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A Note on the Decision of a Sales Maximizer in Response to the Increase of Per Unit Cost

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Abstract

In this paper the author compares the behaviors of a sales revenue maximizer with that of a profit maximizer in their response to an increase of the per unit production cost or a per unit tax being imposed. A mistake in a currently used textbook is pointed out, and a new proposition is proposed for replacing a false statement there.

Keywords: profit maximization, sales maximization, transformation, translation

JEL Classification: D21, M21

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A NOTE ON THE DECISION OF A SALES MAXIMIZER IN RESPONSE TO THE INCREASE OF PER UNIT COST

1. The Background and a False Statement

To compare the decision-making of a profit-maximizing firm with a sale-maximizing firm, in his well-known book "Economic Theory and Operation Analysis" [Baumol (1961, 1972, 1977)], Baumol suggested three main differences between these two types of firms' behaviors. Baumol's theoretical results have been very influential, and have been quoted by many textbooks. In Moschandreas (1994), these results are quoted as three predictions:

Prediction 1. Sales maximizers produce more compared with profit maximzers and charge a lower price for their product.

Prediction 2. Sales revenue maximizers will tend to spend more than profit maximizers on advertising and other sales promotional activities.

Prediction 3. Sales revenue maximizers will increase price and reduce production in response to increases in fixed cost or lump sum taxes.

Of course the above results are generally correct, and Moschandreas has given very good arguments for them together with elegant geometric explanations. In addition to the above results, Moschandreas (1994) tries also to examine the situation when not the fixed cost but the variable cost is increased, which has not been analysed in detail by Baumol. The result is presented as follows: [1]

Prediction 4. Sales maximizers will reduce output and increase price BY MORE THAN profit maximizers when variable costs increase or per unit taxes are imposed.

Unfortunately in general this proposition is false. In fact, in response to the increase of per unit cost, while two different types of firms all reduce their outputs, as to which type reduces more, it depends on the net profit function and the minimal profit contraint the sales maximizer subjected to, it may also depend on the magnitude of the change of the per unit cost.

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[1] Please refer to Moschandreas (1994), pp. 279-280.

In Section 2 we will construct a counter example against Prediction 4. In Section 3, we examine some special cases where Prediction 4 does hold.

2. A Counter Example

We now construct a counter example to show that Prediction 4 may be false. For simplicity, we choose piecewise smooth cost function in the example. However it is easy to see that the cost function can be made differenetiable in the whole range of its argument after some small modifications are made in the small neighborhoods of the corner points.

Assume the inverse demand function for the firm being

$$p = 5 - 0.01q$$

where q is the quantity produced by the firm and p is the market clearing price. Assume that this firm originally has the piecewise linear cost function

$$\mathrm{TC} = \begin{cases} q; & q \in [0, 1] \\ 4.75q - 3.75; & q \in [1, 5] \\ 7.5q - 17.5; & q \in [5, \infty) \end{cases}$$

It is easy to check that this cost function is convex. Please also note that, while there are two constants -3.75 and -17.5 appearing in the cost function, the fixed cost is actually zero. The piecewise linear structure of the cost function only reflects the fact that the marginal cost of production increases as the output increases from one interval to another interval. The cost function is depicted in Figure 1.

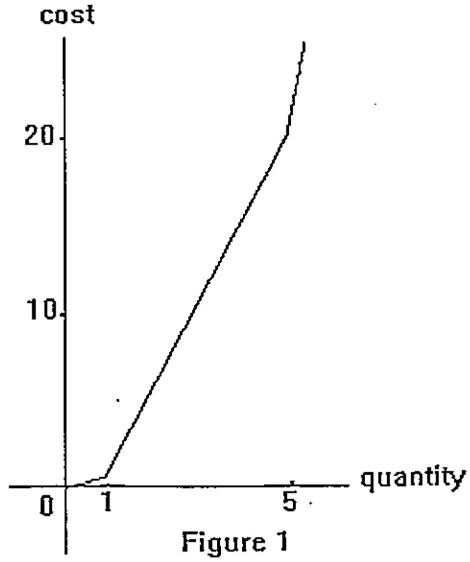

Figure 1

Now the profit function is given by

$$\pi = \begin{cases} 4q - 0.01q^2; & q \in [0, 1] \\ 3.75 + 0.25q - 0.01q^2; & q \in [1, 5] \\ 17.5 - 2.5q - 0.01q^2; & q \in [5, \infty) \end{cases}$$

It is not difficult to check that $\pi = \pi(q)$ is increasing in $[0, 1]$ and $[1, 5]$, but decreasing in $[5, \infty)$. Thus π attains its maximum at $q^* = 5$. So a profit maximization manager will produce a quantity of 5. In this case the profit is 4.75.

