The GSBGM Working Paper Series

WORKING PAPER 13/95

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ISSN 1173-4523
ISBN 0-475-11494-9

The Graduate School of Business and Government Management Working Paper Series 13/95 July 1995.
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Printed by The Victoria University of Wellington Printers.

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Stephen Burnell, Lewis Evans and Shuntian Yao

Abstract: This paper considers the performance of a market in which a network supplies retail firms with an essential input. There is imperfect competition in the retail market. The regulatory environment is one of light-handed regulation in which the same network contract must be offered to each retail firm. The optimal contract from the perspective of the network is characterised for Nash equilibria in three cases: where the network operates as a separate firm, where it operates as a conglomerate with devolved decision making to its retail division and where a conglomerate structure fully integrates the network and a retail firm. Because of a key characteristic of networks, it is shown that, given the possibility of bypass, unfettered oligopolistic competition under light handed regulation will generally be approximately efficient. These results are placed in the context of the literature concerned with regulating network industries.

JEL Classifications: C70, D43, L51

Acknowledgments: John McMillan and participants of the International Telecommunications Society Conference, Wellington, April 1995, made helpful comments on early versions of this paper.

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1. Introduction

The theory and practice of regulation have undergone significant change in the last decade. The possibilities and limitations of regulating near monopoly oligopolistic industries have evolved to reflect asymmetric information, principal-agent issues, rent seeking and the political economy of regulatory institutions. In the late 1970s Baumol, Bailey and Willig (1977) developed the concept of contestability and showed that under very special circumstances the threat of competition was sufficient to induce a monopolist to achieve a second-best welfare maximising outcome. Starting from the presumptions that regulators have their own objectives and that regulatory institutions reflect the political pressures of various constituencies, the public choice literature emphasises the effects of rent seeking, regulatory capture and balancing of special interest groups, for the efficiency of regulatory institutions.[1] The models of regulatory process have revealed the, usually very considerable, amount of information which the regulator should possess to be effective [see the review of Berg and Tschirthart (1988, Chs. 3 and 8), and the access pricing formulae of Laffont and Tirole (1994, 1679-1690)]. These strands of literature have been influential in that they have predicated significant changes in the regulatory policies of various countries. Usually these changes have led to reductions in regulation. Certain markets have been slow to change, however, and it remains common to have industry-specific regulation in network industries.[2]

Indeed, the increased questioning of the efficacy of regulation has predicated little change in the approach to analysing interconnection contracts in network industries. Many (for a recent example see Laffont and Tirole (1994)) start with the social planner's problem and maintain a significant role for an ever-present regulator. In contrast, our starting point is the operation of private markets, and we enquire into the efficiency of light-handed regulation.[3]

By light-handed regulation we mean that there is no price regulation at all, but owners of a network must offer access to any (potential) user on the same terms and conditions as any other

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[1] Examples are Peltzman (1976) and Weingast and Moran (1983).

[2] New Zealand is an exception. Excluding financial markets, it has no industry-specific regulatory bodies. In the example of the telecommunications market, players act with knowledge of the rules facing any firm under the Commerce Act 1986. The Act contains very significant penalties for actions restricting competition, and the ability to invoke price control. The incumbent is required to reveal essentially all customer prices, and to provide network access to entrants under transparent, reasonable terms and conditions. When the company was privatised the Government retained one share which had attached to it, the requirement, to provide a free residential local calling option, and to satisfy a price cap on the prices of residential local calls.

[3] The approach to regulation wherein any regulation should first be judged against the outcomes of private markets before any regulatory steps are taken is the philosophy on which New Zealand's industry regulation has been based since 1986.

(potential) user. Under this arrangement we characterise the privately-determined contract where one firm provides network services as an intermediate input to retail firms which require access to the network. The dominant firm owning the network also owns a retail firm. While the distinguishing characteristic of the network is that it owes its dominant position to high fixed, and low marginal costs, the network cost function is not constrained to this form. The retail firms may enjoy product differentiation while competing in the same market for final consumers. The optimal contract is determined in a one-shot game among the firms.

The issue of an optimal interconnect contract has long been the subject of work by Baumol and Willig whose (1991) "efficient component pricing" (ECP) rule determines a price for entry to the network.[4] Entry, is presumed to be at the expense of network services used by the incumbent. The ecp price will generally exceed average incremental cost by an amount to cover fixed costs, any cross-subsidisation imposed on the firm by regulation and loss of profit due to the entrant's use of the network. Indeed, setting aside the cross-subsidy issue, if Ramsey pricing by a natural monopoly firm is sustainable and the market is perfectly contestable the efficient and ecp price charged to any entering retail firm will simply be the prevailing price.[5] This presumes that the Ramsey network price can be separately identified, and this is, of course, very difficult where the incumbent also owns the downstream retail firm. We address the optimal pricing contract under alternative ownership structures where a conglomerate owns the network and a retail firm.

Empirical studies of deregulated industries suggest that actual entry, rather than potential entry, is important in affecting pricing behaviour.[6] Also, it is well-known that the preconditions for efficient, sustainable Ramsey pricing are exceedingly stringent (see Dixit (1982) and Brock (1983)). An outcome of our work is to suggest that there are particular characteristics of network industries which can generate outcomes normally associated with perfect contestability but in oligopolistic markets. Another is to suggest that there are certain characteristics of connection contracts for bypass which may be indicative of industry performance.

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[4] See also Laffont and Tirole (1993, ch.5).

[5] Waterson (1987, 69-70) provides a review of the links between Ramsey pricing, sustainability, contestability and efficiency. Laffont and Tirole (1994, 1693-1696) discuss the efficiency of the ecp rule.

[6] See Whinston and Collins' (1992) analysis of the airline industry.

2. The Model

Consider a retail industry with I (potential) firms. Let $p = [p_1, p_2,..., p_I] \in R^I_{++}$, be the vector of industry prices, and $q = [q_1, q_2,..., q_I] \in R^I_{++}$, the vector of industry outputs. Suppose the service demand functions for the retail firms are given by:

$$q_i = D_i[p], \text{ for all } i \in I.$$

Assumption 1: $D[p] = \{D_1[p],...,D_I[p]\}$ is one-to-one.

Hence, there exists $H[\cdot] = D^{-1}[\cdot]$, such that $p = H[q]$, and so $p_i = H_i[q]$. Further, $q_i = 0$, will be interpreted as equivalent to Firm i not entering the retail market, at least for the function, $H[\cdot]$, as this enables $H[\cdot]$ to be constructed independently of the number of firms that actually decide to enter the retail market.

Throughout this discussion we assume that Firm N provides the network and, further that a single institution owns both Firms 1 and N, which we refer to as the *conglomerate*. If Firm i decides to enter the market, then for it to produce q_i units of service the non-network cost is given by:

$$c_i = c_i[q_i] \text{ for all } i \in I.$$

Further, the amount of network required by retail Firm i to produce q_i units of service is given by:

$$g_i = g_i[q_i]$$

Assumption 2: The functions $c_i: R_+ \to R_+$ and $g_i: R_+ \to R_+$ are strictly increasing; in particular the inverse of $g_i[\cdot]$ exists.

Assumption 3: It is assumed that $H[\cdot]$, $c_i[q_i]$, and $g_i[q_i]$ are common knowledge (to all firms).

The contract, , $f[g_i(q_i)]$, specifies the payment which firm i makes to Firm N for the use of the network when i produces q_i, and it is described in

Definition 1: A network contract, $f: R_+ \rightarrow R_+$, is a continuous increasing function mapping any firm's usage of the network into a payment to Firm N.

Under light-handed regulation, Firm N is required to offer access to any firm under terms and conditions of the contract $f[\cdot]$. It will be apparent from the analysis which follows that if the conglomerate was not so constrained it would generally use the network contract to preserve a monopoly position in the retail market, especially where the demand functions of the retail firms are similar.

When $f[\cdot]$ is announced by Firm N, the I retail firms decide whether to enter the market or not. If Firm i enters, it chooses an output strategy, q_i, yielding a profit of:

$$\pi_i = H_i[q] \cdot q_i - c_i[q_i] - f[g_i(q_i)] \tag{1.1}$$

Now, given the network contract, $f[\cdot]$, let $I^*[f]$ be the set of firms with $q_i^*[f] > 0$, and the profits for Firm N can now be written as:

$$\pi_N = \sum_{i \in I^*[f]} f[g_i(q_i)] - c_N\{ \sum_{i \in I^*[f]} g_i[q_i]\}, \tag{1.2}$$

where $c_N[\cdot]$ is the cost function for Firm N. The profits for the conglomerate - Firms 1 and N combined - are then given by:

$$\pi_1 + \pi_N = H_1[q] \cdot q_1 - c_1[q_1] + \sum_{i \in I^*[f], i \neq 1} f[g_i(q_i)] - c_N\{ \sum_{i \in I^*[f]} g_i[q_i]\}. \tag{1.3}$$

Now, there are three basic ways that Firms 1 and N can be related to each other. We characterise these alternative institutional arrangements as:

Case 1: The contract shares network fixed costs between Firms 1 and 2 and its rate of usage cost is the marginal cost of the network, so $\pi_N = 0$.

