.dolan@vuw.ac.nz

*Roger J Bowden

School of Economics and Finance

Victoria University of Wellington

PO Box 600

Wellington

New Zealand

Printed by Victoria University of Wellington Printers.

November 1997

CAPM and Empirical Embedding: When is 'near enough', good enough ?

by Roger J. Bowden*

Abstract

Empirical testing for the existence of a CAPM relationship among a group of securities has been hampered by the lack of structure under the alternative hypothesis. This paper examines what happens when the chosen or available group is embedded within a larger group (the global set). A risk premium process defined with respect to the more embracing group allows securities to be priced, and those pricing relationships are embodied in the chosen set. If one then fits a CAPM with respect to a market index constructed from the chosen set, the resulting betas will be biased even up to a factor of proportionality. By demonstrating the form of the bias, however, we can show how to correct it and demonstrate conditions under which the more limited market index can suffice for such purposes as cost of capital studies. A simple test for spanning emerges, which can be reinterpreted as providing an alternative hypothesis for the CAPM itself. The resulting empirical methodology is a straightforward two step modification of OLS, from which tests of structural stability over time can also be derived. The methods are applied to monthly data on a set of New Zealand stocks.

Key words: *Beta, CAPM, conditional expectation, cost of capital, risk premium process, security market, spanning.*

* School of Economics and Finance, Victoria University of Wellington, New Zealand. The author is grateful to Ross Tucker of the University of Waikato for capable research assistance.

I Introduction

Recent years have seen a reinterpretation of the CAPM model of security market equilibrium in terms of arbitrage or semi arbitrage models. If the market is complete in the sense that securities are available that span all sources of stochastic variation, then there exists a risk premium process that embodies the market price of risk. Individual security returns have an instantaneous beta with respect to this process, the effect of which is to modify the market risk premium according to their particular risk characteristics. In turn, the market risk premium process can be embodied into the required risk premium on a market portfolio, so that betas are then those defined with respect to the market portfolio, more or less in the usual way. For reviews of the complete theory see, for example Duffie (1992) or Magill and Quinzii (1996). Where markets are not complete, the risk premium process is not necessarily unique. However, all securities can nonetheless be priced with respect to one chosen version of the market risk premium process, according to the 'law of one price' of Hansen and Jagannathan (1996). In the latter methodology, betas are established in discrete time with respect to stochastic discount factors, whose expected values are the yields on default free zero coupon bonds. Bowden (1997) has shown that this approach remains compatible with the more traditional formulation in terms of the market portfolio of risky assets, but where the risk free rate, which is generally unobservable, is replaced by the yield on the zero coupon bond.

The implications of this reinterpretation are far reaching. In the first place, many of the restrictive assumptions of the original CAPM model, such as homogeneous agent expectations, quadratic preferences or normal densities, have proved to be unnecessary. Secondly, it has become clear that in econometric terms, we are looking at a latent variable structure, with the market risk premium process as the latent process to be modelled. As remarked by Eun (1994), various anomalies such as the firm size effect or the Friend/Blume effect may in fact be viewed as consequences of the way that the beta is to be estimated, and disappear when the problem is viewed in the light of a latent variable structure. The maximum likelihood theory attached to such a model therefore provides - in principle - all the information that the data can reveal about the betas, so that the distinctly ad hoc methodology of the early CAPM literature, riddled as it was with controversy if not actual dissension, is replaced by something in which the precise limits of what can or can not be deduced are transparent. Moreover critiques such as that of Roll (1977) can be placed more firmly in context. For methodology based on dynamic latent variable

representations such as the Kalman filter, see e.g. Fisher and Kamin (1985), Ferson and Harvey (1991), or Bansal and Viswanathan (1993).

However, some problems remain. In practice, one often has available only a limited set of all the assets that are traded, bought or sold. Thus a study limited to the stockmarket might not be able to identify or even proxy the global market risk premium, for the simple reason that this might contain contributions from the fixed interest markets, the real estate markets, offshore markets, and so on, the sort of objection that became associated with the Roll critique referred to above. In other words, the stock market might not by itself be able to span important sources of stochastic variation. But would this matter? Suppose the task was to determine the cost of capital for a given firm. For such a purpose the beta defined with respect to the market portfolio for the stock market (the Dow Jones index, for example) might be perfectly adequate. One might regard the Dow Jones index, suitably corrected for the risk free rate, as simply replacing the global risk premium process. If one can estimate the more limited beta with respect to the index, and perhaps model the determinants of the index itself, then one apparently has an acceptable methodology for determining the required cost of capital.

This paper asks whether and under what conditions such a procedure has validity. In the process, it provides a testing procedure for the completeness of the available set of securities, or more precisely whether the problem matters at all. It turns out that in general, the usual CAPM style regression methodology to determine the betas even with respect to the available index, is biased. The nature of the theoretical bias can be understood in terms of the covariance structure of the available set, i.e. those from which the index is formed, and the unavailable remainder; or equivalently, whether reference need be made to anything outside the available set in order to form the optimum portfolio of available securities. Empirically, however, the bias can be corrected by means of a simple two step least squares procedure as a way of tackling an estimation problem that is nonlinear in the parameters. The resulting bias-corrected betas with respect to the available index are proportional to the betas with respect to the global underlying risk premium process. One can also use the methodology to test the CAPM model as a whole, for the presence of incompleteness bias provides a simple alternative hypothesis to be set off against the CAPM as the null.

The scheme of the paper is as follows. Section II sets up the problem to be addressed, and establishes the basic conditioning result: The conditional expectation of the security excess

returns, given the available index, in general includes an extraneous term, that represents an infection- as it were - from the remaining sources of risk, and securities, in the economy. Thus the usual OLS regression will not correctly identify the betas with respect to the available index. However it can be corrected. This section explores also the conditions under which the incompleteness bias vanishes. Section III sets up the econometrics, presenting the two step estimator and test for incompleteness bias. Also discussed are issues such as the problem of testing for temporal stability, and the maximum likelihood solution problems. Section IV is an illustration using NZ data, which suggests that incompleteness bias is not a pervasive problem in the NZ stock market, and also that the time series betas are temporally stable. Section V contains some concluding comments.

II Conditioning and the equity beta

2.1 Specifications

The basic assumptions of this section are as follows.

(i) There exists an extended set of securities in the economy. These may be combined into the *global market portfolio*, which represents the securitisation of a global risk premium process with respect to which all securities can be priced. The risk premium process may not be unique, so that the global set of securities may or may not be collectively complete with respect to the underlying sources of risk. But all assets can be consistently priced with respect to the spanning portfolio for whichever version of the risk premium process is chosen. The details are standard,

for further detail see Duffie(1992), Hansen and Jagannathan (1996) for an operational treatment of the incomplete case, and for a recent comprehensive treatment, Magill and Quinzii (1996). For the purposes of the present study it is assumed that there is an underlying global market portfolio with respect to which all assets obey a CAPM relationship, so that asset betas are defined, which are constant over the unit time interval, though they may vary over time. The return processes are assumed to follow some underlying Ito process, so that one period returns, conditional on the available information , are Normally distributed. This does not, of course, mean that unconditional return distributions are likewise Normal. For the purposes of this section, the betas

etc are all defined in terms of the conditional distribution of returns, and there is no essential reason why the betas cannot vary over time. Properties of the global portfolio will be distinguished with an asterisk:

R_* = the return on the global portfolio.

