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Modelling and Solution to the Municipal Effluent Management Problem

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Abstract

The aim of this paper is to analyse the municipal effluent management problem; we also show how to solve a problem of that kind.

Municipal effluent economics is a complex problem. There are many agents confined to a small area, their capital can change through the investment and disinvestment processes and they can abate pollution in the abatement facilities or "instantaneously" (*e.g.* by exporting it). The agents pollute a local river. On the other hand, a regional authority would like to preserve the quality of the river's water. The quality can be expressed as a series of constraints to be satisfied locally, globally or on average, in the spatial and temporal sense. For that complicated situation, we propose a multi-criteria hierarchical multilevel optimisation model with a Stackelberg type equilibrium *between* the leader's and the followers' levels, and a Nash type equilibrium *at* the followers' level. We outline the procedure necessary for a solution to a problem of that class. We analytically compute a steady state Nash equilibrium solution for the followers, and determine a Stackelberg equilibrium solution for the leader-followers interactions, through a Decision Support Tool.

1 Introduction

This paper analyses the municipal effluent management problem. We propose a model for that problem and point out difficulties one encounters in solving it. We also discuss under which assumptions the current state of the art in *dynamic games* enables us to solve this problem. We suggest that the approach we are using, can be followed in the modelling and solution of other intertemporal conflict problems in the area of environmental economics. In that sense, the effluent management problem is the vehicle for the presentation of a methodology useful for a solution to a larger class of conflict models.

Our modelling framework is suitable for situations in which the Local Government (LG) acts as an elective representative of different interest groups[1]. That is, we consider the situations where the existing infrastructure permits LG to recognise a few feasible abatement options for which it can build a mathematical model of how the realisation of the options relates to the groups' and its own interests.

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[1]For methodological guidelines for models of ecosystems management and resolution of the arising conflicts see [26].

In the model, there are many agents (commercial, dwelling, transportation) confined to a small area. Their production capacities can change through the investment and dis-investment processes. The agents can take decisions about abating pollution "instantaneously" (*e.g.* by exporting it) and/or investing in the abatement capacities. The production and abatement should be such that the quality of the local river's water satisfies a series of constraints locally, globally or on average, in the spatial and temporal sense. This potential diversity of structures makes the model very complex.

In [17] a framework for the description of the municipal effluent economics was proposed. Here, we recall the main result of that paper, which consists of a multi-criteria hierarchical optimisation model with a Stackelberg type equilibrium *between* the leader's and the followers' levels, and a Nash type equilibrium *at* the followers' level. We outline a procedure necessary for a solution to a problem of that class, under an appropriate "dynamic" solution concept. As that ideal solution cannot be analytically, or even numerically, easily computed, in this paper, we find a (closed form) "limit" solution which is a steady state Nash equilibrium solution for the followers. For the leader-followers interactions, we determine a Stackelberg equilibrium solution through a Decision Support Tool, in an analogous manner to [18].

The paper is organised as follows. In Section 2, a complete mathematical model of the effluent management problem is formulated. In Section 3, we discuss strategies relevant to the economic goals of the players and consider the solution concepts. It is there that we conclude that obtaining solutions in the "ideal" class of strategies presents a lot of mathematical problems, that in the present state of the art cannot be overcome (unless using heuristics). A decision support tool (*DST*) which leads to a satisfactory solution to this simplified problem is provided in that section. It is in that framework in which we obtain, in Section 4.2, a solution to the municipal effluent problem specified in details in Section 4.1. The paper ends with the concluding remarks.

2 A mathematical model

2.1 The model dynamics

The model illustrates typical interactions between several economic agents and a community located in a municipality, whose effluent contributes to the pollution of the neighbouring water resources.

Economic agents. We suppose that each economic agent $i, i = 1, 2, ...N$ makes decisions that control his market production, investment, abatement *etc.* Assume that the i-th agent produces pollution within a production period (a month, say) t, $t = 0, 1, 2, ...$, in the volume of $\alpha_i q_i^t$. Symbol q_i^t denotes the i-th agent's production output in period t and $\alpha_i > 0$ can be interpreted as a "technological" coefficient, *i.e.* $\alpha_i = \dfrac{n_1}{n_2}$ where n_1 are m^{3}s of pollution and n_2 kgs of output, *per* month. We will identify the agents' outputs with their production capacities. The agents are supposed to invest v_i^t (or dis-invest, hence v_i^t can be of either sign) so that the production meets the demand. The equation of motion which describes the production capacities reads:

$$q_i^t = (1 - \psi_i) q_i^{t-1} + v_i^{t-1} \qquad i = 1, 2, ...N \tag{1}$$

where $0 \leq \psi_i \leq 1$ is the capacity depreciation rate, q_i^0 is known and $q_i^t \geq 0$.

Each agent is supposed to use an abatement plant whose capacity is $\overline{y}_i^t$, $0 \leq \overline{y}_i^t$, in which pollution y_i^t will be stocked and abated. Each period t, the capacity depreciates to $(1-\mu_i)$, $0 \leq \mu_i \leq 1$ and can be rebuilt, augmented or dismantled through investment $u_i^t \geq 0$ or < 0:

$$\overline{y}_i^t = (1-\mu_i)\overline{y}_i^{t-1} + u_i^{t-1} \qquad i = 1, 2, ...N; \tag{2}$$

$\overline{y}_i^0$ is known and $\overline{y}_i^t \geq 0$. We will assume that a natural process of the pollution elimination takes place in the abatement plant. This may be helped through an abatement effort. The stock of pollution y_i^t ($y_i^t \geq 0$), satisfies the following equation of motion, under the adopted nonlinear[2] abatement regime:

$$y_i^t = (1-\kappa_i)(y_i^{t-1} + \alpha_i q_i^{t-1})(1-k_i^{t-1}) - r_i^{t-1}, \qquad y_i^0 \text{ -- given} \tag{3}$$

where k_i^t, $0 \leq k_i^t \leq 1$ is an instantaneous abatement effort (as said, could be the pollution export), r_i^t is the pollution discharge to the city sewage system, y_i^0 is the initial pollution stock and $0 \leq \kappa_i \leq 1$ is the natural elimination coefficient. Obviously, if the pollution stock at t does not exceed the abatement capacity, the discharge r_i^t is null. In general, the discharge is

$$r_i^t = \max\{0, (1-\kappa_i)(y_i^t + \alpha_i q_i^t)(1-k_i^t) - \overline{y}_i^t\}, \tag{4}$$

and can be considered an observable output variable of the i-th agent's production process. Allowing for (4) in (3) yields

$$y_i^t = \min\{(1-\kappa_i)(y_i^{t-1} + \alpha_i q_i^{t-1})(1-k_i^{t-1}), \overline{y}_i^t\} \qquad y_i^0 \text{ -- given.} \tag{5}$$

The last equation is useful to see that the pollution stock cannot exceed the plant's capacity and that the discharge is the pollution stock "surplus" over and above the capacity $\overline{y}_i^t$.

Notice that the changes in the economic prosperity of a city can also imply the changes in the number of economic agents N. However, in this study, we will assume that N remains constant. Nevertheless, through the dis-investment process ($v_i^t < 0$), production of a particular agent i can vanish.

The dwellings. Let M^t be a measure of the size of the dwelling sector[3]. If each unit of M^t generates the volume β, $0 < \beta$ of pollution *per* month, a monthly pollution load S^t from the city households is

$$S^t = \beta M^t. \tag{6}$$

An improvement in the wealth of the municipality attracts immigrants; conversely, a worsening of the economic prosperity causes emigration. We can model these phenomena through the following relationship:

$$M^{(t+1)} = (1+\sigma_1)\left(\frac{q^t}{q^{t-1}}\right)^{\sigma_2} M^t. \tag{7}$$

[2]Where the nonlinearity comes from the max operator, see (4), (12), (13) below.

[3]It can be the sum of the rates. We expect that a higher rate's dwelling generates more pollution than a lower rate's one.

Here, we assume that production is "aggregable", or substitutable, and $q^t = \sum_{i=1}^{N} q_i^t$; σ_1 is the natural reproduction rate; M^0 is known and σ_2 is between zero and one. The expression $\left(\frac{q^t}{q^{t-1}}\right)^{\xi}$ represents the migration propensity.

Storm waters. The storm waters are polluted due to leaks from the agents' production processes, fertilisers, pesticides and herbicides washed from the green areas, tar *etc.* The volume of the storm waters carrying the pollution can be estimated as T^t which will be proportional to the average precipitation in month t.

Government abatement. The pollution carried through the sewage system, and the storm waters, that eventually reaches the river, lake or sea becomes the effluent. For every amount of the "dominant" pollutant[4] s^t dumped into the target waters, the *Dissolved Oxygen*[5] concentration c^t, at a critical area, can be computed as

$$c^t = \mathcal{L}(s^t, c^0), \tag{8}$$

where $\mathcal{L}$ is an operator representing an averaged solution (over a production period t) to a partial differential equation that describes the pollutant transportation, decay and diffusion processes. The quantity s^t is a forcing factor and c^0 a boundary condition for that equation[6].

Suppose that the Local Government wants the concentration of *Dissolved Oxygen* in the target waters to be less than a critical level $\bar{c}$. It is evident that the concentration c^t can be controlled by the agents' decisions on production and abatement. However, c^t can also depend on the government abatement as the Local Government can build their own abatement facilities to purify the sewage and storm waters. Those facilities are a safety measure to help and clean the "mess" which has escaped the agents' and community's abatement.

Suppose that LG uses two abatement plants: $j = 1$ for cleaning sewage, and $j = 2$ for the storm waters (see [9] and [23]). The plants' capacities are $\bar{Y}_1$ and $\bar{Y}_2$, respectively. They can change in the following way:

$$\bar{Y}_j^t = (1 - \nu_j)\bar{Y}_j^{t-1} + U_j^{t-1}, \quad j = 1, 2, \tag{9}$$

where $0 \leq \nu_j \leq 1$ are the sewage and storm waters', treatment plants' depreciation rates, and U_j^t is the investment. As the Local Government will probably not dis-invest, U_i^t will be non-negative (while, as said before, u_i^t and v_i^t could be of either sign). Obviously, the capacities $\bar{Y}_j^t \geq 0, \ t = 0, 1, 2...$

Under the accepted abatement regime, the pollution stock Y_j^t will satisfy the following state equations:

$$Y_1^t = (1 - \kappa_1')(Y_1^{t-1} + R^{t-1} + S^{t-1})(1 - K_1^{t-1}) - P_1^t, \tag{10}$$
$$Y_2^t = (1 - \kappa_2')(Y_2^{t-1} + T^{t-1})(1 - K_2^{t-1}) - P_2^t, \tag{11}$$

[4]In this paper, we will implicitly assume that curbing one pollutant's emission is satisfactory. In particular, *Ammonia-N* usually dominates the total dissolved inorganic nitrogen so it appears to be a candidate to be the "dominant" pollutant, see [1].

[5]*Dissolved Oxygen* is also a good measure of water quality, see [27].

[6]For the general form of the transportation equation, see [5]; also [15], [11], [16] for a few applications.

where $Y_j^0, j = 1,2$ are given, and $R^t = \sum_{j=1}^{N} r_j^t$. The functions K_j^t, $0 \leq K_j^t \leq 1$ are LG's "instantaneous" abatement efforts, $\kappa_j^t, 0 \leq \kappa_j^t$ are the natural elimination coefficients and Y_j^0 represent the initial pollution stock in the abatement plant, all for $j = 1,2$. Pollution which is not abated or retained is leaking into the target river, lake or sea in the amounts:

$$P_1^t = \max\{0, (1-\kappa_1^t)(Y_1^t + R^t + S^t)(1 - K_1^t) - \bar{Y}_1\}, \tag{12}$$

$$P_2^t = \max\{0, (1-\kappa_2^t)(Y_2^t + T^t)(1 - K_2^t) - \bar{Y}_2\}. \tag{13}$$

Pollution amount. The pollutant amount s^t that eventually reaches the target waters can be modelled as

$$s^t = \varphi_1 P_1^t + \varphi_2^t P_2^t, \tag{14}$$

where $\varphi_j^{(\cdot)}$, $j = 1,2$ are the pollutant concentrations in the effluent from the sewage and storm waters, respectively. Notice, however, that while the coefficient φ_1 can be considered constant, φ_2 has to be modelled as a function of the pollution that reaches the storm waters *i.e.*, depending on the amount:

$$\delta M^t + \sum_{i=1}^{N} \delta_i q_i^t,$$

where δM^t represents the pollution from the the residential sector and $\delta_i q_i^t$ is the one which originates from the commercial sector (due to trucks, cars, leaks *etc.*).

We will denote X^t the state variable at time t with

$$X^t = (\mathbf{x}^t, \mathbf{z}^t, M^t, c^t),$$

where $\mathbf{x}^t = (\mathbf{x}_1^t, \mathbf{x}_2^t, \ldots, \mathbf{x}_N^t)$. Symbol $\mathbf{x}_i^t$ denotes the part of the state variable vector related to the i-th economic agent; symbol $\mathbf{z}^t$ refers to the Local Government's part. Precisely,

$$\left.\begin{array}{l} \mathbf{x}_i^t = (y_i^t, q_i^t, \overline{y}_i^t) \\ \mathbf{z}^t = (Y_1^t, Y_2^t, \overline{Y}_1^t, \overline{Y}_2^t). \end{array}\right\} \tag{15}$$

2.2 The management problem

2.2.1 A game theory problem

The effluent producers are potentially in conflict with oneanother. The cause of the conflict can be economic competition, and/or global common constraints on the amount of effluent tolerated by the environment. On the other hand, the producers, including the dwellers, depend on some regional authority, like the Local Government, whose aim is to negotiate, and legislate, agreements on the admissible pollution and abatement policies of each polluter. Features such as the conflicting interests and the possibility of negotiations naturally suggest a game theoretical approach towards solving the municipal effluent management problem. Applications of the theory of non-cooperative dynamic games to environmental management have already been reported in the literature, see [10], [14], [16], [21], [28], [25]. The idea for the model studied in this paper comes from [17].