Case 2: Firms 1 and N are completely independent; there is no conglomerate.

Case 3: the conglomerate exists, although Firm 1 acts as an independent entity when choosing q_1. That is, given $f[\cdot]$, Firm 1 maximises its profits, as defined in Equation (1.1). This will be referred to as a two-division conglomerate.

Case 4: the conglomerate exists, and Firm 1 recognises its place in the conglomerate when choosing q_1. That is, given $f[\cdot]$, the profits of Firms 1 and N, as defined in Equation (1.3), are maximised; This will be referred to as a one-division conglomerate.

Cases 1 and 2 are used as benchmarks in comparisons. Case 1 is the situation where the contract is the network cost function with the exception that the fixed costs are equally shared by the 2 retail firms, that is, where $f[g] = a/2 + b \cdot g$. In this case, each of the three ownership/control structures (no conglomerate, one-division conglomerate, two-division conglomerate), this network contract yields the same Nash equilibrium and consequently the same welfare.

Case 2 should also be viewed as a benchmark because it carries an implicit contract that Firm 1 is committed to use Firm N's network: there is no threat of Firm 1 building an alternative network or using any network built by Firm 2.

For each of the four cases, a Nash equilibrium - given $f[\cdot]$ - can now be defined. If there is no conglomerate, or a two-division conglomerate, a Nash equilibrium is given by an output vector, $q^*[f]$ = $\{q_i^*[f]\}$, such that i's output maximises π_i, given the output levels of the other firms. Further, let $p^*[f] = p_i^*[f]$, be the equilibrium price vector. If there is a one-division conglomerate - Case 4 -, a Nash equilibrium is given by an output vector, $q^*[f]$ = $\{q_i^*[f]\}$, such that i's output maximises π_i, for $i \neq 1$, while Firm 1's output maximises $\pi_1 + \pi_N$.

As i's profit function is common knowledge, each retail firm (as well as Firm N), can calculate the equilibrium, $q^*[f]$ and so decide whether or not to enter (remember, $q_i^*[f] = 0$ is equivalent to Firm i not entering the market, at least from the viewpoint of other retail firms). We will assume that $c_i[0]$ incorporates any rental cost of fixed capital, and so given $q_j[f]\ (j \neq i)$, i will enter only if there exists $q_i > 0$, yielding $\pi_i > 0$.

Notice, the Nash equilibrium can be interpreted as a function from the network contract, $f[\cdot]$, to the equilibrium quantities (and hence prices). As the only restriction (thus far) on $f: R_+ \rightarrow R_+$ is that it be continuous and increasing, the range of the equilibrium mapping, $q^*[f]$, has the potential to be quite a large subset of R_+^I.

Lemma 1. Consider the world with no conglomerate. Given the network contract, $f[\cdot]$, the set of Nash equilibria is the same as the world with a two-division conglomerate.

Assumption 4: $H_i[q]\cdot q_i$ is concave in q_i. Further, it achieves a maximum value at some finite value of q_i. Alternatively, consider the (vector) function, H: $R_+^l \rightarrow R_+^l$. There exists a compact subset of R_+^l, call it $\hat{Q}$, such that for all $q \notin \hat{Q}$, $D_{qi}\{H_i[q]\cdot q_i\} < 0$. That is, marginal revenue - for every retail firm - is negative if q lies outside the compact set, $\hat{Q}$.

This assumption is sufficient to ensure the problem of profit maximisation always has a (finite) solution, even if the cost function (and/or the network contract) is also concave in q_i.[7]

Proposition 1: Given assumptions 1-4, the contract $f[\cdot]$ and that each firm has a convex cost function at least one Nash equilibrium exists for each case.[8]

In general, the set of Nash equilibria depend upon the contract, and the contract itself need not be convex for an equilibrium to exist. The uniqueness of the equilibrium is of considerable interest, for without uniqueness the optimal contract cannot be defined. Uniqueness usually requires some kind of linearity in the reaction functions. Confining attention to the duopoly case which we consider in detail below, we have

Proposition 2: In the class of inverse demand functions, H_i, that are linear in (q_1, q_2), and the quadratic (and linear) firm total cost functions there exists a large set that yield unique Nash equilibria in each case, given the contract.[9]

3. Duopoly Under Linear Contracts

Suppose that Firm N is free to choose the network contract, $f[\cdot]$, without any government regulation excepting that the contract should be the same for all retail firms. For simplicity, suppose there are two retail firms. Irrespective of the variable network contract costs, the fixed cost component will be chosen so as to reduce Firm 2's profit to its lowest possible value. However,

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[7] Notice, without additional assumptions (possibly on the signs of the third-order derivatives of revenue and costs), there is no guarantee that Firm i's optimal strategy is a continuous function of the strategies of the other retail firms (given the network contract).

[8] By the convexity of costs and Assumption 4 each retail firms' profit function will be concave. Also, there will exist an upperbound Q for the q_i such that when the profit of firm i is negative. Thus there is a compact, convex, nonempty strategy set $[0, Q]$ for each firm.

[9] Proposition 2 for Cases 2 and 3 is demonstrated in Appendix 1.

Assumption 5: Firm 2 can build its own network with the same cost structure as that of Firm N.

provides an additional option for Firm 2 that will place an upper bound on the network's - Firm N's - ability to extract profit from Firm 2.

First, Firm N announces a contract, $f[\cdot]$. Firm 2 must then decide whether to enter the retail market or not (we will focus on the case where it does). Given that Firm 2 chooses to enter the retail market, it conjectures that Firm 1 will purchase its network services from Firm N; yielding Firm 1's best-response function. Firm 2 then calculates two Nash equilibria; one in which it builds its own network, and the other in which it purchases its network services from Firm N. The equilibrium with the highest profit level for Firm 2 then indicates whether Firm 2 should build its own network or not. Put another way, if Firm N wishes to ensure that Firm 2 does not build its own network, it must set $f[0]$ small enough.

Hence, Firm N will choose $f[\cdot]$ so as to maximise its objective function - either $\{\pi_1 + \pi_N\}$ or π_N, depending upon the conglomerate's organisational structure - subject to the constraint that Firm 2 will build its own network if $f[0]$ is too large. In the examples that follow, Firm N will choose $f[\cdot]$ so as to make Firm 2 indifferent between building its own network and purchasing from N.

We consider in detail duopoly under linear contracts starting with Case 1 which provides a benchmark in the evaluation of whether it is desirable to raise or lower the marginal cost of the network contract. That is, for each of the three cases we consider whether the network's owners will prefer the contract, $f^*[g] = A + B \cdot g$, to $f[g] = a/2 + b \cdot g$, where A and B (both assumed to be non-negative) are chosen so as to make Firm 2 indifferent between accepting the network contract and building their own network. The equations determining the Nash equilibria for this game are set out in Appendix 3. They establish

Proposition 3: Let $c_N[g] = a + b \cdot g$, and consider a network contract of the form, $f^*[g] = A + B \cdot g$, with initial values $[A, B] = [a/2, b]$. Then

$$\frac{d\pi_N}{dB} = g_1[q_1] \cong g_2[q_2] - 2 \cdot \{D_1 H_2[q] \cdot q_2\} \cdot \{\frac{d\hat{q}_1}{dB} - \frac{dq_1}{dB}\}$$

and if $g_1[q_1] \leq g_2[q_2]$, $D_1 H_2[q] \leq 0$, and $\frac{d\hat{q}_1}{dB} \leq \frac{dq_1}{dB}$, the network will (weakly) prefer $B \leq b$ to $B > b$, for B belonging to a small enough neighbourhood of b.

In Proposition 3, $\hat{q}_1$ is the level of Firm 1's output when Firm 2 has built a network and q_1 the level without the network. It gives sufficient conditions for the network not to make the usage charge exceed the marginal cost of network provision. If Firms 1 and 2 have the same demand functions then $g_1[q_1] \equiv g_2[q_2]$, and $D_1H_2[q] \leq 0$ because the services yielded will be substitutes. Moreover, if the demand function is linear it is straight forward to show that $\frac{d\hat{q}_1}{dB} < \frac{dq_1}{dB}$ which, from Proposition 3, establishes that the usage charge will be less than the marginal cost of the network in these circumstances.

Examples of duopoly under linear contracts are given in Table 1 and in Appendix 2. Consumer surplus is taken to be the area under the two demand curves - above the price. In these examples it is given by $CS = [0.5]\cdot\{[q_1]^2 + [q_2]^2\}$, and welfare is $W = CS + \pi_1 + \pi_2 + \pi_N$.