μ_* = the conditional mean of the global portfolio

σ_*^2 = the conditional variance of R_* .

β_{i*} = the conditional beta of asset i with respect to the global portfolio.

There is a risk free rate ρ , which like all the above, may vary over time. In summary, this is very much a conventional CAPM sort of world and associated methodology. In particular, we do not explore here the precise relationship to the stochastic discount factor approach.

(ii) There is a subset of n assets, which constitute the chosen or *available* set, $i \in A$. For example they may be the set of equities traded on the stock exchange, whereas the global set will also include fixed interest securities, mortgages, real estate etc. With respect to the available set we define the following:

R = the return on an index portfolio constructed from the available set, using a designated set of weights w_i attached to the securities that make up the available set. For the moment we take the weights as given and arbitrary.

μ_R = the mean of R .

σ_R^2 = the variance of R .

β_{iR} = β_i = the beta of asset i with respect to R .

β_{R*} = the beta of R with respect to the full R_* .

(iii) From the above it follows that for each security i in the available set, there is a CAPM relationship with respect to the global portfolio:

$$(1a) \quad \mu_i - \rho = \beta_{i*} (\mu_* - \rho) ; \quad i = 1, 2 \dots n$$

$$(1b) \quad r_i - \rho = \beta_{i*} (R_* - \rho) + \eta_i ,$$

where η_i is the zero mean idiosyncratic term uncorrelated with $R_* - \rho$.

The covariance matrix of the η_i for $i \in A$ will be denoted Ω , assumed nonsingular. Hence the covariance matrix of the returns r_i for $i \in A$ can be written

$$(2) \quad \Sigma = \sigma_i^2 \beta \beta' + \Omega.$$

The available market portfolio R inherits the CAPM structure from its constituent securities:

$$(3a) \quad \beta_{R^*} = \sum_{i \in A} w_i \beta_i.$$

$$(3b) \quad \mu_R - \rho = \beta_R (\mu_* - \rho)$$

$$(3c) \quad R - \rho = \beta_{R^*} (R_* - \rho) + \eta_R; \quad \eta_R = w' \eta$$

$$(3d) \quad \sigma_R^2 = \sigma_*^2 \beta_{R^*}^2 + w' \Omega w.$$

2.2 *The conditional CAPM regression*

It follows from the model specifications that

$$(4) \quad r_i - \rho = \beta_i (R - \rho) + \eta_i - \beta_i \eta_R$$

where $\beta_i = \beta_{iR} = \beta_i / \beta_{R^*}$.

Equation (4) is a starting point for the present contribution. At first sight it looks like a conventional CAPM estimating equation for the β_i . Even if the latter are with respect to the available market R rather than the true or global R_* , they can still be used to construct or estimate the cost of capital for company i , a common application. However, equation (4) is not as it stands a conventional CAPM regression type specification. The reason is that the expectation of the dependent variable $r_i - \rho$, conditional on the right hand variable $R - \rho$, is not equal to $\beta_i (R - \rho)$, so that a regression of the former on the latter will not yield the desired beta coefficient. The reason for the bias is that in general,

$$(5) \quad E(\eta_i - \beta_i \eta_R) | (R - \rho) \neq 0.$$

The true conditional expectation is not trivial to establish and is the subject of the following :

Lemma CE.

If security returns over the unit period, conditional on the available information, are Normally distributed, then

$$(6) \quad E(r - \rho \mathbf{1}) | (R - \rho) = \gamma_1 (\mu_R - \rho) + \gamma_2 (R - \rho),$$

$$\text{where:} \quad \begin{aligned} \gamma_1 &= -(I - \beta w') \Omega w / \sigma_R^2 \\ \gamma_2 &= \beta + (I - \beta w') \Omega w / \sigma_R^2. \end{aligned}$$

Proof.

Rewriting equation (4) in vector form,

$$(4') \quad E[(r - \rho \mathbf{1}) | (R - \rho)] = \beta (R - \rho) + (I - \beta w') E(\eta | R).$$

The problem essentially reduces to finding $E[\eta | (R - \rho)]$, using equations (1) -(4) above. Writing $p(\cdot)$ for the generic univariate or multivariate density, we have

$$(7) \quad E[\eta | (R - \rho)] = \int_{-\infty}^{\infty} \eta \frac{p(\eta, R - \rho)}{p(R - \rho)} d\eta.$$

In what follows the distributions will be Normal, written as densities of the form $n(x; a, b)$ where x is a scalar or vector argument, a is the mean, and b is the variance or covariance matrix. To evaluate (7) we start with the mapping defined by

$$(8) \quad \begin{bmatrix} R - \rho \\ \eta \end{bmatrix} = \begin{bmatrix} \beta_R & w' \\ \sigma & I \end{bmatrix} \begin{bmatrix} R - \rho \\ \eta \end{bmatrix},$$

which has inverse Jacobian equal to $1/|\beta_R|$. Since η and $(R - \rho)$ are independent by construction,

the mapping (8) implies that:

$$\begin{aligned}
 (9) \quad p(R_* - \rho, \eta) &= p(\eta) p(R_* - \rho) \\
 &= n(\eta; 0, \Omega) n(R_* - \rho; \mu_* - \rho, \sigma_*^2) / |\beta_{R_*}| \\
 &= n(\eta; 0, \Omega) n\left(\frac{1}{\beta_{R_*}} (R - \rho - w'\eta); (\mu_* - \rho, \sigma_*^2) / |\beta_{R_*}| \right) \\
 &= n(\eta; 0, \Omega) n(w'\eta; \mu_R - R, \beta_{R_*}^2 \sigma_*^2).
 \end{aligned}$$

The two densities on the RHS of (9) may be condensed into one by completing the square. Using equation (7) and also (3d) we end up with

$$(10) \quad E[\eta | (R - \rho)] = (2\pi)^{-\frac{n}{2}} \det^{-\frac{1}{2}} \Omega \frac{n(R; \mu_R, \beta_{R_*}^2 \sigma_*^2)}{n(R; \mu_R, \sigma_R^2)} \int_{-\infty}^{\infty} \eta e^{-\frac{1}{2}T} d\eta$$

$$\text{where} \quad T = \eta' \left(\Omega^{-1} + \frac{ww'}{\beta_{R_*}^2 \sigma_*^2} \right) \eta + 2 \frac{(R - \mu_R)}{\beta_{R_*}^2 \sigma_*^2} w'\eta.$$

The term T can be rewritten as

$$(11) \quad T = \left(\eta + \frac{R - \mu_R}{\beta_{R_*}^2 \sigma_*^2} D^{-1}w \right)' \left(\Omega^{-1} + \frac{1}{\beta_{R_*}^2 \sigma_*^2} ww' \right) \left(\eta + \frac{R - \mu_R}{\beta_{R_*}^2 \sigma_*^2} D^{-1}w \right) - \left(\frac{\mu_R - R}{\beta_{R_*}^2 \sigma_*^2} \right)' w' D^{-1}w$$

$$\text{where} \quad D = \Omega^{-1} + \frac{1}{\beta_{R_*}^2 \sigma_*^2} ww'.$$

Utilising (11) in (10), we end up with

$$(12) \quad E[\eta | (R - \rho)] = \det^{-\frac{1}{2}} \Omega \det^{-\frac{1}{2}} D \frac{n(R; \mu_R, \beta_{R_*}^2 \sigma_*^2)}{n(R; \mu_R, \sigma_R^2) n\left(R; \mu_R, \frac{\beta_{R_*}^4 \sigma_*^4}{w'D^{-1}w}\right)}.$$

$$\text{Now} \quad D^{-1} = \Omega - \frac{\Omega ww'\Omega}{\beta_{R_*}^2 \sigma_*^2 + w'\Omega w}.$$