2.2.2 Agents' controls and the profit function

The producers' problem is to choose their strategy of abatement and production. The choice is not obvious. On the one hand, investing in an abatement plant and/or abating instantly makes products more costly. This, due to the demand law, diminishes the sales. On the other hand, failing to abate the effluent, can make the producers vulnerable to an environmental levy.

At each instant of time t the economic agents choose actions (k^t, u^t, v^t) using certain strategies (to be described in Section 3). To this set of actions and the amount of tax, with the tax rate τ^t fixed by LG, corresponds an agent's one-period profit function defined as

$$\pi_i(\mathbf{x}^t; \tau^t; k_i^t, u_i^t, v_i^t) = q_i^t p_i \left(\sum_{i=j}^{N} q_j^t \right) - d_i k_i^t (1 - \kappa_i^t)(y_i^t + \alpha_i q_i^t)$$
$$- \tau^t w_i(\alpha_i q_i) - h_i(q_i^t, \alpha_i q_i^t) - g_i(u_i^t, \overline{y}_i^t) - f_i(v_i^t, q_i^t), \tag{16}$$

where $d_i \geq 0$ is a (constant) price of the pollution neutralisation. Function $w_i(\cdot)$ allows for the pollutant diffusion, decay and transportation[7] from the source down to a critical area at which the government wants to enforce a standard. Each agent is levied $\tau^t w_i(\alpha_i q_i^t)$, with $\tau^t \geq 0$ by the Local Government for a unit of the transformed pollutant $\alpha_i q_i^t$. Assuming production is "aggregable" and $q^t = \sum_{i=1}^{N} q_i^t$, function $p_i(q)$ is the i-th agent's inverse demand law and $h_i(\cdot,\cdot), g_i(\cdot,\cdot), f_i(\cdot,\cdot)$ are the production and (abatement and capital) investment cost functions. The dependence of h_i on $\alpha_i q_i$ allows for the fact that although the i-th agent may not realise the cumulative effects of his emission, he may want the clean production in his own interest, if *e.g.* his water intake is below his effluent pipes.

Notice that the polluters are coupled through the inverse demand law $p_i(\cdot)$. Neither this law nor the instantaneous reward depend on the current actions undertaken by other agents. Instead, the coupling is realised through the state variable $\mathbf{x}^t$. In turn, the evolution of $\mathbf{x}^t$ depends on all players' controls.

Define a player's intertemporal profit:

$$\Pi_i(\mathbf{x}^0, \gamma_1, \gamma_2, \ldots, \gamma_N) = \sum_{t=0}^{\infty} \varrho^t \pi_i(\mathbf{x}^t; \tau^t; k_i^t, u_i^t, v_i^t), \tag{17}$$

where a discount factor is ϱ, $0 < \varrho < 1$ and γ_i is the i-th economic agent's strategy, that determines controls k_i^t, u_i^t, v_i^t at each instant in time.

2.2.3 The Local Government's controls and goals

The Local Government is supposed to use a version of *the polluter pays* rule *i.e.*, it levies the environmental damage caused by the agents. Their first, say, instrument is τ which is the environment tax rate. The other controls at LG's disposal are: U_1 — investment in sewage abatement, U_2 — investment in the storm waters abatement, K_1 — sewage abatement effort and K_2 — storm waters abatement effort. These controls are to be chosen so that certain goals of LG are satisfied.

We assume that LG is interested in the environment quality and economic prosperity of the city. This makes the LG's problem *multi-criteria* with $\mathbf{J}$ being its vector criterion,

[7] This function is known to each agent and to the government, and can be a solution to the pollutant's transportation equation, similar to equation (8).

$\mathbf{J} = (\Psi_1, \Psi_2, \ldots, \Psi_n)$. We will concentrate on two components of vector $\mathbf{J}$ only, those which describe the most "natural" objectives of the Local Government:

$$\mathbf{J} = \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \begin{bmatrix} -pollution \\ economic\ prosperity \end{bmatrix} \tag{18}$$

By *pollution*, we understand the numerical value of the dominant water quality measure (*e.g. Dissolved Oxygen*). The measure could be a topological norm of concentration c like $||c^t||_{m^2}$ or $\sup_t c^t$. However, providing the LG decision maker with the time profile of $c^t, t = 0, 1, 2, \ldots$ could give them a better basis for the comparison of the different outcomes of their policies[8]. In this paper, we define Ψ_1 as $\underline{c}$ which is the long term concentration of c^t, or its steady state value,

$$\Psi_1 = \underline{c}. \tag{19}$$

This implies that we expect our system to reach a steady state after a sufficiently long transitory period. We discuss thoroughly the steady state solution concept in Section 3.2.1.

Economic prosperity of the municipality will be measured as a part of the economic agents' sales[9] *plus* the revenue LG collects from the pollution taxation. All LG's state variables, as well as production and pollution outputs can be determined at any time through the state and output equations (see Section 2.1), provided that the relevant initial conditions and previous decisions on investments and efforts are known. In this case, a one-period second objective of the government can be modelled by a function

$$H(X^t, \tau^t, U_1^t, U_2^t, K_1^t, K_2^t) =$$
$$\sum_{i=1}^{N} \left[\zeta q_i^t p_i \left(\sum_{i=1}^{N} q_i^t \right) + \tau^t w_i(\alpha_i q_i^t) \right] - \sum_{j=1}^{2} G_j(U_j^t, \bar{Y}_j^t) - \sum_{j=1}^{2} D_j K_j^t (1 - (\kappa_j^t)')(Y_j^t + C_j^t) \tag{20}$$

where $C_1^t = S^t + R^t$, $C_2^t = T^t$, the coefficient ζ represents a "local" tax rate and D_j, $j = 1, 2$ are prices of the pollution neutralisation in LG's reservoirs. The functions $G_j(U_j^t, \bar{Y}_j^t)$, $j = 1, 2$ are LG's investment cost functions. As in the case of an economic agent, instantaneous reward of the government does not depend on the other players' control variables, and coupling is realised through the state variable. Assuming the Local Government is interested in the long term discounted sum of revenues and that η is its strategy, the LG's second goal function can be defined as

$$\Psi_2(X^0, \gamma_1, \gamma_2, \ldots, \gamma_n, \eta) = \sum_{t=0}^{\infty} (\varrho')^t H(X^t, \tau^t, U_1^t, U_2^t, K_1^t, K_2^t) \tag{21}$$

A strategy η defines the LG's control variables at each instant of time t and is such that its results are "satisfactory", as measured by Ψ_1 and Ψ_2. This strategy will be defined in the next section. Notice that the discount factor ϱ' may be different from ϱ which is the economic agents' discount factor. In particular, as LG may be less myopic than the other players we could expect $\varrho' \geq \varrho$.

[8] See [16] where the local government decision maker was supplied with the time profiles of c.

[9] Assume that a percentage ζ of the GST (Goods and Services Tax) is retained by LG.

3 Problem solution

3.1 Strategies and solution concepts

The "realistic" manner in which the players determine their controls may look like this: at time t LG chooses the tax rate τ^t, the levels of investments U_1^t, U_2^t and abatement efforts K_1^t, K_2^t. The controls, in particular the value of τ^t, are communicated to the economic agents who react to those values by selecting their actions k_i^t, u_i^t, v_i^t. Notice that the information sets available to economic agents and LG are not the same: at time t, LG knows all the state variables X^t, whereas each economic agent knows their own state variable only, *i.e.* x_i^t. In other words, we regard a class of feedback strategies the most appropriate for the situation at hand. We define them in the following way:

- For each economic agent i, a strategy γ_i is a sequence of functions $(\gamma_i^0, \gamma_i^1, \ldots, \gamma_i^t, \ldots)$, where γ_i^t is a mapping from the agent's information set at time t to their set of actions. Thus the controls of the i-th agent chosen at time t are

$$(k_i^t, u_i^t, v_i^t) = \gamma_i^t(\mathrm{x}_i^t, \omega^t)$$

where $\omega^t = (\tau^t, U_1^t, U_2^t, K_1^t, K_2^t)$ is LG's decision vector.

- Similarly, a strategy η of the Local Government is a sequence of functions $(\eta^0, \eta^1, \ldots, \eta^t, \ldots)$, where η^t is a mapping from LG's information set available at time t to its control set. The controls thus selected are

$$\omega^t = (\tau^t, U_1^t, U_2^t, K_1^t, K_2^t) = \eta^t(X^t).$$

Notice that for the $n+1$ strategies thus selected, $(\eta, \gamma_1, \gamma_2, \ldots, \gamma_N)$, the state sequence $(X^0, X^1, \ldots, X^t, \ldots)$ is well defined for every initial state X^0. Consequently, the profit functions $\Pi_i(\eta, \gamma_1, \gamma_2, \ldots, \gamma_N)$, and indices $\Psi_1(\eta, \gamma_1, \gamma_2, \ldots, \gamma_N)$, $\Psi_2(\eta, \gamma_1, \gamma_2, \ldots, \gamma_N)$ are also well defined.

Let us now discuss the solution concept for a game where the set of players comprises economic agents and the Local Government. Notice that the game is multi-level, or hierarchical. It is so because the economic agents *react* to the LG's controls rather than choose them simultaneously. We will call the agents — *followers*, and the Local Government — the *leader*. We assume the followers compete against one another with no cooperation. Hence, the natural concept for the solution to the followers "reaction" game is a Nash equilibrium. We define the followers' optimal reactions to the leader's strategy η as $(\gamma_1^*(\eta), \gamma_2^*(\eta), \ldots, \gamma_N^*(\eta))$ which satisfy the following Nash equilibrium conditions:

$$\Pi_i(\eta, \gamma_1^*(\eta), \gamma_2^*(\eta) \ldots \gamma_i^*(\eta) \ldots \gamma_N^*(\eta)) \geq \Pi_i(\eta, \gamma_1^*(\eta), \gamma_2^*(\eta) \ldots \gamma_i \ldots \gamma_N^*(\eta)) \qquad (22)$$

$$\forall \gamma_i, \qquad i = 1, 2, \ldots, N.$$

Notice that for the Local Government, to be able to enforce an environmental policy, the agents' game has to have a unique equilibrium solution. (If there was no unique solution, LG would not know what the reaction of the firms to a tax policy would be.) Moreover, the unique Nash equilibrium is self-enforcing which makes it an even more attractive solution concept. Indeed, at the equilibrium, no agent can increase their profit by unilaterally deviating from the Nash strategy. Hence the firms become "self-regulated" with no need for monitoring.

However, to guarantee the uniqueness of a (steady state[10]) Nash equilibrium we need players' combined payoffs to be strictly diagonally concave[11] (SDC). In broad economic terms, this mathematical property signifies that the agents' own decisions influence their payoff functions more than the onces of the competitors. As this is not an infrequent situation, the SDC requirement on the agents' payoffs does not greatly limit the applicability of our model.

Now, consider LG's problem. We have to discuss what solution the Local Government can regard as optimal. We have settled for two LG's goals Ψ_1 and Ψ_2, that depend on the strategy η chosen by the government itself, as well as on the followers' reaction strategies $\gamma_1^*(\eta), \gamma_2^*(\eta) \ldots \gamma_N^*(\eta)$. (To simplify notation we will write $\Psi_i(\eta)$ for $\Psi_i(\eta, \gamma_1^*(\eta), \gamma_2^*(\eta) \ldots \gamma_N^*(\eta))$, $i = 1, 2$). The leader will *not* accept a strategy which is dominated. This means that the leader's "optimal" strategy η^* has to satisfy:

$$\forall \eta, \text{ either } \begin{cases} \Psi_1(\eta) > \Psi_1(\eta^*) \\ \Psi_2(\eta) \leq \Psi_2(\eta^*) \end{cases} \text{or } \begin{cases} \Psi_1(\eta) \leq \Psi_1(\eta^*) \\ \Psi_2(\eta) > \Psi_2(\eta^*) \end{cases} \text{or } \begin{cases} \Psi_1(\eta) \leq \Psi_1(\eta^*) \\ \Psi_2(\eta) \leq \Psi_2(\eta^*). \end{cases} \tag{23}$$

Obviously, conditions (23) define the Pareto solution concept which typically does not provide the unique "optimal" strategy. Indeed, (23) determines a set of Pareto efficient solutions $\mathcal{P}$. To enumerate a few of them, it is sufficient to solve the one-criterion optimisation problem embedded in the following definition of $\mathcal{P}_\phi$:

$$\mathcal{P}_\phi = \left\{ \eta_\phi^* = \arg\max_\eta [\phi \Psi_1(\eta) + (1-\phi)\Psi_2(\eta)], \quad \phi \in [0,1] \right\} \tag{24}$$

for several[12] $\phi \in [0, 1]$. If the Local Government believes that one of Ψ_is has to be kept greater than a given number M, as a result e.g. of having been lobbied by the greenies to maintain $\Psi_1 \geq M$, then it (LG) will want to solve the following constrained optimisation problem, which defines $\mathcal{P}_{j,M}$ as :

$$\mathcal{P}_{j,M} = \left\{ \eta_M^* \text{ such that } \eta_M^* = \arg\max_{\eta \in \mathcal{P}_\phi} \Psi_{-j}(\eta) \text{ subject to } \Psi_j \geq M \right\} \tag{25}$$

where $j = 1, 2$ and $-j$ means "non" j. Solutions $\mathcal{P}_{j,M}$ constitute a subset of $\mathcal{P}_\phi$. The former is presumably smaller than the latter, which will make it easier to choose from $\mathcal{P}_{j,M}$ a non-dominated solution for implementation.