In the examples of Table 1 the optimal contract entails increasing the usage component and reducing the access component as institutional arrangements move from independent firms (Case 2) to an integrated conglomerate (Case 4). This occurs because of the conglomerate's desire to restrict output, especially that of Firm 2. In the fully integrated case it is free to raise the marginal contract payment to inhibit Firm 2's output. The contract is irrelevant for the conglomerate's decision making because it is simply a transfer payment within the company.

It is noteworthy that welfare, even consumers' surplus, can increase from Case 1 to Case 2. The linear demand functions which are the same for each retail firm fit the preconditions of Proposition 3: thus when the network Firm is free to choose the contract it moves away from the constrained Case 1 to a situation in which it chooses a usage charge rate which is less than the marginal cost of the network. In Examples 1 and 2, of Table 1 the extra output this generates produces a higher consumers' surplus in Case 2 than Case 1. In Example A2.1 of Appendix 2, we have asymmetric demand functions in that Firm 2's demand function has a lower constant term than that of Firm 1. Here we get a usage charge rate in Case 2 that exceeds the marginal cost of the network. Also, in that example we get declining welfare in the move from Case 1 to Case 2, but the highest welfare is attained in Case 4, the situation of the fully integrated conglomerate.[10]

[10] Maximum consumer surplus is attained in Case 1, although the difference from Case 4 is quantitatively minimal.

Table 1

Duopoly and Linear Contracts

Example 1.

$I=2$, $H_1[q]=100-q_1-[0.7]\cdot q_2$, $H_2[q]=100-q_2-[0.7]\cdot q_1$, $c_i[q_i]=100+[0.5]\cdot q_i$ $g_i[q_i]=q_i$, and $c_N[g]=500+2\cdot g$.

	Case 1	Case 2	Case 3	Case 4
f[g]	250 + [2]·g	583	328 + [6]·g	[16]·g
q_1	36	37	35	39
q_2	36	37	35	28
p_1	39	37	41	41
p_2	39	37	41	44
π_1	954	675	766	879
π_2	954	675	766	704
π_N	0	518	448	414
π_C	954	2147	1214	1293
Con. Surp.	1304	1358	1194	1156
Welfare	3212	3227	3172	3153

$\pi_C=\pi_1+\pi_N$

Example 2. Same as Example 1, except $c_N[g]=500+g$

	Case 1	Case 2	Case 3	Case 4
f[g]	250 + g	542	323 + [5]·g	[14]·g
q_1	36	37	35	39
q_2	36	37	35	29
p_1	38	37	41	41
p_2	38	37	41	44
π_1	981	716	794	907
π_2	981	716	794	731
π_N	0	510	446	414
π_C	981	1226	1240	1321
Con. Surp.	1331	1358	1217	1182
Welfare	3293	3300	3252	3234

$\pi_C=\pi_1+\pi_N$

Finally, notice that profits are present in all cases. This simply reflects the fact that the market is imperfect. But our comparison of welfare outcomes will follow the comparative institutional approach of comparing welfare outcomes, recognising that no state is first-best efficient. In a dynamic world without institutional restrictions on entry the profits of the retail firms can be expected to attract entry until they are much reduced, if not eliminated.

5. Bypass and the Equilibrium Contract

Thus far, the story could have been about any industry and any input (involving a high fixed cost). Examples (A2.3) and (A2.4) of Appendix 2 attempt to capture the idea that the network involves a high fixed cost but a very small marginal cost. However, they do not encapsulate the possibility that in many situations Firm 2 may construct a sub-network, thereby reducing its contractual payments to Firm N.[11]

Suppose $c_N[g] = a + b \cdot g$, and that Firm 2 constructs a bypass network that carries with it variable cost b and fixed cost $K^* \leq a$.[12] The proportion of the total network covered - by Firm 2 - is therefore given by K^*/a. Let $\delta[.]$ be the function that maps the physical proportion of the network built by Firm 2, namely K^*/a, into the proportion of the market actually used by the bypass network. Then we invoke

Assumption 6: Usage of Firm 2 and Firm N's network, respectively, are given by

$$\delta[K^*].g(q_2) \text{ and } (1-\delta[K^*]).g(q_2).$$

Thus the choice of K^* by Firm 2 will determine the distribution of Firm 2's network traffic between the bypass and Firm N's networks. In the case of bypass, Firm 1 is not constrained to pay Firm 2 for the use of any network constructed by the latter firm, and Firm 1 is restricted to the use of Firm N's network, as formerly. As will become apparent, the nature of $\delta[.]$ is critical to the equilibria of the network market. The approach is directly applicable to telecommunications markets, but it will apply to other network industries where bypass is feasible.

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[11] We assume that the firms have access to the same network technology at the same installed cost.

[12] This assumes that, whereas the extent of network coverage (duplication) is determined by the ratio of fixed costs K^*/a, the marginal cost of use of each network is the same. Arguments can be adduced for scaling down the marginal cost of the bypass network, in which case the threat of bypass will gain more force from that represented here. It could also be represented that by not reducing the marginal cost of the bypass network we have provided some recognition of interconnection costs not represented elsewhere in the model.

We need to distinguish between the bypass network's coverage of customers and the actual usage of the network.[13] Let the function $\gamma[K^*]$ map the physical proportion of the bypass network into the coverage obtained. It will reflect the fact that Firm 2 will optimise in constructing its network. Customers' volumes of use are far from identical and they are not randomly distributed over the network; in consequence Firm 2 will construct the bypass network to maximise network traffic volumes for any given K^*.[14] Thus $\gamma[K^*]$ will satisfy $\gamma[0]=0, \gamma[a]=1$, and be strictly increasing and concave in K^*. We use the specific form $\gamma[K^*]=(K^*/a)^{\alpha}$ for $\alpha \in (0,1)$.

Returning to the proportion of Firm 2's network use that uses the bypass, $\delta[K^*]$, consider the case where, given $\gamma[K^*]$, customers randomly contact other customers. In this event, the distribution of network use will be

$\gamma[K^*]^2$	within Firm 2's network, and
$[1-\gamma[K^*]]^2$	within Firm N's network, and
$2\gamma[K^*][1-\gamma[K^*]]$	between the networks.

Thus the proportion of Firm 2's network usage that employs bypass is

$$\begin{aligned}\delta[K^*,\alpha] &= \gamma[K^*]^2 + (K^*/a)2\gamma[K^*][1-\gamma[K^*]] \\ &= (K^*/a)^{2\alpha} + 2(K^*/a)^{\alpha+1} - 2(K^*/a)^{2\alpha+1}\end{aligned}$$

which is increasing, and for values of α greater than 1/2 defines

$$U \equiv \{K^*/a;\ \ \delta[K^*,\alpha] < K^*/a\}, \text{ and}$$
$$V \equiv \{K^*/a;\ \ \delta[K^*,\alpha] > K^*/a\}$$

where the sets are partitioned at the fixed point $(K^*/a)_{fp} = \{K^*/a; \delta[K^*,\alpha] = K^*/a\}$. The function $\delta[K^*,\alpha]$ is convex on the set U and concave on V. In the case where $\alpha = 0.8$, the fixed point occurs at $(K^*/a)_{fp} = 0.34$, and hence for bypass below 34 percent of the full network, coverage will be less that the proportion of the physical network built. Retaining $\alpha = 0.8$, when 40 percent of the network is bypassed 48 percent coverage is obtained and 43

[13] Here the term customer includes coverage of network traffic or volume.

[14] This optimisation problem poses an interesting direction for further work. Note that in telecommunications bypass is predominantly directed towards large volume customers such as those within and between central business districts.

percent of Firm 2's network usage can be handled by bypass. If $\alpha > 0.5$ then the set U is nonempty and this will predispose Firm 2 to build a bypass network of a size that at least reaches the fixed point if it builds at all. For $\alpha < 0.5$ the concavity of $\gamma[K^*]$ ensures that $\delta[K^*,\alpha]$ is concave for all $K^* \in [0,a]$ and that the set U is empty. If, for example, if $\alpha = 0.25$ and if 40 percent of Firm N's network is bypassed then 80 percent customer coverage is provided by bypass and 76 percent of Firm 2's usage will be via the bypass network.

Assumption 7: Given the contract, $f[g]$, if Firm 2 chooses a fixed cost of K^*, then its payment function is given by:

$$f[g,K^*] = \{\frac{a-K^*}{a}\} \cdot f[0] + f\{[1-\delta(K^*)] \cdot g\}$$

Firm N remains free to choose the contract $f[g]$, but there is an additional requirement that if an access fee is included it is rebated in proportion to the size of Firm 2's network relative to that of Firm N.[15] It is noteworthy that the equilibria which flow from Assumption 7 will be invariant to multiplication of $(a-K^*)/a$ by any positive scalar.