Using expression (3d) we get

$$(13) \quad w'D^{-1}w = \beta_{R_*}^2 \sigma_*^2 \left(1 - \frac{\beta_{R_*}^2 \sigma_*^2}{\sigma_R^2} \right).$$

Also

$$(14) \quad \det^{-1/2} \Omega \det^{-1/2} D = |\beta_{R^*}| \sigma_* / \sigma_R .$$

Substituting into expression (12) and simplifying yields the desired result (6). \square

From expression (6), we see that $\beta = \gamma_1 + \gamma_2$. The term γ_1 is the bias; if this term is zero then we can identify the betas with respect to the chosen portfolio R; or with respect to the global portfolio, R_* up to a factor of proportionality given by the beta of R with respect to R_* . Note that $\gamma_1' w = 0$. In other words, the weighted average bias is zero. But this does not mean that the bias is zero for each individual security. The next result gives a necessary and sufficient condition for the latter to be true.

Proposition 1

The bias term is zero if and only if the weights w applied to the available securities in the formation of R are proportional to $\Sigma^{-1} \beta_$ or equivalently $\Sigma^{-1} \beta$, where Σ is the covariance matrix of the available security returns.*

Proof

(a) First we show that the bias is zero if and only if w is such that $\Omega w = \lambda \beta_*$ for some scalar λ . Note that for any weights w , it follows from equations (3a) and (4) above that

$$(15) \quad \gamma_1 = \left(I - \frac{\beta_* w'}{\beta_*' w} \right) \Omega w / \sigma_R^2$$

\Rightarrow Suppose that $w = \lambda \Omega^{-1} \beta_*$, for some scalar λ . Direct substitution shows that $\gamma_1 = 0$.

\Rightarrow Conversely, suppose that γ_1 as defined by expression (15) is zero. Then Ωw must be the sole zero eigenvector of the rank $(n-1)$ matrix $\left(I - \frac{\beta_* w'}{\beta_*' w} \right)$. But we already know that this is β_* , so the two must coincide up to a factor of proportionality.

(b) With the equivalent condition (a) established we can show that all is unchanged if in fact Ω

is replaced by Σ .

= Suppose that w is proportional to $\Sigma^{-1}\beta$. Then apart from a multiplicative scalar,

$$\begin{aligned}\Omega w &= (\Sigma - \sigma_R^2 \beta \beta') \Sigma^{-1} \beta \\ &= (1 - \sigma_R^2 \beta' \Sigma^{-1} \beta) \beta\end{aligned}$$

so that condition (a) is satisfied.

= Conversely, suppose that the bias vanishes, so that from condition (a), $(\Sigma - \sigma_R^2 \beta \beta') w = \lambda \beta$, for some scalar λ . Hence

$$(\Sigma - \sigma_R^2 \beta \beta') w = \lambda \beta,$$

which implies

$$w = (\lambda - \sigma_R^2 \beta' w) \Sigma^{-1} \beta,$$

so that w is proportional to $\Sigma^{-1}\beta$ as required.

□

The bias may be interpreted in terms of the global portfolio weights w_* for all possible securities. If the global covariance matrix Σ_* has full rank, then the optimal global portfolio has w_* proportional to the vector $\Sigma_*^{-1}\beta_*$. In turn, the available portfolio, as a subportfolio of the optimal global portfolio, would have a set of weights proportional to the appropriate subvector of w_* . In general this is not proportional to the portfolio indicated by the unbiasedness condition of the Proposition, which is of the same form but utilises a submatrix Σ of the global Σ_* to form the required inverse. The inverse of a matrix partition is not the partition of the inverses, so that the natural portfolio weights generated off the global portfolio allocation will be biased in the sense indicated by the Lemma.

Indeed, suppose that the global covariance matrix is partitioned into the available securities, indexed by A, and the rest, indexed by B:

$$\Sigma_* = \begin{bmatrix} \Sigma_{AA} & \Sigma_{AB} \\ \Sigma_{AB}' & \Sigma_{BB} \end{bmatrix}$$

It can then be shown that the bias term is proportional to $\Sigma_{AB}z_*$, where z_* denotes the weights

allocated in the global portfolio to securities from the non available group B. Only if the securities in group A are uncorrelated with those in group B can one be confident that the bias will vanish.

The implication in practice is that use of market indexes to form R for the available group may very well not provide the right proportions to make the availability bias vanish. For, the capitalisation weights used to form the market index may result from portfolio choices conditioned or influenced by the availability to the investing public of a wider class of assets than those encompassed by the index. In other words, the particular capitalisation proportions used in, say, the market index for the stockmarket may reflect pricing that is determined by wider portfolio considerations such as the availability of bonds or real estate. One way to escape this problem might be to redefine the weights used in the computation of the index R of available assets, in the manner suggested by Proposition 1. However this needs a prior estimate of the betas as well as a good estimate of the covariance matrix of available securities. A better procedure would simply be to devise corrections to the bias that can in principle be applied no matter what index R is used for the available market. This is the subject of the next section.

In preparation, we note that by incorporating equation (4), equation (6) can be cast as a theoretical regression relationship:

$$(16) \quad \mathbf{r} - \rho \mathbf{1} = \gamma_1 (\mu_R - \rho) + \gamma_2 (\mathbf{R} - \rho) + \epsilon,$$

where $E \epsilon | \mathbf{R} = \mathbf{0}$. One can show that

$$\Phi = E \epsilon \epsilon' = (\mathbf{I} - \beta \mathbf{w}') \Omega (\mathbf{I} - \mathbf{w} \beta') - \sigma_R^2 \gamma_1 \gamma_1'.$$

As $\gamma_1' \mathbf{w} = 0$ and $\beta' \mathbf{w} = 1$, it follows that $\Phi \mathbf{w} = \mathbf{0}$. The covariance matrix of the disturbances in the model (16) is therefore singular; in general it has rank $n-1$.

III Econometrics

3.1 *The estimating equations*

Equation (6) of Lemma CE is cast in terms of the conditional expectation of the excess return for each security, and it is therefore natural to approach parameter estimation in this framework by means of OLS, although full information maximum likelihood will also be considered in due course. However apart from the natural connection with conditional means, OLS can be expected to have the usual property of robustness with respect to specification error elsewhere in the system, and is also a useful source of initial values for numerical solutions of maximum likelihood and similar techniques. In what follows we shall assume that the underlying betas are invariant over the sample period, although the risk premium process is not, though at a later point we do consider explicitly the problem of testing for temporal stability of the betas.