3.2 The difficulties

3.2.1 The followers' game

One problem in solving the hierarchical optimisation problem (22), (24), (25) comes from the difficulties in calculating numerically, or analytically, the Nash equilibria (22) in the followers' game[13]. A computable solution option are open loop strategies. In real life,

[10] We explain in Section 3.2.1 why we decide to solve the game using a "simple" steady state equilibrium as the solution concept.

[11] As formulated by Rosen [22] and quoted in the Appendix A.

[12] We could have all Pareto efficient solutions if we solved (24) for *all* $\phi \in [0, 1]$.

[13] See [12] for a continuous time dynamic game.

the state observations could be difficult or costly and we could eventually assume that neither player observes the state variables. In that case open loop strategies would have to be considered, for which turnpike theory (à la [4]) would tell us if and to what steady state the system converges. From now on, we will restrict our attention to sytems whose long term behaviour is the "turnpike" steady state equilibrium. This *should* be true for systems with strict diagonal concavity of payoffs and linear decoupled state equations[14].

Unfortunately, there is no general methodology how to compute a turnpike Nash equilibrium. In particular, solving *implicit programming problems*, as in the optimal control case (see [8]), is not a proved method to determine a turnpike equilibrium in games. We will not pursue the problem of computing turnpike equilibria in this paper, as it would lengthen it substantially. Instead, we will compute a "simple" steady state Nash equilibrium. That kind of a solution might also be useful for the Local Government.

What we know about this solution is that, for $\varrho = 1$, the sufficient conditions [4] for the turnpike (or extremal) steady state equilibrium become the "simple" steady state equilibrium conditions. So, if we admit a hypothesis about monotonicity of the "evolution" of the Nash equilibrium as the discount factor tends to 1, and combine it with those on turnpike equilibrium existence and SDC of payoffs, then our "simple" steady state equilibrium payoffs will provide us with lower bounds to the agents' optimal payoffs. Consequently, the controls supporting the simple steady state equilibrium will be indications on how "myopic" agents might react to the levy τ. We assume that the above hypotheses are satisfied and proceed to define a steady state Nash equilibrium.

We look for the steady states[15] $\underline{\mathrm{x}}_i^*$, and associated controls $\underline{u}_i(\underline{\mathrm{x}}_i^*)$, $\underline{v}_i(\underline{\mathrm{x}}_i^*)$ and $\underline{k}_i(\underline{\mathrm{x}}_i^*)$ such that the controls stabilise the state $\underline{\mathrm{x}}_i$ and

$$\begin{array}{l}\pi_i(\underline{\mathrm{x}}_1^*, \underline{\mathrm{x}}_2^*, \ldots \underline{\mathrm{x}}_i^* \ldots, \underline{\mathrm{x}}_N^*; \tau) \geq \pi_i(\underline{\mathrm{x}}_1^*, \underline{\mathrm{x}}_2^*, \ldots \underline{\mathrm{x}}_i \ldots, \underline{\mathrm{x}}_N^*; \tau) \\ \forall x_i \;\; \forall i = 1, 2, \ldots, N.\end{array} \tag{26}$$

Assuming uniqueness of the stabilising controls, we shorten the notation and write $\pi_i(\underline{\mathrm{x}}_1^*, \underline{\mathrm{x}}_2^*, \ldots \underline{\mathrm{x}}_i^* \ldots, \underline{\mathrm{x}}_N^*; \tau)$ instead of $\pi_i(\underline{\mathrm{x}}_1^*, \underline{\mathrm{x}}_2^*, \ldots \underline{\mathrm{x}}_i^* \ldots, \underline{\mathrm{x}}_N^*, \tau; \underline{k}_i, \underline{u}_i, \underline{v}_i)$, where $\underline{k}_i, \underline{u}_i, \underline{v}_i$ are controls which maintain the steady state $\underline{\mathrm{x}}^*$.

Having decided on the computation of the followers' steady state equilibrium reactions, we consider the LG's problem (also static) with the following goal functions:

$$\left.\begin{array}{l}\Psi_1 = \underline{c} \\ \Psi_2 = H(\underline{X}; \tau)\end{array}\right\} \tag{27}$$

where $H(\underline{X}; \tau)$ was written instead of $H(\underline{X}; \tau; U_1(\underline{X}), U_2(\underline{X}), K_1(\underline{X}), K_2(\underline{X}))$, and where LG's controls are stabilising the state $\underline{X}$.

3.2.2 The hierarchical game

Another difficulty in solving the hierarchical optimisation problem (22), (24), (25) comes from the fact that the Local Government's goal is non unique, see (18). Even if we get a closed form solution in the followers' problem, determining analytically the set (25),

[14] Notice that this "theorem" has been inspired by the examples contained in [4] and [13]. It remains to be stricktly formulated anf proved. See the Conluding Remarks.

[15] We underline a variable to denote its steady state value.

is difficult in this hierarchical context. Also, assigning the weights ϕ in problem (24) to determine $\mathcal{P}_\phi$ might be non intuitive for a Local Government decision makers' panel.

We recommend a *satisfactory solution* to this game which will be computed through a Decision Support Tool (*DST*), *cf* [18], [16]. The *DST* described below is an interactive way of using our model to generate a number of solutions to the problem, under the adopted solution concept (here, (26), (27), (23)). The solutions are Pareto efficient and, obviously, fulfill all constraints. The solutions thus obtained are easier to interpret and apply by the Local Government decision makers than other solutions to that problem[16].

The *DST* for the effluent management problem works in steps as follows:

I. The subset of LG's strategies $\mathcal{P}^{sub}$ is initialized to the empty set.

II. LG sets up a tax function $\tau(\xi)$, which is part of strategy η, and where ξ can be the initial state (constant tax), or the time (open loop tax). Notice that followers' reactions do not depend on the other part of η.

III. LG solves the follower's problem and obtains a polluters' reaction to the levies imposed. In particular, index $\mathrm{J}(\mathrm{X}^0;\tau)$ is computed.

IV. LG chooses a value $\phi \in [0,1]$.

V. LG solves the $\mathcal{P}_\phi$ problem (24) in the remaining instruments (*i.e.* $U_1, U2, K_1, K_2$).

VI. The Local Government tests the results to see if they are satisfactory (*e.g.*, the local lobbiests and politicians debate if the pollutant concentration has been confined to an acceptable limit, and whether the economic prosperity level is adequate).
If the answer is <u>not</u>, LG returns to step IV; if <u>yes</u>, LG continues to step VII.

VII. The strategy is placed in the set $\mathcal{P}^{sub}$.

VIII. After having obtained a few strategies for a value of τ fixed in step II, LG returns to that step and modifies τ.

IX. When the set $\mathcal{P}^{sub}$ contains a number of strategies which result in Ψ_1 ranging from low to high pollution, and Ψ_2 from low to high economic prosperity respectively, LG selects one of its elements (e.g., through voting).

4 A steady state solution

4.1 Model specification

In this section we specify the demand and cost functions, as well as LG's prosperity measure, for which we will solve the effluent management problem.

The underlying situation which the model is able to capture remains unchanged. In particular, we envisage a city where economic agents (like factories or big shops) produce commercial goods and effluent as a by-product. The effluent might be treated by the agents partially, or entirely, before it reaches the city sewage pipes. In the pipes, the "commercial"

[16]In particular, the *DST* was successfully used in [18] to solve a "real life" four criteria leader-follower problem; in [16], also through the *DST*, a farm effluent management problem was solved for typical pork farm data.

effluent joins the community human waste and then is processed at a treatment plant and discharged to a neighbouring river, lake or sea. Besides the sewage pipe network, the city has also got a storm water system. This system carries the storm waters into the same river (or lake, or sea).

We assume that the linear inverse demand law $p(q)$ is given as

$$p_i = p = A - B\sum_{i=1}^{N} q_i. \tag{28}$$

The pollution function $w_i(\cdot)$, proportionally to which an agent will be levied, is fixed as r_i, see (4). This means that the abatement effort of each agent is not going to be unnoticed and that the penalty will be imposed on the part of the production which is responsible for the pollution emission.

We assume that cost functions introduced in Section 2.2 satisfy the standard convexity assumptions. We suppose that these functions are given as:

$$\begin{aligned} h_i(q_i, e_i) &= C_i q_i, & (29)\\ g_i(u_i, \overline{y}_i) &= L_i u_i^2 + E_i \overline{y}_i, & (30)\\ f_i(v_i, q_i) &= F_i v_i^2 \quad i = 1, 2, ...N & (31)\\ G_j(U_j, \bar{Y}_j) &= \Lambda_i U_j^2 + \Xi_j \bar{Y}_j, \quad j = 1, 2 & (32) \end{aligned}$$

where all constants $A, B, C_i, L_i, E_i, F_i, \Lambda_i, \Xi_i$ are positive. The above functions define a large class of agents for which a solution of the effluent management problem will be carried out. In particular, accepting a linear production cost function in the form of (29) means that we expect our agents to easily adjust their production to the demand (no "bottle necks" in particular). The form (30) (and (32)) of the investment cost function defines our agents as the ones for whom an extension of the existing abatement capacities costs more than if they started from scratch. Finally, the quadratic form of (31) means that, to our agents, heavy investment into the production capital is costly.

Now, a one-period agent's profit π_i and a one-period economic prosperity measure H can be written respectively as (dropping the index t):

$$\begin{aligned} \pi_i(\mathrm{x}; \tau; k_i, u_i, v_i) = q_i \left(A - B\sum_{i=1}^{N} q_i \right) - d_i k_i (1 - \kappa_i)(y_i + \alpha_i q_i) - \tau\, r_i \\ - C_i q_i - (L_i u_i^2 + E_i \overline{y}_i) - F_i v_i^2. \end{aligned} \tag{33}$$

and

$$\begin{aligned} H(\mathrm{z}, \mathrm{x}, \tau; U_1, U_2, K_1, K_2) = \sum_{i=1}^{N} \left[\zeta q_i \left(A - B\sum_{i=1}^{N} q_i \right) + \tau r_i \right] \\ - \sum_{j=1}^{2} (\Lambda_j U_j^2 + \Xi_j \overline{Y}_j) - \sum_{j=1}^{2} D_j K_j^t (1 - (\kappa_j^t)')(Y_j^t + C_j^t). \end{aligned} \tag{34}$$

To end the model specification we will rewrite total pollution equation (14) as

$$s = \varphi_1 P_1 + \varphi_2 \left(M + \sum_{i=1}^{N} \delta_i q_i \right) P_2, \tag{35}$$

where φ_1 is constant, and P_1, P_2 are given by equations (12) and (13), respectively.

4.2 A steady state Nash equilibrium solution

4.2.1 Notation and profit segmentation

We simplify the notation to suit the definition of a steady state equilibrium (26). From (1), we obtain straightforwardly that, for any constant control $\underline{v}_i$, the state

$$\underline{q}_i = \frac{\underline{v}_i}{\psi_i},$$

is constant. For brevity of the notation we introduce:

$$V_i = \frac{1}{\psi_i}\underline{v}_i;$$

consequently

$$\underline{q}_i = V_i.$$

As the production has to be non negative we have $\underline{v}_i \geq 0$ and $V_i \geq 0$. Similarly, we denote

$$W_i = \frac{1}{\mu_i}\underline{u}_i$$

and then, from (2) we have that for any value of W_i we have a steady state

$$\underline{\overline{y}}_i = W_i.$$

Again, W_i must be non negative. Using the new variables V_i and W_i, the steady state conditions for the pollution stock (5) and discharge (4) can be rewritten as

$$\begin{cases} \underline{y}_i = \min\{(1-\kappa_i)(1-\underline{k}_i)(\underline{y}_i + \alpha_i V_i), W_i\} \\ \underline{r}_i = -W_i + \max\{(1-\kappa_i)(1-\underline{k}_i)(\underline{y}_i + \alpha_i V_i), W_i\} \end{cases} \tag{36}$$

Now assume that we are dealing with two economic agents $i = 1, 2$ only and that the levy rule is constant, *i.e.*

$$\tau(c) \equiv \tau.$$

from (36), it is easy to see that the i-th agent's pollution discharge $\underline{r}_i$ can be:

I. $\underline{r}_i > 0$, if the abatement capacity W_i and/or cleaning efforts $\underline{k}_i$, are not large enough to contain the pollution. In this case the abatement capacity will be fully utilised

$$\underline{y}_i = \underline{\overline{y}}_i = W_i.$$

This is when the following condition is satisfied :

$$(1-\kappa_i)(1-\underline{k}_i)(W_i + \alpha_i V_i) - W_i \geq 0. \tag{37}$$

Consequently the discharge (or "leakage") into the sewage system is

$$\underline{r}_i = (1-\kappa_i)(\underline{\overline{y}}_i + \alpha_i q_i)(1-\underline{k}_i) - \underline{\overline{y}}_i = (1-\kappa_i)(W_i + \alpha_i V_i)(1-\underline{k}_i) - W_i. \tag{38}$$

For this "high" pollution case, the profit function can be written as $\pi_i^h(\cdots)$ and defined as follows

$$\left.\begin{aligned}&\pi_i(\underline{\mathbf{x}}_i;\tau;\underline{k}_1,W_1,V_1,\underline{k}_2,W_2,V_2)=\pi_i^h(\tau;\underline{k}_i,W_i,V_1,V_2)\\&=V_i(A-B(V_1+V_2))-C_iV_i-F_iV_i^2\\&-(1-\kappa_i)(W_i+\alpha_iV_i)(d_i\underline{k}_i+\tau(1-\underline{k}_i))+\tau W_i-L_iW_i^2-E_iW_i.\end{aligned}\right\}\tag{39}$$

where $\underline{\mathbf{x}}_i$ is the i-th agent's vector of his steady state stocks.