The sequence of events can now be imagined as follows. First, Firm N announces the contract, $f[g]$. Second, Firm 2 simultaneously chooses her fixed cost (or capital stock), K^*, and her retail output, q_2. Also, Firm 1 chooses her output level, q_1. It will be assumed that $[K^*,q_1,q_2]$ is chosen so as to yield a Nash equilibrium. As usual, the network contract, $f[g]$, is chosen so as to maximise profits - of either the network or the conglomerate - taking the equilibrium process into account.[16] Finally, in a world with no conglomerate both retail firms will choose a capital stock and an output level in response to the contract, $f[g]$. The proof of

[15] In some optimal contracts $f[0] = 0$ making the extra requirement problematical. It requires further exploration.

[16] The assumption that Firm 2 chooses K^* and q_2 simultaneously is made purely for technical convenience. For if K^* is chosen before retail output levels, Firm 2 must construct a function from K^* - and $f[g]$ - to the Nash equilibrium in outputs (thereby making the first-order conditions for an optimal value of K^* rather difficult to solve analytically).

Table 2

Linear Contracts in the Presence of Bypass

Example 3:

$I=2,\ H_1[q]=100-q_1-[0.7]\cdot q_2,\ H_2[q]=100-q_2-[0.7]\cdot q_1,\ c_i[q_i]=100+[0.5]\cdot q_i$
$g_i[q_i]=q_i,\ \ c_N[g]=500+2\cdot g$, and $\delta(K^*/a)=(K^*/a)^{0.5}$

	Case 1	Cases 2-4
$f[g]$	250 + 2·g	500 + 2·g
q_1	36	36
q_2	36	36
p_1	39	39
p_2	39	39
π_1	954	704
π_2	954	704
π_N	0	500
π_C	954	1204
Cons. Surp.	1304	1304
Welfare	3212	3212

$\pi_C=\pi_1+\pi_N$

Example 4:

$I=2,\ H_1[q]=100-q_1-[0.7]\cdot q_2,\ H_2[q]=100-q_2-[0.7]\cdot q_1,\ c_i[q_i]=100+[0.5]\cdot q_i$
$g_i[q_i]=q_i,\ \ c_N[g]=500+2\cdot g$, and $\delta(K^*/a)=(K^*/a)^{0.9}$

	Case 1	Cases 2-3	Case 4
$f[g]$	250 + 2·g	500 + 2·g	17·g
q_1	36	36	39
q_2	36	36	28
p_1	39	39	41
p_2	39	39	44
π_1	954	704	826
π_2	954	704	668
π_N	0	500	469
π_C	954	1204	1295
Cons. Surp.	1304	1304	1155
Welfare	3212	3212	3119
K*	0	0	34
δ	0	0	0.09

$\pi_C=\pi_1+\pi_N$

Proposition 4: Given a linear contract and Assumptions 1-7 there exists at least one Nash equilibrium in the presence of the possibility of bypass.

is presented in Appendix 3. The Nash equilibrium for this game and for Firm N's choice of a contract, is set out in Appendix 4. Two examples are provided in Table 2.

In Example 3 the proportion of Firm 2's traffic which uses Firm 2's network is approximated by $\delta[K^*,\alpha]=(K^*/a)^{\beta}$ where $\beta=0.5$.[17] The contract, $f[g]=a+bg$, is certainly optimal when $A\leq a$ (and $B\geq b$). To see that the integrated conglomerate will never set $\{A>a,B<b\}$ notice that for $B<b$, A will be chosen by the conglomerate to just ensure that Firm 2 does not build a bypass network, because if it pays firm 2 to build any network it will pay to build a complete network. Furthermore, B below b will result in larger output from Firm 2 at a cost of Firm 1 sales and profits. Hence, the integrated conglomerate will not choose $\{A>a,B<b\}$.

In Example 3 we see that the introduction of bypass has markedly reduced the market power of the conglomerate, and that its optimal contract entails setting the usage charge rate at network-marginal cost to all retail firms, and there is no bypass. In this event, all of Cases 2-4 collapse to yield the same outcome: namely the relatively efficient outcome of Case 1 in Example 1. This occurs because output decisions are based on the same marginal cost in each case.

The network profit stems from a higher access fee than that required to cover its fixed costs, but it is not high enough to induce bypass. To examine whether this outcome of our game is specific to demand and cost parameters assumed in Example 3 we explore alternatives. Under the scenarios of Table 3 we get exactly the same qualitative conclusions as Example 3: the integrated conglomerate privately chooses the relatively efficient contract.

There will exist specific parameter values which yield contracts, chosen by the conglomerate, which differ from pricing usage at the marginal cost of the network. In particular, as the exponent of $(K^*/a)^{\beta}$ approaches 1 we would expect the contract to revert to Case 4 of Example 1, and, indeed, Example 4 demonstrates this outcome. As β approaches 1 coverage of network traffic approaches the proportion of the physical network covered, thus increasing

[17] This approximation has been used for analytical convenience. It represents the envelope of $\delta[k^*,\alpha]$ and it captures the increasing and concave nature of this function for $\alpha\leq 1/2$, but the approximation will not enable representation of the set U. The effect of this is to make building a small $\{i.e.\ (K^*/a)\in U\}$ bypass network somewhat more profitable for Firm 2 (the sets U and V having been defined in the text). However, as practical matter we expect $\alpha\leq 1/2$, and as a consequence U to be empty.

Table 3
Specified Departures From Example 3

i) Cost:

A lower network marginal cost as in $c_N[g] = 500 + g$.

ii) Demand:

Asymmetric Demand:	$H_1[q] = 100 - 0.7q_2 - q_1$ $H_2[q] = 85 - q_2 - 0.7q_1$
Reduced Substitutability:	$H_1[q] = 100 - 0.4q_2 - q_1$ $H_2[q] = 100 - q_2 - 0.4q_1$
Increased Substitutability:	$H_1[q] = 100 - 0.9q_2 - q_1$ $H_2[q] = 100 - q_2 - 0.9q_1$

iii) Coverage:

$\delta[K^*] = (K^*/a)^{0.8}$

the cost to Firm 2 of bypass.[18] It is likely, however, that, particularly for low values of K^*/a, small proportions of the physical network will cover large proportions - in our examples represented by $\beta < 1/2$ - of the traffic, making the threat of bypass to the conglomerate credible.

A comparison of Case 3 in Examples 3 and 4 reveals the importance of strategic reactions between the firms. Although Firm 2's profit is greater in Case 3 of Example 3, where Firm 2 essentially faces the network cost function, than in Example 4, the situation of the latter example is indeed a Nash equilibrium given the contract. If Firm 2 did behave as if in Example 3 then Firm 1 would react by producing 36 (as in Example 3), but it would then not be rational for Firm 2 to build a complete network given the network contract 17g: it would pay to use the conglomerate's network rather than build a second network. It can be shown that 17 is the marginal cost to Firm 2 of an additional unit of network so that K*=34 is the optimal amount of bypass network. The network contract is chosen as if the conglomerate is a Stackleberg leader, and if this assumption is relaxed then other contracts might emerge. In Example 4, the resulting equilibria may include that of Example 3.

5 Duopoly and Nonlinear Contracts

The contracts we have considered so far have been linear, and under bypass these have optimally included an access fee. It is natural explore a wider class of contracts to assess the consequent outcomes and the importance of an access fee. We restrict attention to contracts of the form:

$$f[g] = A + B \cdot g + C \cdot g^{\phi};\ \ A \geq 0, B \geq 0, C \geq 0, \phi \in (0,1)$$

This contract space confers more instruments on Firm N, and hence the conglomerate, than do linear contracts, and in consequence the conglomerate can be expected to produce larger profits. The choice of the optimal contract is complicated by the fact that the functional form of the cost function may also be generalised. Indeed, to provide a long-run marginal cost - as many analyses require - of the network when the network cost function is linear requires approximating $c_N[g] = a + bg$ with an increasing, concave function. The addition of nonlinear cost functions and contracts to our model introduces nonlinear reaction functions and increased

[18] In our standard example (as specified in Example 3) the integrated conglomerate switches to a contract with a usage charge rate which exceeds network marginal cost when $\beta > 0.8$.

possibility of non-unique Nash equilibria.

We restrict analysis to the case where bypass is feasible. First, suppose that the network cost function has high fixed and low marginal cost, then it will be possible to mimic the relatively efficient contract of bypass in, say, Example 3, with a nonlinear contract which has no access fee, of the form $f[g] = B \cdot g + C \cdot g^{\phi};\ 0, B \geq 0, C > 0$ where $\phi \in (0,1)$ is sufficiently small for the second term to act as an approximate access fee. Such a contract may have practical uses but, for the evaluation of light-handed regulation the optimal contract for the conglomerate is much more useful: the linear contract equilibrium may not obtain when the contract space is generalised.