Consider the decision of a sales maximization manager, assuming that the minimum profit level is 2. It is easy to see that the revenue $R = (5 - 0.01q)q$ is being maximized at $q = 250$, with the corresponding profit $\pi(250)$ being less than 0. Thus under the minimal profit constraint, the quantity to be produced is determined by

$$17.5 - 2.5q - 0.01q^2 = 2$$

The only root in $[5, \infty)$ for the above equation is 6.05. Thus a sales maximizer will produce a quantity of 6.05.

We now consider the case that the per unit cost has an increment of 0.5. Thus for q to be produced, the increase of the variable cost is $0.5q$. The total cost function becomes

$$TC' = \begin{cases} 1.5q; & q \in [0, 1] \\ 5.25q - 3.75; & q \in [1, 5] \\ 8q - 17.5; & q \in [5, \infty) \end{cases}$$

The profit function now is given by

$$\pi' = \begin{cases} 3.5q - 0.01q^2; & q \in [0, 1] \\ 3.75 - 0.25q - 0.01q^2; & q \in [1, 5] \\ 17.5 - 3q - 0.01q^2; & q \in [5, \infty) \end{cases}$$

Now π' is increasing in $[0, 1]$, and decreasing in $[1, 5]$ and $[5, \infty)$. Thus the profit maximizer will produce a quantity of 1, earning a profit of 3.49.

For the sales maximizer, again the constaint $\pi \geq 2$ becomes a binding one:

$17.5 - 3q - 0.01q^2 = 2$

The only root in $[5, \infty)$ is 5.08. Therefore he will produce a quantity of 5.08.

To compare the above two situations, in response to the increase of the per umit cost, the profit maximizer has reduced his production by $5 - 1 = 4$, wheras the sales maximizer his reduced by only $6.05 - 5.08 = 0.97$. Obviously Prediction 4 is not true in this example! This is illustrated by Figure 2 below.

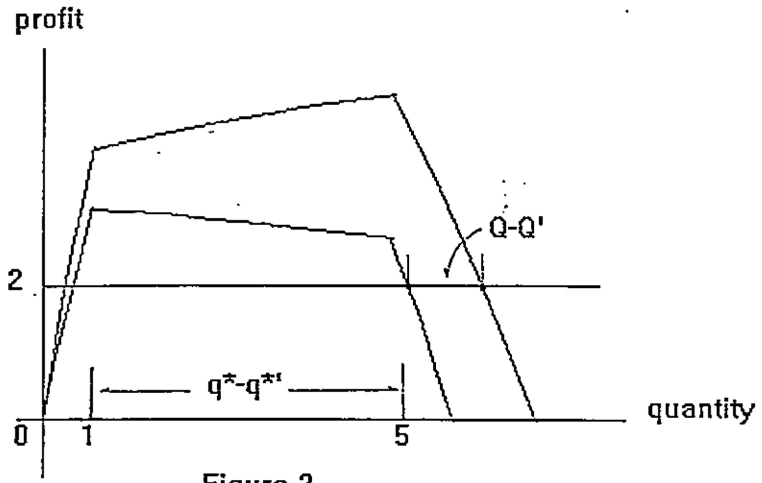

Figure 2

Remarks.

(1). The geometry implication behind this example, as we may have already seen, is that, if the profit curve ascends very slowly in a sufficiently large interval before it reaches the peak, then the transformation $(q,\pi) \rightarrow (q,\pi\text{-}kq)$ may cause a substantial change of the abcissa of the peak point.

(2). For a counter example with concave differentiable profit function, one can check $\pi = -(q-5)^4 + 5$. (say, with inverse demand $p = 500-q$, and TC = $q^4-20q^3+150q^2+620$). It is easy to verify that if the minimal profit required is 4, then any <u>small</u> increase of the per unit cost will result with a larger deduction of output by the profit maximizer than by the sales maximizer.

3. The Case with Quadratic Cost Function and Linear Demand Function

We have seen that Prediction 4 in general is false. But if we restrict our attention to some special classes of cost functions and demand functions, we may guarantee the result of Prediction 4. In particular, we do have

Prediction 4'. Assume that the firm has a linear inverse demand function $p = A - Bq$ and a quadratic total cost function $TC = C + Dq + Eq^2$, ($A > 0$, $B > 0$, $D > 0$, $C \geq 0$, $E \geq 0$ are all constants). Then a sales maximizer will reduce output and increase price BY MORE THAN *a profit maximizer when the per unit cost increases or a per unit tax is imposed.*