Alternatively,

II. $\underline{r}_i = 0$ (no pollution), if the abatement capacity W_i and/or cleaning efforts $\underline{k}_i$, are sufficiently large, relative to the production output. In this case, k_i and W_i satisfy the inequality

$$(1-\kappa_i)(1-\overline{k}_i)(W_i+\alpha_iV_i)-W_i<0,\tag{40}$$

the capacity will be filled up to the level

$$\underline{y}_i=\frac{(1-\kappa_i)(1-\underline{k}_i)\alpha_iV_i}{1-(1-\kappa_i)(1-\underline{k}_i)},\tag{41}$$

and the "low" pollution case profit function $\pi_i^\ell(\cdots)$ can be written as

$$\left.\begin{aligned}&\pi_i(\underline{\mathbf{x}}_i;\tau;\underline{k}_1,W_1,V_1,\underline{k}_2,W_2,V_2)=\pi_i^\ell(\tau;\underline{k}_i,W_i,V_1,V_2)\\&=\quad V_i(A-B(V_1+V_2))-C_iV_i-F_iV_i^2\\&-\quad\frac{d_i\underline{k}_i(1-\kappa_i)\alpha_iV_i}{1-(1-\kappa_i)(1-\underline{k}_i)}-L_iW_i^2-E_iW_i\end{aligned}\right\}\tag{42}$$

The equilibrium conditions (26) can be interpreted in the new variables as follows: we are looking for the agents' decisions $\hat{k}_1,\hat{k}_2,\hat{W}_1,\hat{W}_2,\hat{V}_1,\hat{V}_2$ such that the Nash equilibrium conditions are satisfied, that is

$$\left.\begin{aligned}&\underline{\pi}_1(\underline{\mathbf{x}}_1;\tau;\hat{\underline{k}}_1,\hat{W}_1,\hat{V}_1,\hat{\underline{k}}_2,\hat{W}_2,\hat{V}_2)=\max_{\underline{k}_1}\max_{W_1}\max_{V_1}\underline{\pi}_1(\underline{\mathbf{x}}_1;\tau;\underline{k}_1,W_1,V_1,\hat{\underline{k}}_2,\hat{W}_2,\hat{V}_2),\\&\underline{\pi}_2(\underline{\mathbf{x}}_2;\tau;\hat{\underline{k}}_1,\hat{W}_1,\hat{V}_1,\hat{\underline{k}}_2,\hat{W}_2,\hat{V}_2)=\max_{\underline{k}_2}\max_{W_2}\max_{V_2}\underline{\pi}_2(\underline{\mathbf{x}}_2;\tau;\hat{\underline{k}}_1,\hat{W}_1,\hat{V}_1,\underline{k}_2,W_2,V_2).\end{aligned}\right\}\tag{43}$$

In computing the optimal values, we will exploit the fact that in the adopted model the i-th player's cost function $\underline{\pi}_i$ does not depend on the other player's decisions concerning the abatement effort $\underline{k}_{(-i)}$ and capacity $W_{(-i)}$. Moreover, we can swap the order of the maximisations in (43). Hence we will be able to deduce the optimal values $\hat{\underline{k}}_i$ and $\hat{W}_i$, as functions of $V_i, V_{(-i)}$, for each player independently. To do that, we will fix $V_i, V_{(-i)}$ and maximise $\underline{\pi}_i$ sequentially in $\underline{k}_i$ and W_i. Finally, we will solve the interrelated maximisation problem, that is we will find the Nash equilibrium payoffs, and compute decisions $\hat{V}_1, \hat{V}_2$.

4.2.2 Optimal abatement effort $\underline{k}_i$.

Consider player i. Suppose that W_i, V_1 and V_2 are given. The profit is a piecewise continuous function of $\underline{k}_i$. Let us denote $k_i^{cr}(W_i)$ a critical value of the abatement effort defined as

$$k_i^{cr}(W_i) = \max(0, k_i^{cr\prime}(W_i)),$$

where $k_i^{cr\prime}(W_i)$ is such that the equality is attained in (37). In other words, abatement efforts $k_i^{cr}(W_i)$ and more, guarantee no effluent leaks into the river.

To compute the optimal abatement effort, we have to determine the shape of the profit function for the following two regions of $\underline{k}_i$:

- for $\underline{k}_i \leq k_i^{cr}(W_i)$, π_i is given by equation (39) and is an affine function with slope

$$(\tau - d_i)(1 - \kappa_i)(W_i + \alpha_i V_i).$$

  If the tax is large relative to the price of the abatement effort ($\tau > d_i$), the slope is positive and the function grows in $\underline{k}_i$. If $d_i > \tau$ the slope is negative, and the function is decreasing in $\underline{k}_i$. If $d_i = \tau$, then π_i is constant in k_i.

- for $\underline{k}_i \geq k_i^{cr}(W_i)$, π_i is given by equation (42) and decreases in $\underline{k}_i$.

Note that $\pi_i^\ell(k_i^{cr}(W_i)) = \pi_i^h(k_i^{cr}(W_i))$, which proves that the profit function is continuous in $\underline{k}_i$.

from the above, we can deduce that the "whole" function π_i attains a maximum in $\underline{k}_i$ either at zero if $\tau < d_i$ (see Figure 1a), or at $k_i^{cr}(W_i)$ if $\tau > d_i$ (see Figure 1b and 1b). If $\tau = d_i$ any value in $[0, k_i^{cr}(W_i)]$ leads to the same profit. We will make an (optimistic) assumption that, in this case, the agent will choose the "good" environmental control, that is $\hat{\underline{k}}_i = k_i^{cr}(W_i)$.

Notice finally that if $\tau < d_i$ the maximum is attained in the first segment of the function, which means that there will be some pollution if $k_i^{cr}(W_i) > 0$. Conversely, if $\tau \geq d_i$, the maximum is attained in the second segment of the function and there will be no pollution leakage into the sewage system.

Table I displays the current results.

Conditions	Optimal k	Figure	Pollution
$\tau < d_i$	$\hat{\underline{k}}_i = 0$	1.a	Yes, if $k_i^{cr}(W_i) > 0$
$\tau \geq d_i$	$\hat{\underline{k}}_i = k_i^{cr}(W_i)$	1.b-c	No

Table I: Optimal abatement effort.

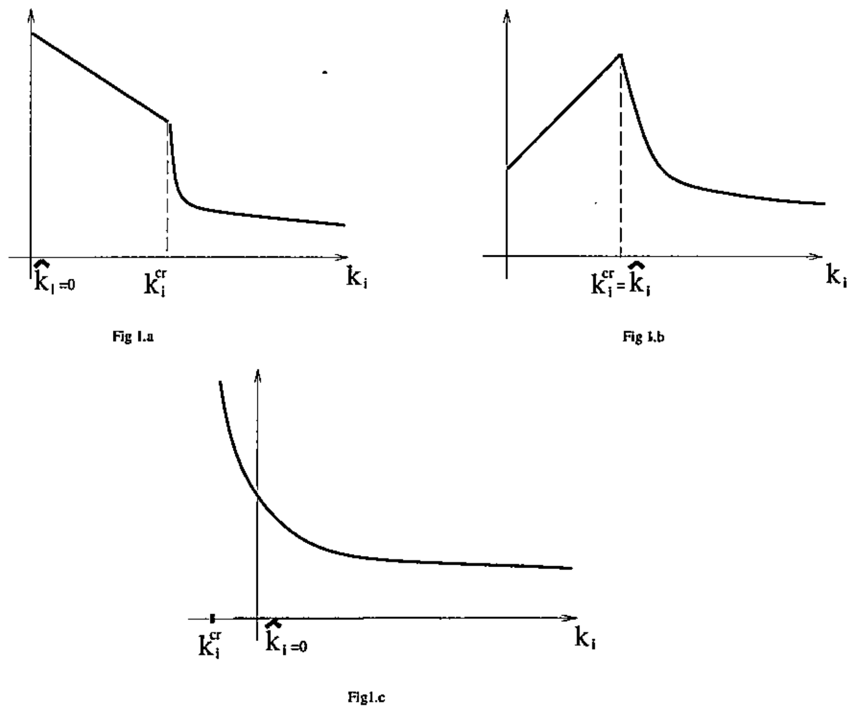

Figure 1: Computation of optimal abatement efforts.

4.2.3 Optimal abatement capacity W_i.

Here too, the computation of $\hat{W}_i$ can be done independently for each player. Let us fix the players' decisions V_1 and V_2 and analyse the resulting optimisation problems. Table I tells us that for the computation of the optimal abatement capacity we need to distinguish between the situations in which the tax value τ (fixed by the local authority), is more - or less - than d_i (which is the marginal cost of the ith agent's abatement effort).

<u>First</u> consider $\tau < d_i$.
From Table I we see that $\hat{\underline{k}}_i = 0$. Let us define a critical value W_i^{cr} of W_i as

$$W_i^{cr} = \frac{(1-\kappa_i)\alpha_i V_i}{\kappa_i}.$$

The value W_i^{cr} is such that equation (37) with $\hat{\underline{k}}_i = 0$, holds with equality, *i.e.*, any capacity greater or equal than critical, guarantees no pollution leaks into the river, no matter how small the abatement effort is. Let us maximise W_i for $W_i \leq W_i^{cr}$ and $W_i > W_i^{cr}$ separately.

- $W_i < W_i^{cr}$. In this case the profit function is given by (39) with $\hat{\underline{k}}_i = 0$, and is a concave parabola in W_i, with the maximum at
$$W_i^h = \frac{\tau\kappa_i - E_i}{2L_i} \quad \text{if} \quad 0 \le W_i^h \le W_i^{cr};$$
or at 0, if $W_i^h \le 0$. If, however, $W_i^h \ge W_i^{cr}$ then the minimum is at W_i^{cr}.

- $W_i \ge W_i^{cr}$, *i.e.* the facility contains all the pollution. The profit function, given by (42) is a concave parabola in W_i, with the "global" maximum attained at a negative value of W_i. In that case a local maximum is achieved at W_i^{cr}.

The above observations, including the one on the profit function continuity in W_i, allow us to conclude that, for the low tax case ($\tau < d_i$), the optimal size of the abatement capacities will be (see Figure 2):
$$\hat{W}_i = \min\left\{\max\left(0\ ,\ W_i^h\right)\ ,\ W_i^{cr}\right\}.$$
or equivalently
$$\hat{W}_i = \min\left\{\max\left(0\ ,\ \frac{\tau\kappa_i - E_i}{2L_i}\right)\ ,\ \frac{(1-\kappa_i)\alpha_i V_i}{\kappa_i}\right\}. \tag{44}$$

<u>Now</u> let $\tau \ge d_i$.
From Table I the profit function is given by equation (42) and $\hat{\underline{k}}_i = k_i^{cr}(W_i) = \max(0, k_i^{cr\prime}(W_i))$ and there is no pollution.

- If $W_i \le W_i^{cr}$, then $k_i^{cr} = k_i^{cr\prime}(W_i) \ne 0$. If we substitute k_i for k_i^{cr} in equation (42) we can see that the profit function is a concave parabola with a local maximum attained at $W_i^l = \dfrac{d_i\kappa_i - E_i}{2L_i}$, or 0, or W_i^{cr}.

- If $W_i > W_i^{cr}$, then $k_i^{cr} = 0$. The function given by (42) with $k_i = 0$ is a concave parabola decreasing for $W_i \le W_i^{cr}$. Consequently, a "local" maximum is attained at W_i^{cr}.