As a proxy for the equity risk premium we specify a linear function of a set of $K + 1$ predetermined variates x_t which will include the intercept in case the premium is in fact constant. The system to be estimated can be written as

$$(17a) \quad R_t - \rho_t = x_t' \alpha + \epsilon_{Rt} \quad ; \quad t = 1, 2 \dots T$$

$$(17b) \quad r_{it} - \rho_t = \gamma_{i1} x_t' \alpha + \gamma_{i2} (R_t - \rho_t) + \epsilon_{it} \quad ; \quad i = 1, 2 \dots n ; \\ t = 1, 2 \dots T .$$

The defining characteristics of the zero mean disturbances are that for all periods t , $E\epsilon_{Rt} | (R_t - \rho_t) = 0$, and that ϵ_{Rt} and ϵ_{it} are uncorrelated. The first property captures the specification of the equity risk premium $\mu_{Rt} - \rho_t$ as a function of the predetermined variables, and the second the conditioning process involved in Lemma CE. In the basic estimation theory that follows we shall assume that the disturbances are serially uncorrelated and homoscedastic, though the latter property in particular is not necessary for the consistency of the technique, and extensions of White type estimators appear to be feasible. Once the γ_i have been estimated the β_i can be recovered as $\gamma_{i1} + \gamma_{i2}$ and the variance of the betas obtained from the output covariance matrix of the γ_i by using the same relationship.

3.2 The two step OLS estimator

The fitting of equations (17a and b) proceeds in two steps.

Step 1: Regress the market excess return $R_t^d = R_t - \rho_t$ on \mathbf{x}_t , including an intercept. Compute the fitted values \hat{R}_t^d , the estimated residual variance σ_R^2 , and also the coefficient of determination, which is the multiple correlation coefficient uncorrected for the means, defined as

$$c^2 = \frac{\hat{R}^d \hat{R}^d}{\hat{R}^d \hat{R}^d} = \frac{\sum_t \hat{R}_t^{d2}}{\sum_t R_t^{d2}} .$$

Step 2: For each security i , regress the excess return r_{it}^d on both R_t^d and \hat{R}_t^d ie the fitted and actual value of the market excess return. The step 2 equations should have no intercept. The formula for the asymptotic covariance matrix of the step 2 estimators is given below. Its estimation requires the insertion of estimated values of the residual variance and also γ_{ii} from the same regression.

The procedure is therefore a simple two step OLS and can be performed with standard packages plus a little algebraic manipulation of the regression outputs. The econometric justification can be stated as follows.

Proposition 2

Suppose that the market risk premium $\mu_R - \rho \neq 0$, and that all security returns and predetermined variables are Caesaro summable (e.g. stationary) as the sample size $T \rightarrow \infty$. The disturbances ϵ_{Rt} and ϵ_{it} are mutually independent (by construction) and i.i.d. over time. Then the two step procedure is consistent and $\sqrt{T} (\hat{\gamma}_{i1} - \gamma_{i1}, \hat{\gamma}_{i2} - \gamma_{i2})'$ is asymptotically Normal with mean zero and covariance matrix given by:

$$(18) \quad \epsilon_{\gamma} = \underset{T \rightarrow \infty}{plim} T [\sigma_i^2 (H'H)^{-1} + \frac{\gamma_{ii}^2}{T - K - 1} \begin{pmatrix} \frac{1-c^2}{c^2} & 0 \\ 0 & 0 \end{pmatrix}]$$

where $H = [\hat{R}^d : R^d]$ is the $T \times 2$ data matrix of the RHS step 2 variables, and $\sigma_i^2 = E \epsilon_{it}^2$. A consistent estimator of σ_i^2 is

$$(19) \quad \hat{\sigma}_i^2 = \frac{1}{T-2} (\hat{v}_i' \hat{v}_i) - \frac{K}{T-2} \hat{v}_{ii}^2 \hat{\sigma}_R^2,$$

where \hat{v} is the fitted residual from the step 2 regression.

Proof.

Let $\gamma_i' = (\gamma_{i1}, \gamma_{i2})'$. In data matrix form, the step 1 and step 2 regressions can be written

$$(20) \quad \mathbf{R}^d = \mathbf{X}\alpha + \epsilon_R$$

$$(21) \quad r_i^d = H \gamma_i - \gamma_{ii} P_x \epsilon_R + \epsilon_i; \quad P_x = X(X'X)^{-1}X'$$

The step 2 estimator is $\hat{\gamma}_i = (H'H)^{-1} H' r_i^d$ and

$$\hat{\gamma}_i - \gamma_i = (H'H)^{-1} H' (-\gamma_{ii} P_x \epsilon_R + \epsilon_i).$$

The elements of ϵ_i are by construction independent of \mathbf{R} and standard OLS arguments then suffice to ensure consistency and asymptotic normality, with asymptotic covariance matrix of $\sqrt{T}(\hat{\gamma}_i - \gamma_i)$ as the limit in probability of

$$(22) \quad \sqrt{T} (H'H)^{-1} [\gamma_{ii}^2 \sigma_R^2 H' P_x H + \sigma_i^2 H'H] (H'H)^{-1}.$$

Since $P_x H = [P_x \hat{R} : P_x R] = [\hat{R} : R]$

and $\hat{R}' R = \hat{R}' \hat{R}$,

it follows that

$$(23) \quad H' P_x H = H'H + (\hat{R}' \hat{R} - R'R) \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Also

$$(24) \quad (H'H)^{-1} = \begin{bmatrix} \hat{R}' \hat{R} & \hat{R}' R \\ \hat{R}' R & R'R \end{bmatrix}^{-1} = \frac{1}{SSE_R} \begin{bmatrix} 1 & -1 \\ c^2 & 1 \\ -1 & 1 \end{bmatrix},$$

where $c_2 = \frac{\hat{R}'\hat{R}}{R'R}$ and $SSE_R = R'R - \hat{R}'\hat{R}$. The latter can be replaced in probability $(T - K - 1) \sigma_R^2$, in a shorthand but obvious sense. Equation (18) follows by substitution of (23) and (24) into (22).

The step 2 fitted residual is

$$\hat{v}_i = (I - P_H) (-Y_{it} P_x \epsilon_R + \epsilon_i),$$

where P_H is the OLS projection matrix $H(H'H)^{-1}H'$.

Hence $(1/T) \hat{v}'\hat{v}$ can be replaced in probability for large T by

$$(25) \quad \frac{1}{T} Y_{it}^2 \sigma_R^2 \text{trace} [P_x (I - P_x) P_x] + [(T-2)/T] \sigma_i^2.$$

Now

$$\begin{aligned} \text{tr.} [P_x (I - P_H) P_x] &= \text{tr.} P_x - \text{tr.} [(H'H)^{-1} H' P_x H] \\ &= K + 1 - (\hat{R}'\hat{R}) \text{tr.} (H'H)^{-1} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\ &= K + 1 - (\hat{R}'\hat{R}) \frac{1}{\hat{R}'\hat{R}} \\ &= K. \end{aligned}$$

Inserting this into (25) provides the consistent estimator (19) for $\hat{\sigma}_i^2$.

□

As earlier remarked, the betas can be estimated in terms of the γ 's, with associated asymptotic variances and Student t values. The conditional security risk premiums can then be estimated if required as:

$$\mu_{it} - \rho_t = \hat{\beta}_i (\hat{\alpha}' x_t - \rho_t).$$

Some further remarks are as follows:

(a) In the stage 1 regression, the object is to get as precise an estimate of the conditional expectation of the market risk premium as possible. To that end, one could try a variety of different predetermined variables x_t according to data availability and some sort of hypothesising about how the proposed variables are thought to proxy the market risk premium. In doing so it might very well be the case that a degree of multicollinearity arises among the different series. However this need not be of special concern if the underlying purpose is simply to obtain a good instrument(loosely speaking) for use in the step 2 regression in the manner indicated by the procedure, and the parameters α are not in themselves of prime interest. What is important is to try to get a reasonable overall equation fit for step 1 . One possible indicator for this is the overall F statistic for the equation. However it should be noted that the methodology will continue to work if the market risk premium is simply a non zero constant, and for such a case the F statistic, which picks up only the influence of proper variates, would be insignificant. An alternative indicator is just the coefficient of determination, as defined above, uncorrected for the mean.