We now turn to the determination of the optimal contract under bypass using the basic symmetric demand function of Example 3, and searching over $\{A, B, C, \phi\}$ in the contract. Because the network coverage function is concave and bypass will be carried out until the cost saving from another unit of bypass equals the cost of this investment, a concave contract will inhibit larger amounts of bypass and duplication of the network. In addition, because of the interaction between the demand functions a profit-maximising conglomerate will want to restrict Firm 2's output . Its tool for this purpose is the marginal payment of the contract, that is, $\partial f[g] / \partial g$: the higher is this quantity the lower will be output of Firm 2.[19] These two factors lead to there being no access fee, thus we maintain $A = 0$.

Further bounds on the contract should be established to prevent the entrant committing to building a parallel network. The situation differs as between the one and two division conglomerates. The conglomerate's two division and one division, respectively, choices of output are determined by the first-order conditions, as

$$q_1^{3*} = \{q_1;\ H_1(q_1) - q_1 - D_{q_1} c(q_1) - D_{q_1} f[g(q_1)] = 0\} \text{ and}$$

$$q_1^{4*} = \{q_1;\ H_1(q_1) - q_1 - D_{q_1} c(q_1) - b D_{q_1} g(q_1) = 0\}.$$

The sequence of moves is that the conglomerate announces a contract and Firm 2 determines whether, given that contract, it will commit to build a complete network. If it has the incentive to commit

[19] Subject to the requirement that total contract payments are not large enough to induce Firm 2 to build its own network.

Table 4
Nonlinear Contracts in the Presence of Bypass

Example 5:

$I=2,\ H_1[q]=100-q_1-[0.7]\cdot q_2,\ H_2[q]=100-q_2-[0.7]\cdot q_1,$
$c_i[q_i]=100+[0.5]\cdot q_i g_i[q_i]=q_i,\ c_N[g]=500+2\cdot g,$ and $\delta(K^*/a)=(K^*/a)^{0.5}$

	Case 3	Case 4
$f[g]$	$190g^{.31}$	$128g^{.43}$
q_1	35	37
q_2	35	33
p_1	41	40
p_2	41	41
π_1	728	748
π_2	734	680
π_N	492	499
π_C	1220	1247
k	8.2	24
δ	0.13	0.22
Cons. Surp.	1221	1240
Welfare	3175	3167

$\pi_C=\pi_1+\pi_N$

to build the full network the conglomerate will adjust the contract to (just) eliminate the incentive. Thus we require that the contracts satisfy

for Case 3: $\pi_2(q_1^{3*}, F[.], K^* \in [0,a)) \geq \pi_2(q_1^{3*}, F[.], K^* = a)$, and
for Case 4: $\pi_2(q_1^{4*}, F[.], K^* \in [0,a)) \geq \pi_2(q_1^{4*}, F[.], K^* = a)$.

The conglomerate imposes these constraints in choosing $\{B, C, \phi\}$.

The optimal contracts are described in Table 4 where, for the two-division conglomerate $\{B = 0, C = 190, \phi = 0.31\}$ and for the integrated conglomerate $\{B = 0, C = 128, \phi = 0.43\}$. Both contracts carry heavy discounting: in the example of the integrated conglomerate a 10 per cent increase in network usage yields a 5.7 percent reduction in the average cost of a unit of network services. Under both structures there is some bypass. It is interesting that the integrated conglomerate produces the larger consumer surplus, but has a lower total welfare, in part resulting from the greater investment in bypass. The larger consumer welfare results from the use of the marginal contract payment to restrict Firm 2's output. In the two-division conglomerate it also reduces Firm 1's output. The fact is that without an access fee the whole cost of the network has to be met by usage charges, and this makes the usage charge rate higher than in the linear contract case. The wider class of contracts permits the conglomerate to make higher profits in the nonlinear contract cases. However, the absence of an access charge also means that Firm 2 does better under nonlinear than linear contracts in the two-division conglomerate case.

Given the demand and retail cost functions, the linear network cost function of our example offers most advantage to the conglomerate to exercise market power. The cost function of a network industry cannot be expected to be convex - that is, have increasing marginal cost. Also, the increasing, concave, network-traffic coverage function means that the conglomerate will never choose a convex network contract. Thus the design of the concave contract trades off the degree of concavity of the cost function against that of the network coverage function. The linear cost function of our examples is the least concave cost function that can be expected in network industries. It offers the conglomerate the sharpest difference in marginal cost (payment) between the network and the contract: and thus the most scope to cause a divergence between the two retail firm's output levels. Even here however there is little welfare or consumers' surplus lost by the private determination of the contract.[20] If the network cost

20 The fall in consumers' surplus falls of just 4.9 percent can be attributed to the chosen absence of an access fee rather than reduced output in the case of the integrated conglomerate. Notice that consumers' surplus actually

function was strictly concave then the welfare cost of any privately determined strategic pricing of the network services will carry even less welfare cost under bypass.[21] We conjecture that the network contract chosen by the integrated conglomerate will more closely approximate the network cost function as the degree of concavity of this function increases, thus leading to even less efficiency loss in the private determination of network contracts under bypass.

6 Regulation and Strategic Games

Our game theory analysis of imperfect competition in network industries presumes only light regulation wherein all firms must be offered the same contract for use of the network, and, in the case of potential bypass, that the contract should have the feature that any access fee be abated as bypass grows as a proportion of the incumbent's network. To the extent it provides a theory of regulation it is a positive theory because it predicts the behaviour of market participants in the context of light regulation, and it does not require industry-specific subsidisation or taxation. Our comments are necessarily drawn from the examples and work is continuing to extend their domain.[22] Nevertheless, they do illustrate certain points.

Without bypass, as the structure moves from firms behaving independently to integrated control by a conglomerate there is a redistribution of profits among the firms and consumers' surplus declines. The optimal contract chosen by the network shifts from a high access fee and low per-unit usage cost to one in which the usage charge is high for the integrated conglomerate as it chokes off the second firm's supply of output. The high usage charge is irrelevant to the conglomerate's retail firm as it is simply a transfer payment within the conglomerate and this firm will not base decisions upon it.[23]

This situation changes completely with the advent of bypass. Our analysis of bypass is particularly germane to telecommunications, but will be applicable to other industries where bypass is feasible. Bypass enables other retail firms to gain access to a larger proportion of

increases between Case 3, where both retail firms face the same marginal cost, and Case 4.

[21] When commentators refer to long-run marginal cost in network industries they must have in mind an increasing strictly concave cost function. We reason that if our fixed and variable cost function was approximated by a strictly concave cost function that there would be much reduced scope for the integrated conglomerate to write a contract which induces different output levels for firms with symmetric demand and cost functions.

[22] Furthermore, our model, in common with virtually all access pricing and evaluations of ecp, does not incorporate dynamic adjustment and concomitant strategic decisionmaking. There are natural extensions in the timing of network investments vis-a-vis that of output choices which we are following up. We conjecture that they will leave our general conclusions intact.

[23] Of course this would appear as a poor profit performance of the retail division.

network traffic than the proportion of the physical coverage of the network provided by bypass. In this case we get a sort of contestability result in an oligopolistic game with only two entities: the conglomerate and Firm 2. Under linear contracts the conglomerate does raise the access fee over and above that which would cover the network's fixed costs, and thus the network makes a profit, but the usage charge is set at the marginal cost of the network, there is no bypass, and welfare and consumers' surplus are maximised given the imperfect nature of competition. We conjecture that this result will hold for a wide class of retail firm cost and demand functions, in particular where these functions are likely to be similar across retail firms, as in telecommunications. We are exploring the generality of this conclusion.

Baumol and Willig's (1991) ecp principle is simply a pricing rule that yields the efficient price in certain circumstances. As indicated earlier, it is that a price should be charged for the network which covers average incremental cost of provision of the network services plus an allowance to cover opportunity costs from lost output, fixed costs, and cross-subsidisation obligations.[24] The rule has been most controversial, mainly on the basis that to equate the efficient price with the ecp as measured by existing prices struck for the network is to charge for any surplus profits, or inefficiencies characterising the network.[25] Much attention has been focussed on these issues within the context of network bottlenecks.[26] Our work suggests that where bypass is feasible bottlenecks become irrelevant: there will be little in the way of surplus profits and the network owner will have every incentive to reduce costs. Furthermore the threat posed by bypass means that if there are surplus profits to be generated they will not be imposed in the usage charge of linear contracts: it will be set at the efficient level, namely the marginal cost of the network.

Our work suggests that there may be efficiency conclusions which can be drawn from the shape of the contracts themselves, without investigating prices or costs. The absence of an access fee when it is known that the network cost function consists of fixed costs and constant marginal costs suggests a contract which is less in the interest of consumers than it is in the interest of both the conglomerate and retail firms.

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[24] In the presence of other retail suppliers the ecp should cover its incremental cost plus the opportunity cost incurred when a rival supplies the retail product.