From the above, we can conclude that the size of the optimal abatement capacity is:
$$\hat{W}_i = \min\left\{\max\left(0\ ,\ \frac{d_i\kappa_i - E_i}{2L_i}\right)\ ,\ \frac{(1-\kappa_i)\alpha_i V_i}{\kappa_i}\right\}. \tag{45}$$

Tables II and III summarise the current results. Notice that $\pi_i^1, \pi_i^2, \ldots, \pi_i^5$ are defined as follows:
$$\left.\begin{aligned}
\pi_i^1(V_1,V_2) &= -(B+F_i)V_i^2 + (A - C_i - \tau\alpha_i(1-\kappa_i))V_i - BV_1V_2 + \tfrac{(\tau\kappa_i - E_i)^2}{4L_i},\\
\pi_i^2(V_1,V_2) &= -(B+F_i+L_i\tfrac{\alpha_i^2(1-\kappa_i)^2}{\kappa_i^2})V_i^2 + (A - C_i - E_i\tfrac{\alpha_i(1-\kappa_i)}{\kappa_i})V_i - BV_1V_2,\\
\pi_i^3(V_1,V_2) &= -(B+F_i)V_i^2 + (A - C_i - \tau\alpha_i(1-\kappa_i))V_i - BV_1V_2,\\
\pi_i^4(V_1,V_2) &= -(B+F_i)V_i^2 + (A - C_i - \tau\alpha_i(1-\kappa_i))V_i - BV_1V_2 + \tfrac{(d_i\kappa_i - E_i)^2}{4L_i},\\
\pi_i^5(V_1,V_2) &= -(B+F_i)V_i^2 + (A - C_i - d_i\tfrac{\alpha_i(1-\kappa_i)}{\kappa_i})V_i - BV_1V_2.
\end{aligned}\right\} \tag{46}$$
We remark that all profit functions defined in Tables II and III are continuous.

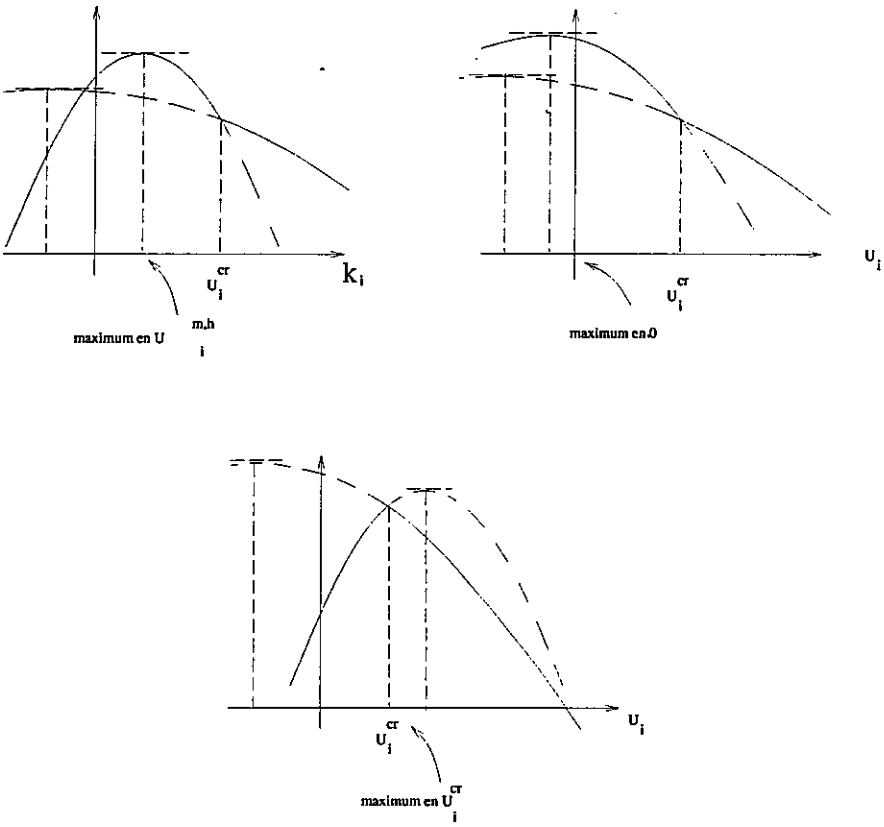

Figure 2: Computation of an optimal abatement capacity.

4.2.4 Economic and ecological interpretations

Before determining the equilibrium decisions on the production capacities $\hat{V}_i, i = 1, 2$ let us economically interpret the results (44), (45).

Let us follow the agents' behaviour as the pollution tax τ increases. If τ is small (even zero), the maximum in (44) has the zero value. Consequently, the optimal capacity's size and abatement effort are $\hat{W}_i = 0$ and $\hat{\underline{k}}_i = 0$, respectively. As expected, if pollution is not taxed there is no incentive for the agents to take this problem seriously: they will not abate the pollution at all. That situation will continue until the tax τ is more penalising, relative to the building and maintenance cost of the abatement capacities (*i.e.* τ has to be in certain relationship with the variables E_i and L_i). The firms will now "stock" part of the produced pollution which is $\hat{W}_i = W_i^h$ and for which the respective facilities will have been built. If the tax still increases, the capacity's size also increases, to allow more pollution to be contained. If the tax increases further, two situations can happen: either the capacities (reservoirs, filters *etc.*) are sufficiently large to stock *all* the produced pollution, or the agents begin to abate the pollution instantaneously in the amount of $\hat{\underline{k}}_i = k_i^{cr}$, which is the part of the pollution impossible to contain.

Conditions	Optimal $(\underline{k}_i, W_i)$	Pollution	Profit function
$E_i \leq \tau\kappa_i$ $V_i \leq \dfrac{\kappa_i(\tau\kappa_i - E_i)}{2L_i(1-\kappa_i)\alpha_i}$	$\left(0, \dfrac{(1-\kappa_i)\alpha_i V_i}{\kappa_i}\right)$	NO	π_i^2
$E_i \leq \tau\kappa_i$ $V_i \geq \dfrac{\kappa_i(\tau\kappa_i - E_i)}{2L_i(1-\kappa_i)\alpha_i}$	$\left(0, \dfrac{\tau\kappa_i - E_i}{2L_i}\right)$	YES	π_i^1
$E_i \geq \tau\kappa_i$	$(0,0)$	YES	π_i^3

Table II: Optimal decisions for $\tau < d_i$.

Notice that similar results would be obtained if the natural cleaning propensity κ_i could increase. In short, if the natural abatement was complete, *i.e.* $\kappa_i = 1$ then, from (45) and (44), $\hat{W}_i = 0$. This is to be interpreted that, obviously, there is no necessity to build reservoirs or filters if there is no pollution. If, on the contrary, there was no natural abatement of pollution (*i.e.* $\kappa_i = 0$), the firms would face the problems described above with the only difference being that the agents would react "earlier" to the levy instrument τ.

4.2.5 Interrelated production capacities

The analysis carried out in Sections 4.2.2 - 4.2.3 provides us with optimal decisions $\hat{\underline{k}}_i$ and $\hat{W}_i$, $i = 1, 2$. If we substitute them in the agents' profits, the latter become functions of (V_1, V_2) only. Tables II and III, together with definitions (46), determine what an agent's profit function is, for a given set of exogenous parameters τ, d_i, E_i. Notice that each function π_i^j, $i = 1, 2$, $j = 1, \ldots, 5$ is of the following form:

$$\pi_i^j(V_1, V_2) = \beta_{i1}^j V_i^2 + \beta_{i2}^j V_i + \beta_{i3}^j V_1 V_2 + \beta_{i4}^j, \tag{47}$$

with

$$\beta_{i1}^j < 0; \;\; \beta_{i3}^j = -B; \;\; \beta_{i4}^2 = \beta_{i4}^3 = \beta_{i4}^5 = 0.$$

The functions are concave parabolas, twice continuously differentiable. Notice also that for $\tau < d_i$ and $E_i \leq \tau\kappa_i$ or $\tau \geq d_i$ and $E_i \leq d_i\kappa_i$ the profit function is piecewise defined, ı is in the first case:

$$\pi_i(V_1, V_2) = \begin{cases} \pi_i^2(V_1, V_2), & V_i \leq V_i^{c\tau}(\tau) \\ \pi_i^1(V_1, V_2), & V_i > V_i^{c\tau}(\tau), \end{cases}$$

Conditions	Optimal (k_i, W_i)	Pollution	Profit function
$E_i \leq d_i\kappa_i$ $V_i \leq \frac{\kappa_i(d_i\kappa_i - E_i)}{2L_i(1-\kappa_i)\alpha_i}$	$\left(0, \frac{(1-\kappa_i)\alpha_i V_i}{\kappa_i}\right)$	NO	π_i^2
$E_i \leq d_i\kappa_i$ $V_i \geq \frac{\kappa_i(d_i\kappa_i - E_i)}{2L_i(1-\kappa_i)\alpha_i}$	$\left(\frac{(1-\kappa_i)2\alpha_i L_i V_i - \kappa_i(d_i\kappa_i - E_i)}{(1-\kappa_i)(d_i\kappa_i - E_i + 2\alpha_i L_i V_i)}, \frac{d_i\kappa_i - E_i}{2L_i}\right)$	NO	π_i^4
$E_i \geq d_i\kappa_i$	$(1, 0)$	NO	π_i^5

Table III: Optimal decisions for $\tau \geq d_i$.

and in the second case:

$$\pi_i(V_1, V_2) = \begin{cases} \pi_i^2(V_1, V_2), & V_i \leq V_i^{cr}(d_i) \\ \pi_i^4(V_1, V_2), & V_i > V_i^{cr}(d_i), \end{cases}$$

with

$$V_i^{cr}(\xi) = \frac{\kappa_i(\xi\kappa_i - E_i)}{2L_i(1-\kappa_i)\alpha_i}.$$

Table IV describes all relations between the tax and cost parameters that may happen, and the corresponding expressions for the profit functions. The notation $(\pi_i^j; \pi_i^k - \xi)$ means that the firm i's profit function is equal to π_i^j if $V_i < V_i^{cr}(\xi)$, and equal to π_i^k for $V_i \geq V_i^{cr}(\xi)$

Lemma 1 *The agents' steady state game (43) is concave and "strictly diagonally concave"*[17] *hence the unique Nash equilibrium exists.*

Proof: We use definitions (46) and those given in Tables II and III.
We first prove that the profit functions π_i are concave in V_i, for each case noted in Table IV. This, applying Theorem 1 of Appendix A, guarantees existence of an equilibrium. Then, we prove the equilibrium uniqueness through showing that the "combined payoff" to the game, of any pair (π_1, π_2) of the profit functions (see the Appendix A), is strictly diagonally concave, which is sufficient for the uniqueness of an equilibrium.

The non-piecewise defined profit functions are obviously concave. Thus we show concavity for the two cases where the profit functions are piecewise defined, *i.e.* $(\pi_i^2; \pi_1 - \tau)$ and $(\pi_i^2; \pi_4 - d_i)$. Here, the only point to verify is that each function π_i is concave at

[17]See Appendix A where we provide relevant definitions and results.

Agent 1	Agent 2	Profit function form (π_1, π_2)
$\tau < d_1$	$\tau < d_2$	
$E_1 < \tau\kappa_1$	$E_2 < \tau\kappa_2$	$((\pi_1^2; \pi_1^1 - \tau), (\pi_2^2; \pi_2^1 - \tau))$
$E_1 < \tau\kappa_1$	$E_2 > \tau\kappa_2$	$((\pi_1^2; \pi_1^1 - \tau), \pi_2^3)$
$E_1 > \tau\kappa_1$	$E_2 > \tau\kappa_2$	(π_1^3, π_2^3)
$\tau < d_1$	$\tau > d_2$	
$E_1 < \tau\kappa_1$	$E_2 < d_2\kappa_2$	$((\pi_1^2; \pi_1^1 - \tau), (\pi_2^2; \pi_2^4 - d_2))$
$E_1 < \tau\kappa_1$	$E_2 > d_2\kappa_2$	$((\pi_1^2; \pi_1^1 - \tau), \pi_2^5)$
$E_1 > \tau\kappa_1$	$E_2 < d_2\kappa_2$	$(\pi_1^3, (\pi_2^2; \pi_2^4 - d_2))$
$E_1 > \tau\kappa_1$	$E_2 > d_2\kappa_2$	(π_1^3, π_2^5)
$\tau > d_1$	$\tau > d_2$	
$E_1 < d_1\kappa_1$	$E_2 < d_2\kappa_2$	$((\pi_1^2; \pi_1^4 - d_1), (\pi_2^2; \pi_2^4 - d_2))$
$E_1 < d_1\kappa_1$	$E_2 > d_2\kappa_2$	$((\pi_1^2; \pi_1^4 - d_1), \pi_2^5)$
$E_1 > d_1\kappa_1$	$E_2 > d_2\kappa_2$	(π_1^5, π_2^5)

Table IV: Profit function pairs.

$V_i^{cr}(\xi)$. This is true because each profit function is continuously differentiable. The proof of that is a straitforward computation of the derivatives. The limit condition of Theorem 1 is also verified. Hence, we obtain existence of a Nash equilibrium.

Next, we demonstrate the uniqueness of the equilibrium.