(b) The step 2 OLS regression may be schematically represented as:

$$(26a) \quad r_{it} - \rho_t = \gamma_{i1} (\widehat{R_t - \rho_t}) + \gamma_{i2} (R_t - \rho_t) + \text{residual}_t$$

where the residual is asymptotically uncorrelated with the right hand regressors. The hat denotes the OLS fitted values from step 1. Now this could also be written

$$(26b) \quad r_{it} - \rho_t = (\gamma_{i1} + \gamma_{i2})(\widehat{R_t - \rho_t}) + \gamma_{i2} \widehat{\epsilon_{Rt}} + \text{residual}_t ,$$

where $\widehat{\epsilon_{Rt}}$ is the fitted OLS residual from step 1. The latter equation has orthogonal regressors, by construction from step 1. So one should expect that a simple OLS regression of individual security returns upon the fitted excess market return from step 1 would yield a consistent estimator for $\beta_i = \gamma_{i1} + \gamma_{i2}$, even though the OLS residual properties from such a step 2 procedure would not be those appropriate for hypothesis testing. Apart from this, however, the orthogonal version (26b) presents the difficulty that both right hand regressors take the form of constructed variables, outputs of the step 1 regression. This rests a great deal of weight in obtaining beta on the correct specification in step 1, and as earlier remarked this may be a problem. The two step method (26a), as proposed in the form of Proposition 1, uses the

observable regressor $R - \rho$ to identify the object of primary interest, namely the betas -even if biased- and the fitted value of $R - \rho$ in step 1 is used in step 2 more or less as an instrument to locate and correct for sources of incompleteness bias. When no such bias exists, for instance, it makes much more sense to utilise the actual rather than the fitted value of $R - \rho$. Formally speaking, there is no statistical difference between the two formulations. The real problems are those of interpreting the results. Suppose, for instance, that the step 1 regression was so badly mis-specified (ie bad choice of X) that all the fitted alphas were practically zero. In step 2, the coefficients of the first RH regressor would in such circumstances be virtually meaningless. However in version (26b) this coefficient is regarded as the direct estimate for the beta, so that one could end up with the presentation of either thoroughly wild or perhaps statistically insignificant estimates for the betas. But version (26a), the coefficient of $(R - \rho)$, while it might possibly incorporate some incompleteness bias that could not be identified, is nevertheless likely to be in the right ball park.

(c) The proposed methodology may be given a more informal interpretation, in terms of an unstructured alternative hypothesis to the CAPM model. Thus suppose that we specified the following:

$$\begin{aligned} r_{it} &= \mu_i + \beta_i (R_t - \mu_{Rt}) + \epsilon_{it} , \\ \mu_{it} - \rho_t &= \alpha_i + \beta'_i (\mu_{Rt} - \rho_t) , \end{aligned}$$

where β'_i is not necessarily equal to β_i . The first equation is just the ordinary theoretical regression of security excess returns on the market, with no particular information content, beyond linearity. The second equation is CAPM-like in form but uses an arbitrary coefficient β' . Combining the two, we get a single equation:

$$r_{it} - \rho_t = \alpha_i + (\beta_i - \beta'_i)(\mu_{Rt} - \rho_t) + \beta_i (R_t - \rho_t) + \epsilon_{it} .$$

Thus a test of the CAPM is that in step 2, the intercept and coefficient of $(R - \rho)$ are both zero. The latter term is equivalent to the incompleteness bias term in the formulation associated with Proposition 1.

3.2 Temporal stability

One is frequently interested in testing whether the fitted betas are constant over time. As the model is nonlinear in the parameters, Chow and similar linear regression based tests cannot be employed. However one can use asymptotic tests based on structural shift parameters in the betas, a procedure which has the obvious advantage that any shifts over time can be examined on a security by security basis.

Accordingly, we specify for each security i ,

$$(27) \quad \beta_{it} = \beta_{i0} + \beta_{i1} d_t,$$

where d is a (0:1) dummy variable indicating some chosen regime. For example, one could have $d=1$ in the second half of the sample period, with the first half chosen as the base period. In that case, $\beta_{i1} \neq 0$ would be a test for a structural break in the beta for security i .

Inserting the specification (27) into the model (6) yields the following generalisation of the estimating equation (17):

$$(28) \quad \begin{aligned} r_{it} &= \gamma_{i1} \hat{R}_t^d + \gamma_{i2} R_t^d + \gamma_{i3} d_t \hat{R}_t^d + \gamma_{i4} d_t R_t^d + \epsilon_{it} \\ \beta_{i0} &= \gamma_{i1} + \gamma_{i2} \\ \beta_{i1} &= \gamma_{i3} + \gamma_{i4} \end{aligned}$$

The step 1 regression is as before. In step 2, the right hand data matrix is redefined as

$$H = [\hat{R}^d, R^d, d\hat{R}^d, dR^d],$$

where for example dR^d is a column vector whose t^{th} element is $d_t R_t^d$, with the OLS projection matrix P_H correspondingly redefined. Also define

$$D = \text{diag}(d_1, d_2, \dots, d_T),$$

$$G = (\hat{\gamma}_{i1} I + \hat{\gamma}_{i2} D) P_x (\hat{\gamma}_{i1} + \gamma_{i3} D).$$

Then the estimated asymptotic covariance matrix of the γ_i is given by

$$(29a) \quad \hat{\Sigma}_{\gamma_i} = \hat{\sigma}_i^2 (H'H)^{-1} + \hat{\sigma}_R^2 (H'H)^{-1} (H'GH) (H'H)^{-1} ,$$

with

$$(29b) \quad \hat{\sigma}_i^2 = \frac{1}{T-4} \{ \hat{\varphi}_i' \hat{\varphi}_i - \hat{\sigma}_R^2 \text{tr} [G (I - P_H)] \} .$$

3.4 Maximum likelihood

The system defined by (17a,b) can be given a maximum likelihood (ML) treatment, though an unusual difficulty arises that will be pointed out in due course. A preliminary problem is that as pointed out at the end of section II in connection with representation (16), the disturbance vector ϵ has rank only $n-1$. Effectively this means that one of the set of equations (17b) can be obtained from the others. Similar rank problems occur in consumer demand and international trade, and we shall adopt here the usual expedient, which is simply to drop one of the equations in the estimation process, though more symmetric alternatives do exist. The resulting covariance matrix we shall write as Φ , and assume that it has full rank. Although Φ itself has a structure indicated in the specification (16), this may be ignored for the purposes of estimating the coefficients α , γ_1 and β , as the structure provides no further restrictions.