[25] This was the basis of the challenge of Clear Communications Ltd. (the entrant) to Telecom Corporation of New Zealand Ltd's network pricing rule (*Clear Communications, Ltd. v Telecom Corporation of New Zealand, N.Z. High Court, Decision Dec. 22 1992, Court of Appeal, Decision Dec 17 1993, Privy Council, Decision Oct. 1994*). The case went as far as the Privy Council where the ecp rule was upheld, the ruling being that the (potential) existence of monopoly profits an network inefficiencies were issues separate from the rule, and that, in particular, monopoly profits had to be demonstrated for the Clear case to be supported.

[26] Tye and Lapuerta (1995) make the notion of a bottleneck a key part of their critique of ecp.

An interesting feature of the discipline of bypass is that it renders the institutional structures of cases equally efficacious from all perspectives; but particularly that of the conglomerate. Given the possibility of bypass, optimal linear contracts make operation in two divisions and as an integrated conglomerate yield the same outcome, and this will approximately hold for nonlinear contracts in many situations. Where there is no market advantage in vertical integration the conglomerate may then choose that organisational structure which delivers, and integrates the delivery of, products or services at low cost. It may be that this has been anticipated in telecommunications. Zielinski (1995) reports that Rochester Telephone's Open Market Plan has the objectives of supplying customers with integrated services at low cost and that it entails no vertical integration: instead, it calls for the breakup of their conglomerate into stand-alone businesses sharply focussed on their customers. The Company's restructuring has led the regulators to introduce changes to the regulatory environment.

An active regulator faces additional conundrums. For example, in the absence of bypass our analysis of duopoly provides examples where welfare, even consumers' surplus, need not be a maximum under Case 1 where the network is forced to earn zero profits. This arises because, where the retail firms have very similar demand functions, the network makes more profit out of charging a usage fee which is less than network marginal cost. The extra output thus generated generates extra profit and improvements in consumers' surplus.

A positive analysis of more active regulation would require that the regulator be incorporated in the game. In this context, it is apparent from all the examples we have considered that the major differences between the various cases are the distributions of rents between companies and between profits and consumers. Even in the case of effective bypass there exist inter-company rents or profits which will provide an incentive for companies to devote resources to rent seeking. In this environment the regulator is part of the game, thus introducing the possibility of companies using the regulator to reallocate rents. A regulator will use resources, and, in the case of effective bypass, at best effect no change in consumer welfare.

Appendix 1: Uniqueness

For simplicity only we consider the duopoly case. The following proof can be easily generalised to the case of n retail firms.

Let firm i's inverse demand and cost functions, respectively, be

$$H_i(q) = A_i - B_i q_i - C q_j \quad and \quad T_i(q_i) = D_i + E_i q_i + F_i q_i^2$$

where the (A_i, B_i, E_i) are positive and (C_i, D_i, F_i) are non-negative. Profits for firm i are then

$$\pi_i = -(B_i + F_i)q_i^2 + (A_i - E_i - C_i q_i)q_i - D_i$$

and its reaction function is

$$q_i = -(A_i - E_i - C_i q_j) / [2(B_i + F_i)].$$

Combining the two reaction functions yields

$$\begin{bmatrix} 2(B_1 + F_1) & C_1 \\ C_2 & 2(B_2 + F_2) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} A_1 - E_1 \\ A_2 - E_2 \end{bmatrix}$$

which has the unique solution

$$q_i^* = [2(A_i - E_i)(B_j + F_j) - (A_j - E_j)C_i)] / [4(B_i + F_i)(B_j + F_j) - C_i C_j] \quad i, j = 1,2, i \neq j$$

when $B_i \geq C_i$. Obviously there is a large set of parameter values for which positive outputs are produced.

Appendix 2: duopoly and linear contracts

Case 2: No conglomerate.

Given that Firm 2 does not build a network:

$$\pi_1 = H_1[q]\cdot q_1 - c_1[q_1] - A - B\cdot g_1[q_1]$$
$$\pi_2 = H_2[q]\cdot q_2 - c_2[q_2] - A - B\cdot g_2[q_2]$$
$$\pi_N = 2A - a + [B-b]\cdot[g_1(q_1) + g_2(q_2)]$$

The Nash equilibrium is given by the first-order conditions:

$$\frac{d\pi_1}{dq_1} = D_1H_1[q]\cdot q_1 + H_1[q] - D_1c_1[q_1] - B\cdot D_1g_1[q_1] = 0 \qquad \text{(A2.1.1)}$$
$$\frac{d\pi_2}{dq_2} = D_2H_2[q]\cdot q_2 + H_2[q] - D_2c_2[q_2] - B\cdot D_2g_2[q_2] = 0 \qquad \text{(A2.1.2)}$$

Next, if Firm 2 decides to build their own network, the Nash equilibrium is given by, $[\hat{q}_1, \hat{q}_2]$, such that:

$$\frac{d\hat{\pi}_1}{d\hat{q}_1} = D_1H_1[\hat{q}]\cdot \hat{q}_1 + H_1[\hat{q}] - D_1c_1[\hat{q}_1] - B\cdot D_1g_1[\hat{q}_1] = 0 \qquad \text{(A2.1.3)}$$
$$\frac{d\hat{\pi}_2}{d\hat{q}_2} = D_2H_2[\hat{q}]\cdot \hat{q}_2 + H_2[\hat{q}] - D_2c_2[\hat{q}_2] - b\cdot D_2g_2[\hat{q}_2] = 0 \qquad \text{(A2.1.4)}$$

Now, as B rises, equations (A2.1.1) and (A2.1.2) explain how $[q_1,q_2]$ adjusts, while equations (A2.1.3) and (A2.1.4) describe how $[\hat{q}_1,\hat{q}_2]$ adjusts. Further, A adjusts so as to ensure $\pi_2 = \hat{\pi}_2$. Hence, when $[A,B] = [a/2, b]$:

$$\frac{d\pi_N}{dB} = g_1[q_1] + g_2[q_2] + 2\cdot\{\frac{dA}{dB}\} + [B-b]\cdot\{D_1g_1(q_1)\cdot[\frac{dq_1}{dB}] + D_2g_2(q_2)\cdot[\frac{dq_2}{dB}]\}$$

where

$$\frac{d\pi_2}{dB} = \{D_1H_2[q]\cdot q_2\}\cdot\{\frac{dq_1}{dB}\} - \{\frac{dA}{dB}\} - g_2[q_2] = \frac{d\hat{\pi}_2}{dB} = \{D_1H_2[q]\cdot q_2\}\cdot\{\frac{d\hat{q}_1}{dB}\}$$

and so $\frac{dA}{dB} = -g_2[q_2] - \{D_1H_2[q]\cdot q_2\}\cdot\{\frac{d\hat{q}_1}{dB} - \frac{dq_1}{dB}\}$.

This then yields:

$$\frac{d\pi_N}{dB} = g_1[q_1] - g_2[q_2] - 2\cdot\{D_1H_2[q]\cdot q_2\}\cdot\{\frac{d\hat{q}_1}{dB} - \frac{dq_1}{dB}\}$$

which establishes Proposition 2 of the text.

Case 2: A two-division conglomerate.

The only difference between cases 1 and 2 is that the network contract is now chosen so as to maximise $\pi_1 + \pi_N$, rather than solely π_N. Firm 1 continues to base decisions on the network contract. Given that Firm 2 does not build a network:

$$\pi_1 = H_1[q]\cdot q_1 - c_1[q_1] - A - B\cdot g_1[q_1]$$
$$\pi_2 = H_2[q]\cdot q_2 - c_2[q_2] - A - B\cdot g_2[q_2]$$
$$\pi_1 + \pi_N = H_1[q]\cdot q_1 + [B-b]\cdot g_2[q_2] + [A-a] - c_1[q_1] - b\cdot g_1[q_1]$$

The Nash equilibrium is given by the first-order conditions:

$$\frac{d\pi_1}{dq_1} = D_1H_1[q]\cdot q_1 + H_1[q] - D_1c_1[q_1] - B\cdot D_1g_1[q_1] = 0 \qquad \text{(A2.2.1)}$$
$$\frac{d\pi_2}{dq_2} = D_2H_2[q]\cdot q_2 + H_2[q] - D_2c_2[q_2] - B\cdot D_2g_2[q_2] = 0 \qquad \text{(A2.2.2)}$$

Next, if Firm 2 decides to build their own network, the Nash equilibrium is given by, $[\hat{q}_1,\hat{q}_2]$, such that:

$$\frac{d\hat{\pi}_1}{d\hat{q}_1} = D_1H_1[\hat{q}]\cdot \hat{q}_1 + H_1[\hat{q}] - D_1c_1[\hat{q}_1] - B\cdot D_1g_1[\hat{q}_1] = 0 \qquad \text{(A2.2.3)}$$
$$\frac{d\hat{\pi}_2}{d\hat{q}_2} = D_2H_2[\hat{q}]\cdot \hat{q}_2 + H_2[\hat{q}] - D_2c_2[\hat{q}_2] - b\cdot D_2g_2[\hat{q}_2] = 0 \qquad \text{(A2.2.4)}$$