To show that the combined payoff is stictly diagonally concave we need the "pseudo-Hessian" matrix

$$H = \begin{pmatrix} \frac{\partial^2 \pi_1^j}{\partial V_1 \partial V_1}(V_1, V_2) & \frac{\partial^2 \pi_1^j}{\partial V_2 \partial V_1}(V_1, V_2) \\ \frac{\partial^2 \pi_2^k}{\partial V_1 \partial V_2}(V_1, V_2) & \frac{\partial^2 \pi_2^k}{\partial V_2 \partial V_2}(V_1, V_2) \end{pmatrix} + \begin{pmatrix} \frac{\partial^2 \pi_1^j}{\partial V_1 \partial V_1}(V_1, V_2) & \frac{\partial^2 \pi_2^k}{\partial V_1 \partial V_2}(V_1, V_2) \\ \frac{\partial^2 \pi_1^j}{\partial V_2 \partial V_1}(V_1, V_2) & \frac{\partial^2 \pi_2^k}{\partial V_2 \partial V_2}(V_1, V_2) \end{pmatrix} \tag{48}$$

to be negative definite, for all V_1, V_2. Using expression (47) we have:

$$H = \begin{pmatrix} 4\beta_{11}^j & \beta_{13}^j \beta_{23}^k \\ \beta_{13}^j \beta_{23}^k & 4\beta_{21}^k \end{pmatrix}$$

The eigenvalues are computed as

$$\left.\begin{array}{ll}\lambda_1 = 2(\beta^j_{11}+\beta^k_{21}) + \sqrt{4(\beta^j_{11}-\beta^k_{21})^2+(\beta^j_{13}+\beta^k_{23})^2} & \text{(a)} \\ \text{and} & \\ \lambda_2 = 2(\beta^j_{11}+\beta^k_{21}) - \sqrt{4(\beta^j_{11}-\beta^k_{21})^2+(\beta^j_{13}+\beta^k_{23})^2} & \text{(b)}\end{array}\right\} \quad (49)$$

The second eigenvalue λ_2 is clearly negative since $\beta_{i1} < 0$. For λ_1 to be negative, the following inequality has to be satisfied:

$$16\beta^j_{11}\beta^k_{21} \geq (\beta^j_{13}+\beta^k_{23})^2. \quad (50)$$

It is easy to see that this is true using expressions (46). Indeed, we have

$$\begin{array}{l}\beta^{(\cdot)}_{i1} = -B - F_i \text{ or } \beta^{(\cdot)}_{i1} = -B - F_i - L_i\dfrac{\alpha_i^2(1-\kappa_i)^2}{\kappa_i^2}, \\ \beta^j_{13} = \beta^k_{23} = -B,\end{array}$$

in any case, which assures us that the previous inequality is satisfied, and ends the proof of the lemma.

◇

4.2.6 Computation of V_1 and V_2

Non piecewise defined profit functions

Suppose that either player has a non piecewise defined profit function (*i.e.* either π_i^3 or π_i^5). We have to compute a constrained Nash equilibrium, as $V_i, i = 1, 2$ have to be non negative.

The condition for equilibrium productions $\hat{V}_1, \hat{V}_2$ is

$$\left.\begin{array}{l}\beta^j_{11}\hat{V}_1^2 + \beta^j_{12}\hat{V}_1 + \beta^j_{13}\hat{V}_1\hat{V}_2 \geq \beta^j_{11}V_1^2 + \beta^j_{12}V_1 + \beta^j_{13}V_1\hat{V}_2, \quad \forall\ V_1 \geq 0 \\ \beta^k_{21}\hat{V}_2^2 + \beta^k_{22}\hat{V}_2 + \beta^k_{23}\hat{V}_1\hat{V}_2 + \geq \beta^k_{21}V_2^2 + \beta^k_{22}V_2 + \beta^k_{23}\hat{V}_1V_2, \quad \forall\ V_2 \geq 0.\end{array}\right\}$$

Equivalently, a pair $\hat{V}_1, \hat{V}_2$ is such that it is the fixed point of the "best reply" functions defined as follows. For V_2 fixed, the first player's best reply is the decision V_1^{best} which maximises his firm's profit:

$$V_1^{best}(V_2) = \max\left(0, -\frac{1}{2}\frac{\beta^j_{12}+\beta^j_{13}V_2}{\beta^j_{11}}\right). \quad (51)$$

Symmetrically, for V_1 fixed, the second player's best reply is decision V_2^{best} which maximises his firm's profit:

$$V_2^{best}(V_1) = \max\left(0, -\frac{1}{2}\frac{\beta^k_{22}+\beta^k_{23}V_1}{\beta^k_{21}}\right). \quad (52)$$

Substituting V_2 in (51) for (52) and taking into account that $V_1^{best}(V_2)$ is a decreasing function of V_2, yields the following equation which $\hat{V}_1$ has to satisfy

$$0 = \min\left(\hat{V}_1\ ,\ \max\left(\hat{V}_1 + \frac{1}{2}\frac{\beta^j_{12}}{\beta^j_{11}}\ ,\ \frac{1}{2}\frac{\beta^j_{12}}{\beta^j_{11}} - \frac{1}{4}\frac{\beta^j_{13}\beta^k_{22}}{\beta^j_{11}\beta^k_{21}} - \frac{1}{4}\frac{\beta^j_{13}\beta^k_{23} - 4\beta^j_{11}\beta^k_{21}}{\beta^j_{11}\beta^k_{21}}\hat{V}_1.\right)\right) \quad (53)$$

Conditions	$(\hat{V}_1, \hat{V}_2)$
$\beta^k_{22} \geq 0$ $\beta^j_{12} \leq 0$ $2\beta^k_{21}\beta^j_{12} \geq \beta^j_{13}\beta^k_{22}$	$\left(0\ ,\ -\frac{1}{2}\frac{\beta^k_{22}}{\beta^k_{21}}\right)$
$\beta^j_{12} \geq 0$ $\beta^k_{22} \leq 0$ $2\beta^j_{11}\beta^k_{22} \geq \beta^k_{23}\beta^j_{12}$	$\left(-\frac{1}{2}\frac{\beta^j_{12}}{\beta^j_{11}}\ ,\ 0\right)$
Other cases	$\left(\frac{\beta^j_{13}\beta^k_{22} - 2\beta^j_{12}\beta^k_{21}}{-\beta^j_{13}\beta^k_{23} + 4\beta^k_{21}\beta^j_{11}}\ ,\ \frac{\beta^k_{23}\beta^j_{12} - 2\beta^k_{22}\beta^j_{11}}{-\beta^k_{23}\beta^j_{13} + 4\beta^j_{11}\beta^k_{21}}\right)$

Table V: Equilibrium production outputs.

The final results of optimal $\hat{V}_1$ and $\hat{V}_2$ are shown in Table V.

<u>Piecewise defined profit functions</u>

Suppose either firm has a piecewise defined profit function (*i.e.* either $(\pi^2_i; \pi_1 - \tau)$ or $(\pi^2_i; \pi_4 - d_i))$. (The case when one firm has a non piecewise defined profit function and the other a piecewise defined profit function can be viewed as a particular case.) We denote $\pi_1 = (\pi^2_1; \pi^j_1 - \xi_1)$ and $\pi_2 = (\pi^2_2; \pi^k_2 - \xi_2)$.

Again, the solution is the fixed point of the best reply functions.

For the decision V_2 fixed, the best reply of the first firm is $V_1^{best}(V_2)$, and is the argument of the maximum of $\pi_1(V_1, V_2)$. Due to the strict concavity of the π_1 this maximum is unique, and we have:

$$V_1^{best}(V_2) \in \{V_1^2(V_2), V_1^j(V_2), 0\}$$

with

$$\left\{ \begin{array}{l} V_1^2(V_2) = -\dfrac{1}{2}\dfrac{\beta^2_{12} + \beta^2_{13}V_2}{\beta^2_{11}} \\ \\ V_1^j(V_2) = -\dfrac{1}{2}\dfrac{\beta^j_{12} + \beta^j_{13}V_2}{\beta^j_{11}} \end{array} \right. \tag{54}$$

In the same way, for V_1 fixed, $V_2^{best}(V_1)$ is the second player's best reply. We have

$$V_2^{best}(V_1) \in \{V_2^2(V_1), V_2^k(V_1), 0\}$$

with

$$\begin{cases} V_2^2(V_1) = -\dfrac{1}{2}\dfrac{\beta_{22}^2 + \beta_{23}^2 V_2}{\beta_{21}^2} \\[2ex] V_2^k(V_1) = -\dfrac{1}{2}\dfrac{\beta_{22}^k + \beta_{23}^k V_2}{\beta_{21}^k} \end{cases} \tag{55}$$

Now the Nash equilibrium is $(\hat{V}_1, \hat{V}_2)$ such that

$$\begin{cases} V_1^{best}(V_2^{best}(\hat{V}_1)) = \hat{V}_1 \\[1ex] V_2^{best}(V_1^{best}(\hat{V}_2)) = \hat{V}_2 \end{cases}$$

or equivalently

$$\begin{cases} V_1^{best}(V_2^{best}(\hat{V}_1)) = \hat{V}_1 \\[1ex] \hat{V}_2 = V_2^{best}(\hat{V}_1). \end{cases} \tag{56}$$

Formulae (54) - (56), together with the definitions of coefficients $\beta_{(\cdot)(\cdot)}^{(\cdot)}$ (see (46)), tell us how to compute the unique Nash equilibrium of a game, for which the parameter values define piecewise differentiable profit functions. If only one profit function is piecewise defined formulae (54) have to be replaced by (52).

4.2.7 Economic and ecological interpretations of the production decisions

The obtained solutions are parametrical in βs rather than in the original problem parameters. Hence the interpretation of the results is not immediate. Instead, numerical simulations are needed. They will show what economic output is optimal for a given tax rate and cost, when production and abatement function parameters are fixed.

4.3 Resolution of the Local Government problem

4.3.1 Computation of the steady state equilibrium

Here we calculate the optimal LG's abatement decisions $\underline{K}_j$ and $\underline{U}_j$ optimising (24), for a given value of $\phi \in [0,1]$. In a steady state, the Local Government states $\underline{Y}_j$ and $\bar{\underline{Y}}_j$ become (see (9), (10) and (11)):

$$\bar{\underline{Y}}_j = (1-\nu_j)\bar{\underline{Y}}_j + \underline{U}_j \qquad j = 1,2 \tag{57}$$

$$\underline{Y}_1 = (1-\kappa_1')(\underline{Y}_1 + R + S)(1-\underline{K}_1) - P_1, \tag{58}$$

$$\underline{Y}_2 = (1-\kappa_2')(\underline{Y}_2 + T)(1-\underline{K}_2) - P_2 \tag{59}$$

where

$$P_1 = \max\{0, (1-\kappa_1')(\underline{Y}_1 + R + S)(1-\underline{K}_1) - \bar{\underline{Y}}_1\}$$
$$P_2 = \max\{0, (1-\kappa_2')(\underline{Y}_2 + T)(1-\underline{K}_2) - \bar{\underline{Y}}_2\}$$

and R, S and T are the steady state, or averaged (see Section 2.1), pollution amounts reaching the government abatement facilities. From (57) we obtain

$$\bar{\underline{Y}}_j = \frac{\underline{U}_j}{\nu_j}.$$

As in the agents case we abbreviate the notation and introduce a new symbol to denote the steady state LG's abatement facility size

$$Z_j = \bar{\underline{Y}}_j.$$

Remember that $C_1 = R + S$, $C_2 = T$ which depend on and the agents' decisions and, implicitly, on the tax rate τ, but not on the other controls of LG. Then we have

$$\left.\begin{array}{l} \underline{Y}_j = \min\{(1-\kappa'_j)(1-\underline{K}_j)(\underline{Y}_j + C_j), Z_j\} \\ P_j = -Z_j + \max\{(1-\kappa'_j)(1-\underline{K}_j)(\underline{Y}_j + C_j), Z_j\} \end{array}\right\} \tag{60}$$

We say that the system (60) possesses the S property if the abatement facility is saturated:

$$(1-\kappa'_j)(1-\underline{K}_j)(Z_j + C_j) - Z_j \geq 0. \tag{61}$$

If the facility is non saturated (which corresponds to the inequality sign $<$ in (61)) we say that the system possesses the property NS. We call $\underline{K}'^{\,cr}_j = \underline{K}'^{\,cr}_j(Z_j)$ if equality is reached in (61):

$$\underline{K}'^{\,cr}_j = 1 - \frac{Z_j}{(1-\kappa'_j)(Z_j + C_j)}. \tag{62}$$

Let us denote $\underline{K}^{cr}_j$ the critical value of the abatement effort defined as $\underline{K}^{cr}_j = \max(0, \underline{K}'^{\,cr}_j)$ and let

$$Z^{cr}_j = \frac{(1-\kappa'_j)C_j}{\kappa'_j}.$$

We obtain:

$$\underline{Y}_j = \frac{(1-\kappa'_j)(1-\underline{K}_j)C_j}{1-(1-\kappa'_j)(1-\underline{K}_j)}, \quad P_j = 0 \quad \text{if} \quad \text{NS}$$

$$\underline{Y}_j = Z_j, \quad P_j = (1-\kappa'_j)(1-\underline{K}_j)(\underline{Y}_j + C_j) \quad \text{if} \quad \text{S}.$$