The data matrix representation of equations (17a, b) is

$$(30a) \quad \mathbf{R} = \mathbf{X} \alpha + \epsilon_R ,$$

$$(30b) \quad \mathbf{S} = \mathbf{X} \alpha \gamma_1' + \mathbf{R} \otimes (\beta' - \gamma_1') + \epsilon ,$$

where for brevity we have redefined market returns r as excess returns $\mathbf{R} - \rho$, and in addition \mathbf{S} is the $T \times (n-1)$ data matrix of individual excess security returns, ie $s_{it} = r_{it} - \rho$. Assuming Normally distributed security returns, the log likelihood function is:

$$(31) \quad \ell = \text{const.} - (\mathbf{R} - \mathbf{X} \alpha)' (\mathbf{R} - \mathbf{X} \alpha) / 2 \sigma_R^2 - (1/2) \text{tr.} [\mathbf{S} - \mathbf{X} \alpha \gamma_1' - \mathbf{R} \otimes (\beta' - \gamma_1')] [\mathbf{S} - \mathbf{X} \alpha \gamma_1' - \mathbf{R} \otimes (\beta' - \gamma_1')] \Phi^{-1} .$$

The normal equations for α , β , and γ , which is all that will concern us here, are obtained by partially differentiating the log likelihood function. As there are no restrictions imposed on Φ , this matrix and σ_R^2 can be concentrated out as usual. Utilising for the most part standard rules of vector differentiation with Kronecker products (eg Bowden and Turkington (1984) pp 209-10), the first order equations for the coefficients are obtained as follows:

$$(32a) \alpha : [1 + \sigma_R^2 \text{tr}(\Phi^{-1}) \gamma_1' \gamma_1] X' X \alpha = X' R + \sigma_R^2 \text{tr}(\Phi^{-1}) X' [S - R \otimes (\beta' - \gamma_1')] \gamma_1 .$$

$$(32b) \gamma_1 : (\mathbf{R} - X\alpha)' (\mathbf{R} - X\alpha) \gamma_1 = -S' (\mathbf{R} - X\alpha) + \mathbf{R}' \mathbf{R} \beta .$$

$$(32c) \beta : \beta = \gamma_1 + [S' \mathbf{R} - (\alpha' X' \mathbf{R}) \gamma_1] / (\mathbf{R}' \mathbf{R}) .$$

If the bias term $\gamma_1 = \mathbf{0}$ in the above, it can be verified that the best estimate of β is just the OLS estimate equation by equation, as one would expect, and the best estimate of α is likewise OLS. However where the bias term is not zero, then equation (17b) provide further information about α , in addition to (17a), and this is taken into account in the normal equations.

By using (32c), equation (32b) for γ_1 can be recast as

$$(33) \gamma_1 = -S' X \alpha / [(\mathbf{R} - X\alpha)' X \alpha] .$$

Suppose that the OLS estimator for α , which is certainly a consistent estimator of such, is used as the initial starting point for the iterative solution to the ML equations. For such a choice, the denominator on the RHS of (33) is zero, from the well known orthogonality property of OLS fitted values and residual. This means that the ML normal equations have a singularity at $\gamma_1 = \mathbf{0}$, for as earlier remarked, in this eventuality the ML estimates are just OLS. Effectively, the case of zero bias is not properly nested within the general model, a similar sort of difficulty to that pointed out in more general contexts by Bontemps and Mizon (1997). In operational terms, this suggests that if preliminary two step testing indicates that the bias is weak, then it is not worthwhile to proceed to a full ML fit. Empirical experiments reported below support this conjecture.

IV Some empirical work

The empirical work reported below is for a set of New Zealand stocks. The NZ capital market is not an extensive one in terms of the number of stocks actively traded, and limitations apply also to the availability of data for the economic state variables. Consequently, primary interest attaches to the illustration of the methodology rather than the particular application itself.

4.1 *Data and sources.*

The returns data consists of the following.

- (a) *Stocks.* Monthly returns on 14 large listed NZ companies over the period Jan. 1990 - Nov. 1995, a total of 83 non overlapped observations. Returns incorporate dividends and capital changes (rights, bonus issues, splits etc) ie. are true accumulation returns. The basic source is the NZ Stock Exchange for the raw numbers and Datex Inc. for the accumulation indexes, to specifications supplied by the author to conform with standard returns calculation procedures. Dividends are imagined to be reinvested immediately on distribution. The period 1990-95 was chosen for maximum availability; some important stocks changed character during 1996; for example Fletcher Challenge split its listing into 4 different companies, each representing a facet of its business. A list of the chosen companies is given in Table 2 below. Criteria for inclusion include temporal stability in company existence and definition, together with reasonably frequent trading. The criterion for the latter was that no stock should have more than 6 days with no trading in any month of the study period.
- (b) *Market index.* This is the official NZSE 40 gross index, which is an accumulation index of the largest 40 companies by market capitalisation.
- (c) *Risk free rate.* This was chosen as the NZ 30 day bank bill rate. As a general matter, the meaning and choice of the risk free rate is one of the most difficult aspects of empirical CAPM studies. Bowden (1997) has shown that the use of a bill rate is an acceptable substitute for the risk free rate, although the corresponding betas then tend to be powered up, in the sense that high betas become even higher and low even lower. One might also remark that the bank bill rate is not technically default free, and with better data availability it might have been preferable to use the NZ treasury bill rate. The underlying data was kindly supplied by the National Bank of NZ.
- (d) *Concordance.* Using daily data on all the above, all three series (a)-(c) were normalised as to the precise day at the start and end of the month so defined. In other words, it was ensured

that all three covered exactly the same number of days, even though there might be minor variation in the number of days per month to allow for holidays etc. Moreover the risk free rate used for any month is that quoted as of the start of the month, to be lined up against the stock or market returns measured as of the end of that month.

(e) *Economic state variables.* The object here is to explain the possibly varying global or market risk premium. The first group of state variables connect with human wealth, incorporating either the marginal disutility arising from the perceived possibility of becoming unemployed, or the consumption possibilities arising from the prospect of higher incomes:

(i) Job Ads : the ANZ job ads index, incorporating the 3 main centres.

(ii) Unemployment: a monthly straight line interpolation of the official NZ registered unemployment series published quarterly. Source is PC Infos. The monthly interpolation is far from optimal, but it was felt that such was the attention paid to this indicator, it had to be included.

(iii) Two building consents series, both available monthly from official Statistics NZ sources, namely 'new dwellings' and 'other new buildings', in each case total numbers over all of NZ.

The second general factor chosen incorporates the link with inflation, as the returns used are defined in nominal rather than real terms:

(iv) The NZ monthly all groups Consumer Price Index, prepared and published by Statistics NZ.

In all cases, the above state variables appear as the lagged value, i.e. the value as recorded for the month prior to the measured returns. This is to capture the ex ante nature of the risk premium. As a concluding remark, it will be observed that the above set of state variables is distinctly limited in scope by the unavailability of National Income and Expenditure series in NZ on a monthly basis.

4.2 *Results.*

The results to be reported are in the main those from the 2 step least squares procedure (2STLS) established in section III, though there is also a short commentary on the performance of maximum likelihood ML.

It will be recalled that 2STLS starts with a preliminary regression of market excess returns on the set of state variables. The set of state variables was augmented with lagged values of the market excess return to allow for dynamic effects. The overall mean of monthly market excess

returns over the period was .0023, equivalent to an annual market risk premium of 2.77%. Overseas estimates suggest a higher market risk premium, but the figure of about 3% is not unreasonable given the low inflation rate over most of the sample period. However there is considerable variability, and the monthly mean is not statistically significant from zero on a standard t test. Table 1 shows the results from the step one regression. Only the CPI shows anything approaching statistical significance. However as earlier pointed out, individual identifiability is not overly important in step 1. The F statistic for the overall equation fit was 1.744, as against the 1% significance level for F (6, 75) of about 0.7, indicating that collectively the state variables do contribute over and above the unconditional mean for the risk premium, even if it is difficult to identify their individual contributions. However the contribution is not strong.