Now, as B rises, equations (A2.2.1) and (A2.2.2) explain how $[q_1,q_2]$ adjusts, while equations (A2.2.3) and (A2.2.4) describe how $[\hat{q}_1,\hat{q}_2]$ adjusts. Further, A adjusts so as to ensure $\pi_2 = \hat{\pi}_2$. Hence, when $[A,B]=[a/2,b]$ (using equation (A2.2.1) to rewrite the coefficient for $\{\frac{dq_1}{dB}\}$):

$$\frac{d\{\pi_1+\pi_N\}}{dB} = [B-b]\cdot D_1g_1[q_1]\cdot\{\frac{dq_1}{dB}\} + \{D_2H_1[q]\cdot q_1 + [B-b]\cdot D_2g_2[q_2]\}\cdot\{\frac{dq_2}{dB}\} + g_2[q_2] + \{\frac{dA}{dB}\}$$

where $\frac{dA}{dB} = g_2[q_2] - \{D_1H_2[q] \cdot q_2\} \cdot \{\frac{d\hat{q}_1}{dB} - \frac{dq_1}{dB}\}$. This then yields:

$$\frac{d\{\pi_1 + \pi_N\}}{dB} = \{D_2H_1[q] \cdot q_1\} \cdot \{\frac{dq_2}{dB}\} - \{D_1H_2[q] \cdot q_2\} \cdot \{\frac{d\hat{q}_1}{dB} - \frac{dq_1}{dB}\}.$$

Case 4: A one-division conglomerate.

Given that Firm 2 does not build a network:

$$\pi_1 + \pi_N = H_1[q] \cdot q_1 + [B - b] \cdot g_2[q_2] + [A - a] - c_1[q_1] - b \cdot g_1[q_1]$$
$$\pi_2 = H_2[q] \cdot q_2 - c_2[q_2] - A - B \cdot g_2[q_2]$$

The Nash equilibrium is given by the first-order conditions:

$$\frac{d\{\pi_1 + \pi_N\}}{dq_1} = D_1H_1[q] \cdot q_1 + H_1[q] - D_1c_1[q_1] - b \cdot D_1g_1[q_1] = 0 \quad \text{(A2.3.1)}$$
$$\frac{d\pi_2}{dq_2} = D_2H_2[q] \cdot q_2 + H_2[q] - D_2c_2[q_2] - B \cdot D_2g_2[q_2] = 0 \quad \text{(A2.3.2)}$$

Next, if Firm 2 decides to build their own network, the Nash equilibrium is given by, $[\hat{q}_1, \hat{q}_2]$, such that:

$$\frac{d\{\hat{\pi}_1 + \hat{\pi}_N\}}{d\hat{q}_1} = D_1H_1[\hat{q}] \cdot \hat{q}_1 + H_1[\hat{q}] - D_1c_1[\hat{q}_1] - b \cdot D_1g_1[\hat{q}_1] = 0 \quad \text{(A2.3.3)}$$
$$\frac{d\hat{\pi}_2}{d\hat{q}_2} = D_2H_2[\hat{q}] \cdot \hat{q}_2 + H_2[\hat{q}] - D_2c_2[\hat{q}_2] - b \cdot D_2g_2[\hat{q}_2] = 0 \quad \text{(A2.3.4)}$$

Notice, $[\hat{q}_1, \hat{q}_2]$ is independent of the choice of $[A, B]$ and so, as B rises, A adjusts so as to ensure $\pi_2 = \hat{\pi}_2$, while equations (1) and (2) explain how $[q_1, q_2]$ adjusts. Hence, when $[A, B] = [a, b]$:

$$\frac{d\{\pi_1 + \pi_N\}}{dB} = \{D_2H_1[q] \cdot q_1 + [B - b] \cdot D_2g_2[q_2]\} \cdot \{\frac{dq_2}{dB}\} + g_2[q_2] + \{\frac{dA}{dB}\}$$

where

$$\frac{d\pi_2}{dB} = \{D_1H_2[q] \cdot q_2\} \cdot \{\frac{dq_1}{dB}\} - \{\frac{dA}{dB}\} - g_2[q_2] = 0.$$

This then yields:

$$\frac{d\{\pi_1 + \pi_N\}}{dB} = \{D_2 H_1[q] \cdot q_1\} \cdot \{\frac{dq_2}{dB}\} + \{D_1 H_2[q] \cdot q_2\} \cdot \{\frac{dq_2}{dB}\}.$$

Table A2

Example A2.1.

$I = 2$, $H_1[q] = 100 - q_1 - [0.7] \cdot q_2$, $H_2[q] = 85 - q_2 - [0.7] \cdot q_1$, $c_i[q_i] = 100 + [0.5] \cdot q_i$ $g_i[q_i] = q_i$, and $c_N[g] = 500 + 2 \cdot g$.

	Case 2	Case 2	Case 2	Case 4
f[g]	250 + [2]·g	391 + [6]·g	309 + [8]·g	[22]·g
q_1	39	38	37	43
q_2	28	26	25	16
p_1	42	44	46	46
p_2	30	32	34	39
π_1	1179	938	946	892
π_2	410	199	230	160
π_N	0	507	503	690
CS	1144	1059	996	1059
W	2733	2703	2675	2801

Thus if Firm 2's demand function shifts to the left (generating an asymmetry between the two retail firms), the equilibrium value of B increases. In particular, for Case 2, it is now profitable for the network to set B > b.

Example A2.2. As in examples 1 and 2, except let $H_2[q] = H - [0.7] \cdot q_1 - q_2$. For each of the three cases, the critical value of A can be found that yields an equilibrium (linear) contract of $f[g] = 500 + 2 \cdot g$. In particular, B < b if and only if A exceeds the critical value.
Case 2: $H = 89.1392$, Case 2: $H = 132.922$, Case 4: $H = 158.848$

Example A2.3. Same as Example 1, except $c_N[g] = 500 + [0.1] \cdot g$

	Case 2	Case 2	Case 2	Case 4
f[g]	250 + [0]·g	504	321 + [4]·g	[13]·g
q_1	37	37	35	39
q_2	37	37	35	29
p_1	37	37	40	40
p_2	37	37	40	43
π_1	1005	754	819	933
π_2	1005	754	819	755
π_N	0	501	446	413
CS	1355	1358	1241	1206
W	3366	3367	3325	3308

Example A2.4: Same as Example 1, except $c_N[g] = 500$

	Case 2	Case 2	Case 2	Case 4
f[g]	250	500	321 + [4]·g	[13]·g
q_1	37	37	35	39
q_2	37	37	35	29
p_1	37	37	40	40
p_2	37	37	40	43
π_1	1008	758	822	936
π_2	1008	758	822	758
π_N	0	500	446	413
CS	1358	1358	1243	1209
W	3374	3374	3333	3316

Example A2.5: Same as Example 1, except $c_N[g] = a + g$

Case 2: If a > 1297.2, then Firm 2 can be driven out of the market. When a = 1297.2, $f[g] = 1112 + [5.5] \cdot g$ is the optimal contract, generating zero profit for Firm 2 (with $q_2 = 34.82$).

Case 4: If a > 1230.9, Firm 2 can be driven out of the market. When a = 1230.9, the optimal contract, $f[g] = 450 + [23.8667] \cdot g$, yields zero profit for Firm 2 (with $q_2 = 23.45$).

Appendix 3: Existence of Nash Equilibrium:linear contracts and bypass

Here we consider only the one division conglomerate case. The proofs for the other cases are very similar.

Note that the conglomerate, Firm 1+N, has only one decision variable q_1 in the duopoly competition. As far as $H_1(q)q_1 - c_1(q_1) - bg_1(q_1)$ is concave in q_1, $\pi_1 + \pi_N$ is concave in q_1, and no difficulty will arise from this quarter. The critical issue concerns Firm 2's decision. Of course q_2 and K^* are two decision variables controlled by Firm 2. While these two variables are independent to some extent with $q_2 \in [0, Q]$ for some Q sufficiently large and $K^* \in [0, a]$, for Firm 2 being rational, it will never choose a very large K^* together with a very small q_2. Thus it is reasonable to assume that the choice of K^* is subject to some constraint such as $K^* \in [0, \phi(q_2)]$[27], where $\phi:[0,Q] \to [0,a]$ is an strictly increasing function. In this situation we will always have $q_2 \geq \phi^{-1}(K^*)$ and hence $g_2(q_2) \geq [\phi^{-1}(K^*)]$. For simplicity we assume that $g_2(q_2) = q_2, \delta(K^*) = (K^*/a)^{\alpha}$. Assume that $\partial^2[H_2(q) - c_2(q_2)]/\partial q_2^2 \leq -m < 0$. By calculation we have

$$\partial^2 \pi_2 / \partial q_2^2 \leq -m; \; \partial^2 \pi_2 / \partial K^{*2} = -(B-b)\alpha(1-\alpha)a^{-\alpha}K^{*\alpha-2}q_2;$$

$$\partial^2 \pi / \partial q_2 \partial K^* = -(B-b)\alpha a^{-\alpha} K^{*\alpha-1}$$

From the above expressions it is easy to derive

Lemma 1. π_2 is concave in the region $R = \{(q_2, K^*): q_2 \in [0,Q], 0 \leq K^* \leq \phi(q_2)]\}$ if for every $K^* \in [0,a]$, $\phi^{-1}(K^*) \geq CK^{*\alpha}$, where $C = [(B-b)\alpha]/[ma^{\alpha}(1-\alpha)]$.