Finally, remember that the Local Government goal functions are:

$$H(\mathbf{z}, \eta) = \sum_{i=1}^{N}\left[\zeta q^i \left(A - B\sum_{i=1}^{N} q_i\right) + \tau r_i\right] - \sum_{j=1}^{2}\Lambda_j Z_j^2 - \Xi_j Z_j - \sum_{j=1}^{2} D_j \underline{K}_j (1-\kappa'_j)(\underline{Y}_j + C_j)$$

$$\underline{s} = \varphi_1 P_1 + \varphi_2 \left(M + \sum_{i=1}^{N}\delta^i q^i\right) P_2,$$

where M is the steady state size of the dwelling sector. According to (24) the LG's optimising criterion can be a weighted sum of H and $-\underline{s}$; we denote

$$\Psi = \phi H - (1-\phi)\underline{s} \tag{63}$$

where ϕ is a parameter given by LG that measures the importance of prosperity *vis-à-vis* pollution. Also, to shorten notation, call:

$$Q = \sum_{i=1}^{N}\left[\zeta q^i \left(A - B\sum_{i=1}^{N} q_i\right) + \tau r_i\right]$$

$$\mathsf{h}_j = -\Lambda_j Z_j^2 - \Xi_j Z_j - D_j \underline{K}_j (1-\kappa_j')(\underline{Y}_j + C_j)$$

$$\underline{s}_j = \varphi_j P_j$$

Depending on whether the property S is satisfied or not, and in what facility, we obtain different forms of (63); *i.e.*:

$$\Psi = \begin{cases} \phi(Q + \mathsf{h}_1^l + \mathsf{h}_2^l) - (1-\phi)(s_1^l + s_2^l) & \text{if} \quad \text{S for} \qquad\qquad j = 1,2 \\ \phi(Q + \mathsf{h}_1^h + \mathsf{h}_2^h) - (1-\phi)(s_1^h + s_2^h) & \text{if} \quad \text{NS for} \qquad\quad j = 1,2 \\ \phi(Q + \mathsf{h}_1^h + \mathsf{h}_2^l) - (1-\phi)(s_1^h + s_2^l) & \text{if} \quad \text{NS for } j=1, \quad \text{S for } j = 2 \\ \phi(Q + \mathsf{h}_1^l + \mathsf{h}_2^h) - (1-\phi)(s_1^l + s_2^h) & \text{if} \quad \text{S for } i=1, \quad \text{NS for } j = 2. \end{cases}$$

Here

$$\mathsf{h}_j^l = -\Lambda_j Z_j^2 - \Xi_j Z_j - D_j \underline{K}_j \frac{(1-\kappa_j')C_j}{1-(1-\kappa_j')(1-\underline{K}_j)}$$

$$\mathsf{h}_j^h = -\Lambda_j Z_j^2 - \Xi_j Z_j - D_j \underline{K}_j Z_j$$

$$s_j^l = 0, \qquad s_j^h = \varphi_j P_j.$$

4.3.2 Optimal abatement effort $\underline{K}_j$ and capacity Z_j

We follow a procedure analogous to that of Section 4.2 and obtain the optimal $\hat{\underline{K}}_j(Z_j)$ and $\hat{Z}_j$.

<u>Case S, $j = 1,2$</u>: In this case Ψ is a decreasing function of $\underline{K}_j$ because

$$\frac{\partial \Psi}{\partial \underline{K}_j} = -\frac{D_j Z_j C_j \kappa_j'}{[1-(1-\kappa_j')(1-\underline{K}_j)]^2} < 0.$$

Then, $\hat{\underline{K}}_j = \min\Big(1, \max(\underline{K}_j^{cr}, 0)\Big)$, $j = 1,2$.

To maximize in Z_j consider:

• $Z_j < Z_j^{cr}$, then $\underline{K}_j^{cr} > 0$; substituting $\underline{K}_j^{cr}$ in the corresponding Ψ yields the maximum at

$$Z_j^l = \frac{D_j \kappa_j' - \Xi_j}{2\Lambda_j}$$

in this "low" pollution case; then,

$$\hat{Z}_j = \min\Big(\max(0, Z_j^{hl}), Z_j^{cr}\Big). \tag{64}$$

• If $Z_j > Z_j^{cr}$, then $\underline{K}_j^{cr} = 0$ and the global maximun of Ψ in Z_j is attained at a negative value. Hence, $\hat{Z}_j = Z_j^{cr}$.

Cases S, $i = 1$, NS, $j = 2$ and NS, j=1,2 can be treated in similarly.

5 Numerical simulation of the LG problem via the *DST*

5.1 Parameter values

We simulate game (24), (26), (27) for the model defined in Section 4.1, and solved in Section 4.2, for the parameter values[18] given below in Tables VI - X.

Player	α	ψ	μ	κ	δ
1	.3	.08	.02	.01	.01
2	.28	.08	.02	.01	.01
	$\mathsf{Par_i[1]}$	$\mathsf{Par_i[8]}$	$\mathsf{Par_i[9]}$	$\mathsf{Par_i[6]}$	$\mathsf{Par_i[10]}$

Table VI: Agents' effluent production parameters.

The tables contain values which reflect a situation in which the Local Government deals with two almost symmetrical players. The asymetry comes from the fact that the second player's technology produces less pollution than the first player's (*i.e.*, $\alpha_1 > \alpha_2$). We assume that maintaining a facility's size is "cheap" relative to changing it *i.e.* $E_{1,2}$ is low compared to $L_{1,2}$, see Table VII.. Regarding the Local Government facilities we assume that they are cheaper to re-build then those of the agents (*i.e.*, $L_{1,2} > \Lambda_{1,2}$, however, $E_{1,2} < \Xi_{1,2}$). Between the sewage abatement and the storm waters' facilities, the latter is cheaper (*i.e.*, $\Lambda_1 > \Lambda_2$).

i/j	C	L	E	F	d	Λ	Ξ
1	.2	1	.1	.1	1	.98	.1
2	.2	1	.1	.1	1	.96	.1
	$\mathsf{Par_i[2]}$	$\mathsf{Par_i[7]}$	$\mathsf{Par_i[4]}$	$\mathsf{Par_i[5]}$	$\mathsf{Par_i[3]}$	$\mathsf{PGar_j[2]}$	$\mathsf{PGar_j[3]}$

Table VII: Cost functions parameters.

We also assume that the municipality size is steady (*i.e.*, $\sigma_1 = 0$).

M^0	T^t	β	σ_1	σ_2	φ_1	δ	ζ
10000	100	.3	.001	.5	.01	.001	.0585
$\mathsf{PGar_1[4]}$	$\mathsf{PGar_1[5]}$	$\mathsf{PGar_1[6]}$	$\mathsf{PGar_1[7]}$	$\mathsf{PGar_1[8]}$	$\mathsf{PGar_1[9]}$	$\mathsf{PGar_1[10]}$	$\mathsf{PGar_1[11]}$

Table VIII: Miscellaneous government's parameters.

[18]The parameters enter a Maple programme, obtainable from the authors on request, as vectors $\mathsf{Par}, \mathsf{Par_1}, \mathsf{Par_2}, \mathsf{PGar_1}, \mathsf{PGar_2}$. The last row of each table, for the convenience of programming, tells us what Par is a parameter part of.

At this stage, we have to specify the $\varphi_2(\cdot)$ rule. We assume that the pollutant's concentration in the storm waters can increase up to 1% of the storm waters volume, according to the following function:

$$\varphi_2(x) = \frac{1}{100}\left(1 - e^{-x}\right) \tag{65}$$

where x is the function $\varphi_2(\cdot)$ argument and symbolises the combined (*i.e.* industry + dwellings) size of the municipality.

	ν	κ'
$j = 1$	.02	.02
$j = 2$	.02	.02
	Par$_j$[11]	PGar$_j$[1]

Table IX: Government's' state equations parameters.

A	B
120	.02
Par[1]	Par[2]

Table X: Inverse demand law function parameters.

Regarding a solution to the transportation equation (8) we assume that it is a time invariant steady state solution (see [11]). If so, the concentration $c^{(\infty)}$ is a linear transformation of s. Hence, we can identify Ψ_1 as $-\underline{s}$, see (35).

5.2 The "current" economic and ecological situation

Here we calculate the agents' payoffs and the Local Government indices which correspond to a no-tax-no-abatement regime.

Player		$\overline{y}^0$	q^0	$\underline{\pi}$
1	0	136.849	460.769	25476.99
2	0	127.725	460.769	25476.99

Table XI: Agents' initial conditions.

Table XI documents the fact that, without taxes, the second player's cleaner technology gives him no financial reward ($\underline{\pi}_1 = \underline{\pi}_2$). The agents are (obviously) not abating pollution ($\underline{k}_1 = \underline{k}_2 = 0$) and have not constructed the abatement capacities ($W_1 = W_2 = 0$). The municipality prosperity, as measuered through Ψ_2, and the pollution concentration are given in Table XII. Notice that in the concentration level of $\underline{s} = 4.6457$, 2 is the "base" pollution concentration which originates from the dwelling sector (1 unit) and the storm waters (also 1 unit). The Local Government abatement facilities are (obviously) non existent.

$\underline{s}$	Ψ_2	$\bar{Y}_1^0$	$\bar{Y}_2^0$
4.6457	5528.309	0	0

Table XII: Government initial conditions.

5.3 Satisfying taxing and abatement options

Here we compute the agents' reaction to a range of tax values. The Local Government is supposed to the make a political decision as to what τ is to be applied, basing its judgemtent on the provided pollution concentration and "prosperity" values.

Here, for simplicity we assume that options of building the common pool abatement facilities are *not* available for the Local Government. In other words, LG is interested in the agents' abatement only. Figure 3 shows what the expected results of raising the taxes are. We see the pollution concetration $\underline{s}$ decreases albeit very slowly from 4.6457 to 4.6394 while τ raises from 0 to 1. Abatement facilities W, built from the middle ragne of τ do not seem to contain a substantial amount of pollution. However, applying $\tau > 1$ (which satisfies the condition $\tau > d_{1,2}$, where d is the price of the abatement effort) motivates the agents to abate "instantaneously": $\underline{k}_1, \underline{k}_2 \approx 1$ *i.e.*, sufficiently to diminish the pollution concentration to the base level. This environmental improvement is associated with a 20 - 25 % drop in the agents' outputs, profits and the regional "prosperity".

6 Concluding remarks

In this paper, we presented a comprehensive model of municipal effluent management which resulted in a hierarchical game with a Nash equilibrium at the lower level. We solved this game by indicating a satisfactory solution obtainable through the use of a Decision Support Tool. That approach can be used to resolve other conflict problems with hierarchy and multiple agents.

Control-through-levies is a market oriented method of compeling economic agents to adhere to environmental standards. Arriving at an acceptable tax through the *DST* means that the authority gets acquainted with a range of feasible taxing options, each implying a different trade-off between the economic and environmental indicators. Through the use of the *DST*, the authority can establish what economic indicators are relevant to desired environmental standards. Also, enforcing non-feasible standards can be avoided.

We will not repeat the economic interpretations of Sections 4.2.4 and 4.2.7 here. However, one qualitative result of importance for the regional abatement services planning is worth quoting. The linearity of the abatement effort cost function results in a bang-bang type of optimal $\hat{\underline{k}}$. As the linearity of this function seems plausible (two track sludge loads probably cost twice as much as a one track load), so does the result. Therefore, the

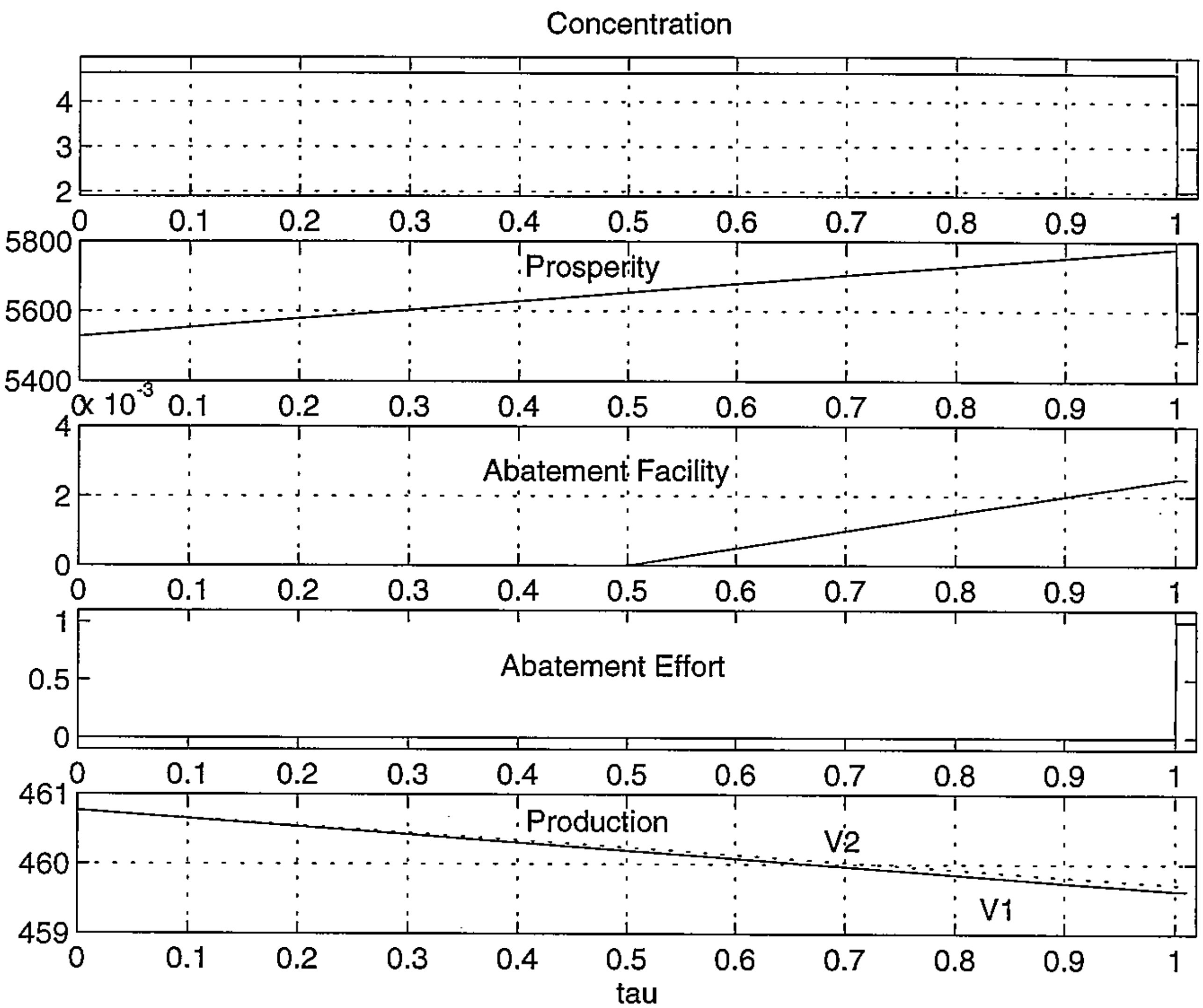

Figure 3: The tax and reactions.