<u>Variable</u>	<u>Estimate</u>	<u>t value</u>
intercept	-0.4243	-1.746
job ads	neg.	-0.439
building consents (dwellings)	neg.	-0.810
building consents (other)	neg.	0.435
unemployment rate	neg.	0.655
CPI	0.00043	1.854
lagged dept variable	-0.085	-0.735

neg. indicates very large ($> 10^5$) rescaling needed to obtain significant digits.

Equation $\sigma = 0.052$; $R^2 = 0.122$; $F(6,75) = 1.744$,

DW = 2.015 (Durbin's correction not computable as $T \cdot \text{var}(b_0) > 1$).

Table 1: OLS fit for step 1

Step 2

The individual security bias and beta terms are presented in Table 2 below. It will be seen that few if any of the bias terms are statistically significant from zero. The nearest is Steel and Tube at a t value of -1.89, and the rest are generally much smaller than this. On the other hand, most of the betas are statistically significant. The magnitudes reinforce prior expectations as to procyclic behaviour. For instance, Air NZ (travel), Fisher and Paykel (consumer whitegoods), and Independent Newspapers all have betas much greater than unity; while the brewers (DB Group and Lion Nathan) have betas less than unity.

<u>Company</u>	<u>γ coeff.</u>	<u>t value</u>	<u>beta</u>	<u>t value</u>
Air New Zealand	-0.329	-0.669	1.497	3.244
Brierley Inv.	-0.363	-0.751	1.268	2.788
Carter Holt Harvey	0.086	0.289	1.160	4.162
Ceramco	-0.453	-0.700	1.435	2.361
DB Group	0.806	1.413	0.155	0.287
Donaghys	-0.062	-0.124	0.741	1.592
Fisher and Paykel	-0.806	-1.515	1.612	3.188
Fernz Corp.	-0.283	-0.596	1.160	2.598
Fletcher Challenge	0.528	1.034	0.732	1.528
Goodman Fielder	-0.227	-0.594	0.869	2.416
Indept. Newspapers	-0.492	-1.104	1.407	3.347
Lion Nathan	0.447	1.087	0.358	0.922
Steel and Tube Holdings	-1.187	-1.891	2.181	3.616
Wilson and Horton	-0.274	-0.656	1.181	3.011

Table 2: Results from the step 2 regressions

Also fitted was the model to test for temporal stability as presented in section 3.2. The observations were divided into two equal periods, the first block the earlier observations, the second the later, and the betas were normalised on the entire set, so that the β_{it} if significant would refer to the marginal effect of the second (later) subperiod. The results will not be

presented in detail, but in the event, none of the β_{11} estimates was statistically significant, indeed only Ceramco with a t value of -1.635 even came close. Correspondingly, the β_{10} values were very similar to those reported in table 2. We conclude that the betas appear to have been rather stable over the period of the study.

Finally, we report a few experiments with maximum likelihood using the SHAZAM package of White (1977 and updates). We found that the computation did not converge inside the limit set of 300 iterations and experienced also problems with the gradient vector and the directions constructed from it. This could have been due to several things such as a lack of identifiability from stage one or perhaps the singularity problem reported in section III. The 2 step methodology is robust against both these problems. While one would not like to rule out ML methodology, which does capture the cross equation restrictions, the correct application remains one for further research.

V Concluding remarks.

The primary purpose of the present paper has been to present some new methodology for testing whether a security pricing is determined solely with reference to a given galaxy of securities, or whether an extended universe is needed. The particular theoretical framework employed, namely the latent variable approach based on the global risk premium process, is not necessary for the important result of the paper, expressed as the Lemma of section II, to hold. It would hold also in a world where all pricing was according to a CAPM model of investor choice under risk and security equilibrium, along more or less conventional textbook lines. Thus the methods are fairly robust with respect to underlying assumptions about the character and scope of the precise capital market equilibrium.

So far as the econometrics are concerned, the specification of beta constancy, or blockwise constancy for the stability testing, is currently a bit restrictive, though the methodology explicitly allows (is indeed advantaged by) the risk premium process to be nonstationary and explained in terms of state variables. One suspects that the problem gets quite demanding when both nonstationary betas and nonstationary risk processes are combined. However, that is a matter for further research, as is the use of the methods for economies where the data availability is less restricting.

REFERENCES

- Bansal, R. and S. Viswanathan (1993). 'No arbitrage and arbitrage pricing: A new approach'. *Journal of Finance*, 48, 1231-62.
- Bontemps, C. and G.E. Mizon (1997). 'Congruence and encompassing'. Invited contribution, *Proc. 1997 Australasian Econometric Society Conference, vol. 3: Macroeconometrics and Finance*. Melbourne: University of Melbourne Press.
- Bowden, R.J. (1997). 'The discrete beta'. Mimeo, Kiwicap Research Ltd, October.
- Bowden, R.J. and D.A. Turkington (1984). *Instrumental Variables*. New York: Cambridge University Press/Econometric Society.
- Duffie, D. (1992). *Dynamic Asset Pricing Theory*. New Jersey: Princeton University Press.
- Eun, C.L. (1994). 'The benchmark beta, CAPM, and pricing anomalies'. *Oxford Economic Papers*, 46, 330-43.
- Ferson W.E. and C.R. Harvey (1991). 'The variation of economic risk premiums'. *Journal of Political Economy*, 99, 385-415.
- Fisher, L. and J.H. Kamin (1985). 'Forecasting systematic risk: Estimates or "raw" beta that take account of the tendency of beta to change and the heteroscedascity of residual returns'. *Journal of Financial and Quantitative Analysis*, 127-49.
- Hansen, L.P. and R. Jagannathan (1991). 'Implications of security market data for models of dynamic economics'. *Journal of Political Economy*, 99, 225-62.
- Harrison, J.M. and D. Kreps (1979). 'Martingales and arbitrage in multiperiod securities markets'. *Journal of Economic Theory*, 9, 245-62.
- Magill, M. and M. Quinzii (1996). *Theory of Incomplete Markets: Volume 1*. Cambridge, Mass.: M.I.T. Press.
- Roll, R. (1977). 'A critique of the asset pricing theory's tests, part 1: On past and potential testability of the theory'. *Journal of Financial Economics*, 4, 129-76.
- Vasicek, O. (1973). 'A note on using cross sectional information in Bayesian estimation of security betas'. *Journal of Finance*, 28, 1233-39.
- White, K.J. (1977). 'A general computer program for econometric methods - SHAZAM'. *Econometrica*, 45, 239-240.

WORKING PAPER SERIES

The main purpose of this series is to reach a wide audience quickly for feedback on recently completed or in progress research. All papers are reviewed before publication.

A full catalogue with abstracts and details of other publications is available, for enquires and to be included in our distribution list, write to:

Angela Dolan
Manager Administration Services
GSBGM, Victoria University of Wellington,
PO Box 600, Wellington, New Zealand
Tel: (04) 471 5366; Fax: (04) 496 5454

Code in bold denotes order number, eg: WP 1/91

--- Group denotes the author's academic discipline Group (note this does not necessarily define the subject matter, as staff's interests may not be confined to the subjects they teach).

1990 - 1994 titles available on request.