Proof. It is easy to derive the profit function

$$\pi = -C + (A-D)q - (B+E)q^2$$

For convenience we introduce the notations:

$$C = \gamma\,; \quad A\text{-}D = \beta\,; \quad B+E = \alpha$$

Then we can write

$$\pi = -\alpha q^2 + \beta q - \gamma$$

It is reasonable to assume that $\beta > 0$ and $\beta^2 > 4\alpha\gamma$ so that there does exist a range of q for positive profits. It is easy to compute the optimal quantity for profit maximization:

$$q^* = (2\alpha)^{-1}\beta \qquad (1)$$

which leads to a profit of $-\gamma+(4\alpha)^{-1}\beta^2$. For sales maximization with minimal profit level m ($0 < m < -\gamma+(4\alpha)^{-1}\beta^2$), the quantity should be chosen as

$$Q = (2\alpha)^{-1}\{\beta + [\beta^2 - 4\alpha(\gamma+m)]^{1/2}\} \qquad (2)$$

Now imagine the per unit cost is increased by δ, ($0 < \delta < \beta$) . The profit function then becomes

$$\pi' = -\alpha q^2 + (\beta-\delta)q - \gamma$$

The profit maximization output is now

$$q^{*\prime} = (2\alpha)^{-1}(\beta-\delta) \qquad (1')$$

the corresponding profit is $-\gamma+(4\alpha)^{-1}(\beta-\delta)^2$. which we still assume being greater than m. And the sales maximization output with minimum profit m is

$$Q' = (2\alpha)^{-1}\{(\beta-\delta)+[(\beta-\delta)^2-4\alpha(\gamma+m)]^{1/2}\} \tag{2'}$$

From (1), (2), (1') and (2'), one easily deduce

$$Q-Q' = (q^*-q^{*\prime}) + (2\alpha)^{-1}\{[\beta^2 - 4\alpha(\gamma+m)]^{1/2}-[(\beta-\delta)^2 - 4\alpha(\gamma+m)]^{1/2}\} \tag{3}$$

It is easy to see that the second term of the right-hand side in (3) is positive, from which we know $Q-Q' > q^*-q^{*\prime}$.
Q.E.D

As for the geometric interpretation, we have a graph as shown in Figure 3. On the (q,π)-plane, $\pi = \pi(q)$ is a parabola with the vertex at $((2\alpha)^{-1}\beta,-\gamma+(4\alpha)^{-1}\beta^2)$. It can be obtained by applying the translation $(x,y) \rightarrow (x + (2\alpha)^{-1}\beta, y - \gamma + (4\alpha)^{-1}\beta^2)$ to the parabola $\pi = -\alpha q^2$. Similarly, $\pi' = \pi'(q)$ is a parabola with the vertex at $((2\alpha)^{-1}(\beta-\delta),-\gamma+(4\alpha)^{-1}(\beta-\delta)^2)$. It can be obtained by applying the translation $(x,y) \rightarrow (x + (2\alpha)^{-1}(\beta-\delta), y - \gamma + (4\alpha)^{-1}(\beta-\delta)^2)$ to the same parabola $\pi = -\alpha q^2$. Thus, as is shown in Figure 3, we can obtain parabola $\pi' = \pi'(q)$ from the parabola $\pi = \pi(q)$ by applying the translation [2]

$$T : (x,y) \rightarrow (x - (2\alpha)^{-1}\delta, y - (4\alpha)^{-1}[\beta^2-(\beta-\delta)^2])$$

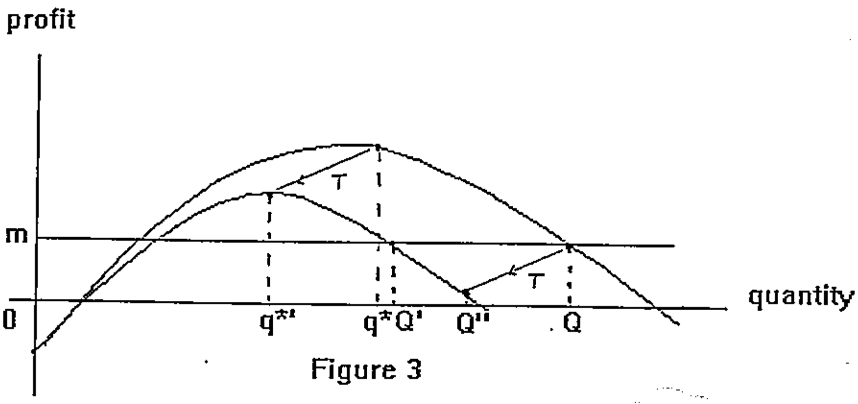

Figure 3

[2]. Though this translation T is different from the transformation $\tau : (q,\pi) \rightarrow (q,\pi-\delta q)$, (i.e. $T(x,y) \neq \tau(x,y)$), they give the same image curve.

Now from Figure 3, we see that

$$Q\text{-}Q' = (Q\text{-}Q'') + (Q''\text{-}Q') = (q^*\text{-}q^{*\prime}) + (Q''\text{-}Q') > q^*\text{-}q^{*\prime}$$

Here we have used the fact that $Q\text{-}Q'' = q^*\text{-}q^{*\prime}$, because they are all equal to the horizontal component $(2\alpha)^{-1}\delta$ of the translation T.

4. Conclusion

From the discussions in the last two sections, one can see that the mistake in Moschandreas (1994) occurs because the result only applying to a special case was claimed to be true for the general situation. Prediction 4 in Moschandreas (1994) might have been derived by a geometric graph similar to Figure 3. Unfortunately geometric intuition sometimes may be misleading, unless it is supported by logical reasoning.

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