Regional Government should expect that the demand for the abatement services will raise sharply, after a certain value of τ has been reached.

Finally, in the course of dicussining the analytical and numerical solution concepts, in particular in Section 3, we identified several issues which in this paper we were able to treat intuitivly rather than with full mathematical rigour. We list them here as topics for further research.

- Continuity of a Nash equilibirium in dynamic games with discounting.
- Easily applicable turnpike equilibrium suffucient conditions.
- Easily applicable turnpike equilibrium necessary conditons.

Appendix

A Existence and uniqueness of the Nash equilibrium

We cite the relevant definitions and results of [22].

Let $i = 1, 2, \ldots N$ be players in a game, where $\pi_i(V_1, \ldots, V_2)$ are the profit functions to be maximised through V_i which is a strategy of player i.

Theorem 1 *Suppose that $\pi_i(V)$, $V = (V_1, \ldots, V_n)$ is continuous in V and concave in V_i, furthermore, let*

$$\pi_i(V_i, V_{-i}) \to -\infty \quad as \quad |V_i| \to \infty \quad \forall V_{-i}, \ i = 1, \ldots N.$$

then a Nash equilibrium point exists.

To obtain uniqueness of an equilibrium we need a stronger condition. Denote $\rho(V, r)$, $r \in \mathbb{R}^n$, $r_i \geq 0, \forall i$ the "combined payoff" as

$$\rho(V, r) = \sum_{i=1}^{N} r_i \pi_i(V), \tag{66}$$

and $g(V, r)$ the pseudo-gradient of $\rho(V, r)$,

$$g(V, r) = \begin{bmatrix} r_1 \dfrac{\partial \pi_1(V)}{\partial V_1} \\ r_2 \dfrac{\partial \pi_2(V)}{\partial V_2} \\ \vdots \\ r_n \dfrac{\partial \pi_N(V)}{\partial V_N} \end{bmatrix}. \tag{67}$$

Definition 1 *The function $\rho(V, r)$ will be called diagonally strictly concave in V, if for every $V = (V_1, V_2, \ldots, V_n)$ and $V' = (V_1', V_2', \ldots, V_n')$, $V_1, V_2 \in I\!R$ and fixed $r \in I\!R_+$ we have*

$$(V - V')^T g(V', r) + (V' - V) g(V, r) > 0,$$

where ^{T} means transposition.

Lemma 2 *A sufficient condition that $\rho(V, r)$ be diagonally strictly concave in V and fixed $r > 0$ is that the "pseudo-Hessian" symmetric matrix*

$$\mathcal{H} = G(V, r) + G^T(V, r)$$

be negative definite in V. Here the matrix $G(V, r)$ is the Jacobian with respect to V of $\rho(V, r)$.

Theorem 2 *If $\rho(V, r)$ is diagonally strictly concave for some $r \in I\!R_+ \backslash \{0\}$, then the Nash equilibrium point is unique.*

B Extensions

We present two lemmas useful for the proof of uniqueness of the equilibrium.

Lemma 3 *Consider two pairs of the payoff functions (π_1^1, π_2) and (π_1^2, π_2). Assume either pair satisfies the property of strict diagonal concavity. Define a new payoff function π_1 of the first player in the following way:*

$$\pi_1(V_1, V_2) = \begin{cases} \pi_1^1(V_1, V_2), & \text{for } V_1 \leq V_1^{cr} \\ \pi_1^2(V_1, V_2), & \text{for } V_1 \leq V_1^{cr} \end{cases} \tag{68}$$

and suppose that π_1 is continuously differentiable at V_1^{cr} for all V_2, and that the following property holds:

$$\frac{\partial \pi_1^1}{\partial V_1}(V_1, V_2) \geq \frac{\partial \pi_1^2}{\partial V_1}(V_1, V_2), \text{ for } V_1 \leq V_1^{cr}. \tag{69}$$

Then the property of strict diagonal concavity holds for the payoff pair (π_1, π_2).

Proof: Let $V = (V_1, V_2)$ and $V' = (V_1', V_2')$ be two pairs of strategies. We must verify that the following property is satisfied:

$$(V - V')^T g_{(\pi_1,\pi_2)}(V') + (V' - V) g_{(\pi_1,\pi_2)}(V) > 0 \tag{70}$$

$g_{(\pi_1,\pi_2)}(V)$ is the pseudo gradient of (π_1, π_2), *i.e.*

$$g_{(\pi_1,\pi_2)}(V) = \begin{pmatrix} \frac{\partial \pi_1}{\partial V_1}(V_1, V_2) \\ \frac{\partial \pi_2}{\partial V_2}(V_1, V_2) \end{pmatrix}.$$

Because each pair of the payoff functions (π_1^1, π_2) and (π_1^2, π_2) possesses the property of strict diagonal concavity, to prove the validity of inequality (70), it is sufficient to check it for V and V' located at the opposite sides of V_1^{cr} i.e., such that $V_1 \leq V_1^{cr} \leq V_1'$. We get

$$(V - V') g_{(\pi_1,\pi_2)}(V') + (V' - V) g_{(\pi_1,\pi_2)}(V) =$$
$$(V - V') g_{(\pi_1^2,\pi_2)}(V') + (V' - V) g_{(\pi_1^2,\pi_2)}(V) + (V_1' - V_1)\left(\frac{\partial \pi_1^1}{\partial V_1}(V_1, V_2) - \frac{\partial \pi_1^2}{\partial V_1}(V_1, V_2)\right) > 0$$

which terminates the proof.

◇

Lemma 4 *Let π_1^1, π_1^2 be two payoff functions of the first player and π_2^1, π_2^2 of the second player. Assume the pairs $(\pi_1^i, \pi_2^j), i, j = 1, 2$ satisfy the strict concavity assumption. Define the i-th player's ($i = 1, 2$) payoff function, π_i as*

$$\pi_i(V_1, V_2) = \begin{cases} \pi_i^1(V_1, V_2), & \text{if } V_i \leq V_i^{cr} \\ \pi_i^2(V_1, V_2), & \text{if } V_i > V_i^{cr} \end{cases}$$

and suppose that π_i^j are such that π_1 and π_2 are continuously differentiable, that the pairs of payoff functions (π_1^i, π_2^j), $i, j = 1, 2$ have the strict diagonal concavity property and that the following two inequalities, for $i = 1, 2$, hold:

$$\frac{\partial \pi_i^1}{\partial V_i}(V_1, V_2) \geq \frac{\partial \pi_i^2}{\partial V_i}(V_1, V_2), \text{ for } V_i \leq V_i^{cr} \tag{71}$$

Then the pair of payoffs (π_1, π_2) satisfies the strict diagonal concavity property.

Proof : Use the previous lemma twice.

◇

References

[1] "Assessment of the effects of the discharge of wastewater from the Waikanae sewage treatment plant on the environment", 1992. *Prepared for* Kapiti Coast District Council. Kingett Mitchell & Associates, Environmental Consultants. September 1992.

[2] BAŞAR T. & G.J. OLSDER, 1982, **Dynamic Noncooperative Game Theory,** Academic Press, London/New York.

[3] BURKHARD, L. P. & JENSON, J. J., 1993, in a Municipal Effluent", *Arch. Environ. Contam. Toxicol.*, **25**, 506-515.

[4] CARLSON, D., A. HAURIE, 1996, "A turnpike Theory for Infinite Horizon Open-Loop Competitive Processes', *SIAM J. Control & Optimization*, .., ...- .... [COMPLETE]

[5] DOMENICO P.A. & F.W. SCHWARZ, 1990, **Physical and Chemical Hydrogeology**, J. Wiley & Sons, New York *etc.*.

[6] BEAVIS, B., I.M. DOBBS, 1986, "The Dynamics of Optimal Environmental Regulation", *Journal of Economic Dynamics and Control*, 10, 415-423.

[7] CLEMHOUT, S., H.Y. WAN JR., 1985, "Dynamic Common Property Resources and Environmental Problems", *JOTA*, vol. 46, no 4, 471-481.

[8] FEINSTEIN C.D. & D.G. LUENBERGER, 1981, "Analysis of Asymptotic Behavior of Optimal Control Trajectories: the Implicit Programming Problem", *SIAM J. Control & Optimization*, 19, 561-585.

[9] HARTIGAN, J. & GEORGE, T., 1992, "The Rain That's Plain Goes Mainly Down The Drain", *American City & County*, September, 22-34.

[10] HAURIE A., 1993, *Environmental Coordination in Dynamic Oligopolistic Markets*, **Group Decision and Negotiation**, to appear.

[11] HAURIE A. & KRAWCZYK J., 1995, *A Game Theoretic Model of River Basin Environmental Management of Identifiable Source Pollution*, Working Paper **14/94** of the Graduate School of Business and Government Management, Victoria University of Wellington, 24 pages.

[12] HAURIE A., KRAWCZYK J., ROCHE M., 1994, "Monitoring Cooperative Equilibria in a Stochastic Differential Games", *JOTA*, vol 81, April, 73-95.

[13] HAURIE A., & ROCHE M., 1994, "Turnpikes and Computation of Piecewise Open-Loop Equilibria in Stochastic Deifferential Games", *JEDC*, vol , , .

[14] HAURIE A. & ZACCOUR G., 1995, *Differential Games Models of Global Environmental Management*, *Annals of Dynamic Games*, (**2**), volume title: **Game-Theoretic Models of the Environment.** Editors: Carraro, C. and Filar J. A.

[15] KANTARDIJEFF, A & JONES, J. P., 1993, "Removal of toxicity and some nonconventional pollutants by a dual power multicellular lagoon system", *Water Environment Research*, **65**, (7), 819-826.

[16] KRAWCZYK, J.B., 1995, "Management of Effluent Discharges: A Dynamic Game Model", *Annals of Dynamic Games*, (**2**), volume title: **Game-Theoretic Models of the Environment**. Editors: Carraro, C. and Filar J. A., 337-356.

[17] KRAWCZYK, J.B., 1996, "Modelling and Economics of Effluent Management in Municipalities", *In:* Modelling Change in Integrated Economic and Environmental Systems. *Editors:* S. Mahendrarajah, A.J. Jakeman and M.J. McAleer, J. Wiley & Sons, *forthcoming.*

[18] KRAWCZYK, J.B., 1995, "Controlling a Dam to Environmentally Acceptable Standards Through the Use of a Decision Support Tool", *Environmental and Resource Economics*, **5**(3), 1-18.

[19] KRAWCZYK, J.B., & G. ZACCOUR, 1995, "Pollution Management Through Levies and Subsidies", presented at the 1995 IFAC/IFORS/SEDC **Symposium on National & Regional Economies**, Gold Coast.

[20] KUSHNER, H.J., 1990, "Numerical Methods for Stochastic Control Problems in Continuous Time", *SIAM J. Control and Optimisation*, vol. 28, No 5, 999-1048.

[21] MARTIN, W.E., R.H. PATRICK & B. TOLWINSKI, 1993, "A Dynamic Game of a Transboundary Pollutant with Asymmetric Players", *Journal of Environmental Economics and Management*, vol. 24, 1-12.

[22] ROSEN J.B., 1965, "Existence and uniqueness of equilibrium points for concave N-person games", *Econometrica*, 520-534.

[23] RUBIN, D. H., POWERS M. B., SETZER S., BRADFORD H., 1992, "U.S. Faces a Draining Experience", *ENR*, vol. 229, Iss: 12, September, 34-38.

[24] SHALL, J., GELLER, R. & HORTON, N., 1993, "New Generation of Solid Waste Plants", *BioCycle*, January, 46-51.

[25] VAN DER PLOEG & DE ZEEUW A., 1992, "International Aspects of Pollution Control", *Environmental and resource Economics*, **2**, 117-139.

[26] B. TOLWINSKI, B. M. KENT & D.J. SHIELDS, 1995 *Models for Conflict Resolution in Ecosystem Management* Technical Report, Rocky Mountain Forest and Range Experiment Station, Fort Collins, CO.

[27] VAN ORDEN, G. N. & UCHRIN, C. G., 1993, "The study of dissolved oxygen dynamics in the Whippany Rever, New Jersey using the QUAL2E model", *Ecological Modelling*, **70**, 1-17.

[28] XEPAPEDAS, A.P., 1992, "Environmental Policy Design and Dynamic Non point-Source Pollution", *Journal of Environmental Economics and Management*, vol. 23, no 1, 22-39.

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