From the above Lemma, we can establish the following

Proposition 1'. Under all the above assumptions, there exists at least one Nash equilibrium for the duopoly competition. What is more, any solution obtained from the first-order conditions of the profit functions corresponds to a Nash equilibrium

Remark. The condition that $\phi^{-1}(K^*) \geq CK^{*\alpha}$ has the meaning: the total usage of the network is at least a constant multiple (C) of some power (α) of the bypass coverage K^*. When C is sufficiently small, (for example, B is very close to b, or/and m is very large), this assumption is sensible.

---

27 Note that any rational choice of K* must yield $H_2(q_2)q_2 - c_2(q_2) - k^* \geq 0$ which will provide an upper bound for . ϕ

Appendix 4: Bypass

Given the network contract

$$f[g] = A + Bg$$

and Firm 1's output, Firm 2 chooses q_2 and K^* to maximise

$$\pi_2 = \{H - q_2 - h \cdot q_1\} \cdot q_2 - \{x + \delta[K^*] \cdot b + [1 - \delta(K^*)] \cdot B\} q_2$$
$$-X - K^* - \left(\frac{A \cdot [a - K^*]}{a} \right),$$

where $\delta[K^*] = \left(\frac{K^*}{a} \right)^{\beta}$. The conglomerate chooses q_1 and terms of the contract A and B, to maximise

$$\pi_1 + \pi_N = \{H - q_1 - h \cdot q_2\} \cdot q_1 - [x + b] \cdot q_1 - X - a$$
$$+[B - b] \cdot \{1 - \delta[K^*]\} \cdot q_2 + \left(\frac{A \cdot [a - K^*]}{a} \right),$$

First-order conditions

Conglomerate choice of q_1:

$$\{H - 2 \cdot q_1 - h \cdot q_2\} - [x + b] = 0.$$

Firm 2's choice of q_2 and K^*:

$$\{H - 2 \cdot q_2 - h \cdot q_1\} - \{x + \delta[K^*] \cdot b + [1 - \delta(K^*)] \cdot B\} = 0, \text{ and}$$

$$\left(\frac{\beta \cdot \delta[K^*] \cdot \{B - b\} \cdot q_2}{K^*} \right) + \left(\frac{A - a}{a} \right) = 0.$$

Notice, that the derivative of $\delta[K^*]$, with respect to K^*, is $\left(\frac{\beta \cdot \delta[K^*]}{K^*} \right)$.

The Nash equilibrium can therefore be characterised by:

$$\{4 - h^2\} \cdot q_1 = [2 - h] \cdot [H - x] - 2 \cdot b + h \cdot B - h \cdot [B - b] \cdot \delta[K^*]$$

$$\{4-h^2\}\cdot q_2=[2-h]\cdot[H-x]-2\cdot B+h\cdot b+2\cdot[B-b]\cdot\delta[K^*]$$

$$\{4-h^2\}\cdot q_2=\{4-h^2\}\cdot\left(\frac{a-A}{B-b}\right)\cdot\left(\frac{K^*}{a}\right)\cdot\left(\frac{1}{\beta\cdot\delta[K^*]}\right).$$

Notice, $\left(\frac{dq_1}{dA}\right)=\left(\frac{[-1]\cdot\beta\cdot h\cdot[B-b]\cdot\delta[K^*]}{K^*\cdot[4-h^2]}\right)\cdot\left(\frac{dK^*}{dA}\right).$

Combining these last two equations enables us to find an implicit expression for K^*:

$$[2-h]\cdot[H-x]-2\cdot B+h\cdot b+2\cdot[B-b]\cdot\left(\frac{K^*}{a}\right)^{\beta}-\left(\frac{[4-h^2]\cdot[a-A]}{\beta\cdot[B-b]}\right)\cdot\left(\frac{K^*}{a}\right)^{(1-\beta)}=0.$$

The derivative of K^*, with respect to A, is therefore given by:

$$\left(\frac{dK^*}{dA}\right)=\left(\frac{[4-h^2]\cdot K^*}{\left\{[1-\beta]\cdot[4-h^2]\cdot[a-A]-2\cdot\beta^2\cdot[B-b]^2\cdot\left(\frac{K^*}{a}\right)^{(2\beta-1)}\right\}}\right)$$

Now,

$$\pi_1+\pi_N=q_1{}^2-X-a+\left(\frac{[a-A]}{\beta}\right)\cdot\left(\frac{K^*}{a}\right)^{(1-\beta)}-\left(\frac{a-A}{\beta}\right)\cdot\left(\frac{K^*}{a}\right)+\left(\frac{A\cdot[a-K^*]}{a}\right)$$

Hence:

$$\left(\frac{d\{\pi_1+\pi_N\}}{dA}\right)=\left(\frac{1}{\beta}\right)\cdot\left(\frac{K^*}{a}\right)+\left(\frac{[a-K^*]}{a}\right)-\left(\frac{1}{\alpha}\right)\cdot\left(\frac{K^*}{a}\right)^{(1-\beta)}$$

and

$$-\left(\frac{2\cdot q_1\cdot\beta\cdot h\cdot[B-b]\cdot\left(\frac{K^*}{a}\right)^{\beta}}{\left\{[1-\beta]\cdot[4-h^2]\cdot[a-A]-2\cdot\beta^2\cdot[B-b]^2\cdot\left(\frac{K^*}{a}\right)^{(2\beta-1)}\right\}}\right)$$

$$+\left(\frac{[1-\beta]\cdot[a-A]\cdot\left[4-h^2\right]}{\beta\cdot\left\{[1-\beta]\cdot\left[4-h^2\right]\cdot[a-A]-2\cdot\beta^2\cdot[B-b]^2\cdot\left(\frac{K^*}{a}\right)^{(2\beta-1)}\right\}}\right)\cdot\left(\frac{K^*}{a}\right)^{(1-\beta)}=0.$$

This equation defines the optimal value of A given B.

Appendix 5: Nonlinear Contracts and Bypass

A contract is now given by:

$$f[g] = B \cdot g + C \cdot g^{\phi}.$$

and profits are:

$$\pi_1 + \pi_N = \{H - q_1 - h \cdot q_2\} \cdot q_1 - [x + b] \cdot q_1 - X - a$$

$$+[B - b] \cdot \{1 - \delta[K^*]\} \cdot q_2 + C \cdot \{[1 - \delta(K^*)] \cdot q_2\}^{\phi}$$

$$\pi_2 = \{H - q_2 - h \cdot q_1\} \cdot q_2 - \{x + \delta[K^*] \cdot b + [1 - \delta(K^*)] \cdot C\} \cdot q_2$$

$$-X - K^* - C \cdot \{[1 - \delta(K^*)] \cdot q_2\}^{\phi}$$

where $\delta[K^*] = \left(\frac{K^*}{a}\right)^{\beta}$.

First order conditions

Conglomerate's choice of q_1:

$$\{H - 2 \cdot q_1 - h \cdot q_2\} - [x + b] = 0.$$

Firm 2's choice of q_2 and K^*, respectively, are

$$\{H - 2 \cdot q_2 - h \cdot q_1\} - \{x + \delta[K^*] \cdot b + [1 - \delta(K^*)] \cdot B\} - \left(\frac{\phi \cdot C \cdot \{[1 - \delta(K^*)] \cdot q_2\}^{\phi}}{q_2}\right) = 0.$$

and

$$\left(\frac{\beta \cdot \delta[K^*] \cdot \{B - b\} \cdot q_2}{K^*}\right) + \left(\frac{\beta \cdot \phi \cdot C \cdot \delta[K^*] \cdot [1 - \delta(K^*)]^{\phi - 1} \cdot q_2^{\phi}}{K^*}\right) - 1 = 0$$

or

$$\beta \cdot \delta[K^*] \cdot \{B - b\} \cdot q_2 + \beta \cdot \phi \cdot C \cdot \delta[K^*] \cdot [1 - \delta(K^*)]^{\phi - 1} \cdot q_2^{\phi} - K^* = 0.$$

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