- WP 1/95** **Management Group**
Gilbertson, D.K., Wright, H., Yska, G, Gilbertson, D.W. and 1994 Students of MGMT 306 'Kiwi entrepreneurs: A study.'
- WP 2/95** **Management Group**
Cavana, R. 'Policy issues related to coastal and international shipping in New Zealand'
Shipping policy issues. *Transport: The Journal of the Chartered Institute of Transport in New Zealand* 1995, Vol 25, No 2, 17-19.
- WP 3/95** **Information Systems Group**
Bonner, Marcus 'On seeing information systems as bridges'
- WP 4/95** **Management Group**
Cavana, Bob, Rob Crozier, Barrie Davis and Perumal Pillai 'A survey of academic staff attitudes towards the system of academic titles used in New Zealand universities'
Attitudes Towards Reclassifying Academic Titles in New Zealand Universities. *The Journal of Higher Education Policy and Management* 1996, Vol 18, No 1.
- WP 5/95** **Econometrics Group**
Krawczyk, J.B. and G. Zaccour 'Pollution management through levies and subsidies'
- WP 6/95** **Marketing Group**
Ashill, Nicholas and Malcolm Wright 'Marketing information systems - A review and reconceptualisation'
- WP 7/95** **Information Systems Group**
Casey, Mary-Ellen 'An exploratory study into the use of information technology as an important enabler of organisational differentiation in the financial sector'
- WP 8/95** **Economics Group**
Boles de Boer, David and Lewis Evans 'The economic efficiency of telecommunications in a deregulated market: the case of New Zealand'
- WP 9/95** **Management Group**
Mabin, Victoria J. 'Using spreadsheet optimisation facilities as a decision aid within the theory of constraints framework'
- WP 10/95** **Economics Group**
M. Khaled, M.D. Adams and M. Pickford 'Estimates of scale and scope economies in the New Zealand life insurance industry.'
- WP 11/95** **Economics Group**
John A. Carlson and Robert A. Buckle 'Price duration with two-sided pricing rules'
In Karl Heinrich Oppenlaender and Guenter Poser (Eds.) *Business Cycle Surveys: Forecasting Issues and Methodological Aspects*. Aldershot, Avebury. 1996: 101-118.

- WP 12/95** **Economics Group**
Ganesh Nana 'Developing a multi-sectoral CGE model of the New Zealand economy.'
- WP 13/95** **Money and Finance Group and Economics Group**
Stephen Burnell, Lewis Evans and Shuntian Yao 'Network games: The optimal network contract and the efficiency of bypass in oligopolistic network industries under light regulation'
- WP 14/95** **Economic History Group**
Gordon Boyce 'The Nickel Syndicate, 1901 - 1939'
- WP 15/95** **Money and Finance Group**
Jan Whitwell 'Monetary disinflation with inflation inertia: Central bank autonomy in an open economy'
- WP 16/95** **Economics Group**
Emery, H. Daniel V. Gordon and Doug McClintock 'On the efficacy of construction site safety inspections.'
- WP 17/95** **Economics Group**
Ganesh Nana 'An inter-temporal CGE model with fiscal and sector balances'
- WP 18/95** **Economics Group**
Jie Zhang 'Government debt, human capital, and endogenous growth'
- WP 19/95** **Accountancy Group**
Zahirul Hoque and Manzurul Alam 'Quality management and accounting in a New Zealand service organisation: Towards an institutional perspective on management accounting'
- WP 20/95** **Economics Group**
Paul Calcott 'Can we test for supplier-induced demand by comparing informed with uninformed consumers?'
- WP 1/96** **Management Group**
M.W. Lee, J. Bennett, R.J. Taylor and R.Y. Cavana 'A dynamic simulation model for possum and gorse control on a farm woodlot.'
R.Y.Cavana, M.W.Lee, J.Bennett & R.J.Taylor. 1996. Possum and gorse control on a farm woodlot in New Zealand: A system dynamics analysis. *Asia-Pacific Journal of Operational Research*, Vol 13, No. 2, pp. 181-207.
- WP 2/96** **Economics Group**
Jie Zhang 'Optimal public investments in education, and endogenous growth'
- WP 3/96** **Economics Group**
Paul Tompkinson 'The benefits of tariff reductions in the presence of psychological adjustments costs.'
- WP 4/96** **Economics Group**
Shuntian Yao 'A note on the decision of a sales maximizer in response to the increase of per unit cost.'
- WP5/96** **Economics Group**
Karen Palmer and Margaret Walls 'Optimal policies for solid waste disposal: taxes, subsidies and standards'
- WP6/96** **Economics Group**
Margaret Walls and Jean Hanson 'Distributional impacts of an environmental tax shift: the case of motor vehicle emission taxes.'
- WP7/96** **Economics Group**
Lloyd S Shapley and Shuntian Yao 'Dynamic properties of the Nash equilibrium.'
- WP 8/96** **Accountancy Group**
Kamran Ahmed, Kazi Feroz Alam and Manzurul Alam 'An empirical study of factors affecting accounting students' career choice in New Zealand'
- WP9/96** **Accountancy Group**
Andrew MC Smith and Paul V Dunmore 'Tax Avoidance and the Financial Structures of Non -Resident Controlled Companies in New Zealand'
- WP10/96** **Econometrics Group**
L Fraser Jackson. Relative Prices and Inflation.'
- WP11/96** **Public Policy Group**
Robert Gregory. 'Reserve Bank independence, political responsibility, and the goals of anti-democratic policy: A political 'Cri de Coeur' in response to an economist's perspective.
- WP12/96** **Economics Group**
Robert A. Buckle and John A. Carlson. 'Inflation and asymmetric price adjustment.'

- WP13/96** **Econometrics Group**
 J.B. Krawczyk, O. Pourtallier and M. Tidball. 'Modelling and solution to the municipal effluent management problem.'
- WP14/96** **Economics Group**
 Viv B. Hall and David Rae. 'Fiscal expansion, interest rate risk premia, and wage reactions: Some illustrative simulations with NBNZ-DEMONZ.'
- WP15/96** **Economics Group**
 Viv. B. Hall, Kunhong Kim and Robert A. Buckle. 'Pacific rim business cycle analysis: Dating, volatility and synchronisation.'
- WP16/96** **Management Group**
 S. Davenport, C. Grimes and J. Davies. 'Collaboration and organisational learning: A study of a New Zealand collaborative research programme.'
- WP1/97** **Information Systems Group**
 Marcus Bonner and Mary-Ellen Casey. 'The informed individual - a consideration of importance to the business organisation.'
- WP2/97** **Management Group**
 R.Y. Cavana, I.G. Harrison, F.E.B. Heffernan and C.C. Kissling. 'Freight transport industry in New Zealand.'
- WP3/97** **School of Economics and Finance**
 Steffan Berridge and Jacek B Krawczyk. 'Relaxation Algorithms in finding Nash Equilibria.'
- WP 4/97** **School of Economics and Finance**
 Margaret Walls and Karen Palmer. 'Upstream Pollution, Downstream Waste Disposal, and the Design of Comprehensive Environmental Policies'
- WP 1/98** **School of Economics and Finance**
 John Haywood and Granville Tunnicliffe Wilson. 'Selection and Estimation of Trigonometric Component Models for Seasonal Time Series'
- WP 2/98** **School of Economics and Finance**
 Roger J Bowden, 'CAPM and Empirical Embedding: When is 'near enough', good enough?'
- WP 3/98** **School of Economics and Finance**
 Roger J Bowden, 'The Discrete Time Beta'
- WP 4/98** **School of Economics and Finance**
 Viv B Hall, 'Assessing Structural Adjustment and Economic Reform: the Case of New Zealand'