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The cross-sectional distributions of financial ratios: Theory, evidence, and implications

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The Cross-Sectional Distributions of Financial Ratios: Theory, Evidence, and Implications

Abstract

Simple assumptions about the joint distribution of two accounting variables lead to the conclusion that the tails of the distributions of financial ratios follow power-law rather than exponential forms. Algebraic constraints on the accounting variables are reflected in different forms of the ratio tail distributions. The conclusions are largely supported for twelve ratios from large samples of U.S. manufacturers during 1974-1992; a shift towards greater long-tailedness occurred around 1980. Several statistical techniques which have been commonly performed using financial ratios are unsound because ratios have such long tails. Redefinitions of most financial ratios are proposed which may mitigate the problems.

KEYWORDS: ratio; distribution; Hill statistic.

1. Introduction

Empirical research on the cross-sectional statistical distributions of financial ratios has not yet led to a coherent model for the distributions. Deakin [1976] reported that financial ratios are not normally distributed, the distributions being flat, skewed, or dominated by outliers. Square-root or logarithmic transformations sometimes made the distributions more nearly normal, but neither transformation was consistently superior to the other. Frecka and Hopwood [1983] started from a gamma distribution as their underlying model, but rejected that model (and an equivalent normal model for the square root of the ratio). After deleting data points that were identified as outliers relative to their model, they found that the remaining data passed tests for normality in about half of the samples; that finding, however, offers no understanding of the distribution of the population itself. McLeay [1986] showed that a *t* distribution was a better model than the normal distribution for the three ratios that he studied, but did not perform a goodness-of-fit test to see if the *t* distribution was itself adequate.

It is important for the methodology of many accounting studies to be able to describe ratio distributions with confidence. Frecka and Hopwood's [1983] evidence on outliers points to a failure to understand the tails of the distribution in particular. Long-tailedness of a distribution typically causes the most severe problems with conventional statistical methods. Researchers have used various procedures to protect against these problems: Kennedy et al. [1992] list the use of logarithmic or square-root transformations, winsorizing, trimming, and (in many studies) ignoring the problem. It is not possible to determine the performance of these procedures without knowing something about the distribution of the population to which they are to be applied.

The essential contribution of this paper is to present a simplified theory which makes testable predictions about the shapes of the tails of the cross-sectional distribution functions for financial ratios. The theory is based on the observation that any ratio is formed by dividing one financial variable by another, so that the distribution of the ratio can be inferred from the joint distribution of the numerator and denominator variables. Five classes of financial ratios are identified, based on different algebraic constraints on the numerator and denominator. A fairly general assumption about the joint distribution function allows specific conclusions to be drawn about the tails of the distribution of ratios in each class.

The theory is incomplete, in that it does not lead to a description of the entire distribution; on the other hand, the tails are the most important parts of the distribution for analyzing certain commonly used methodologies. The theory does not make particular assumptions about accounting rules or about the economic and technological environment, and so its predictions should be capable of application to firms in any industry in any country at any time.

When the value of a ratio is unbounded, the density function is predicted to fall off in the tail according to an inverse-power law. When the value is bounded (as for a ratio which can lie only between 0 and 1), the density function is predicted to follow a power law close to the bounds. Thus, most financial ratios have much longer tails than do the normal distribution, the gamma distribution, and others with exponentially decaying tails. The theory also predicts certain relations between the exponents applying to different ratio tails.

I test the predictions of the theory against ratios for US manufacturing firms for the years 1974-1992. In the great majority of cases, the data bear out the predicted forms of the tails; in several cases, the tail extends virtually to the median without a detectable departure from the predicted form. The predicted relations between the exponents of different ratios are less well supported. It appears that the theory has substantial explanatory power, but that further refinement is needed.

However, the finding that power-law tails are usually appropriate, together with empirical estimates of the exponents involved, justify the following important methodological conclusions.

1 For some ratios, the population mean does not exist; for many, the population variance is infinite; for most, the population skewness and kurtosis are infinite. When a population moment is infinite or undefined, the corresponding sample statistic is inconsistent and meaningless (except as a trivial description of the particular sample).

2 The distributional assumptions of multiple discriminant analysis are fundamentally unreasonable when financial ratios are the discriminating variables.

3 Linear or quadratic combinations of financial ratios cannot generally be optimal predictors (of financial distress, for example).

4 The usual theory behind factor analysis collapses for sets of financial ratios. The results of factor studies are at present uninterpretable.

5 Observed outliers in financial ratios are one manifestation of a non-normality which has much deeper roots. Deleting the outliers and using the trimmed sample is not valid unless the entire procedure is known to be robust under the true population distribution.

6 It is very doubtful that some commonly used statistical procedures are robust when financial ratios are used. Light trimming procedures such as outlier deletion are known to give poor location estimates for distributions which are highly skewed and have very long tails, which is exactly the case of financial ratios; thus trimming or outlier deletion cannot be simply assumed to be robust. Simulation experiments for the effects of non-normality in classification studies have found poor robustness, even though these studies have been very timid in the nature of the non-normality that they considered. Empirical studies based on the jackknife or other resampling procedures are not a sound guide to the true robustness of many techniques, because they cannot explore an important region of the sample space.

7 None of the Box-Cox family of transformations (which includes the square-root, cube-root, and logarithmic transformations) can transform financial ratios to normality. The logarithmic transformation is to be preferred because it produces a short-tailed (but non-normal) distribution.

These difficulties apply to all ratios except those which are bounded on both sides (usually between 0 and 1), such as the ratio of current assets to total assets. The distributions of such ratios are not normal, but all of their moments are finite. Thus, the non-normality should be much less severe in its consequences.

I propose below a simple set of redefinitions of almost any other ratio such that the redefined ratio is guaranteed to be bounded. The redefined ratios may be used instead of the conventional forms, with equivalent information content and much less reason for concern about unsafe inference or uninterpretable results.[1] The only ratios which cannot be redefined are those for which the denominator may take either sign; the use of such a ratio may constitute model mis-specification, which requires correction rather than statistical accommodation.

The remainder of the paper is organized as follows. In Section 2 I present the theory and describe the five classes of financial ratio. In Section 3 I describe an estimator due to Hill [1975], and use it to test the predictions of the theory for a set of twelve ratios for US manufacturing firms. In Section 4 I draw out the methodological consequences of the theory and present redefinitions of financial ratios in order to mitigate the problems. In Section 5, I suggest some directions for further research.

2. A Partial Theory of the Cross-Sectional Distribution

Consider a financial ratio $z = x/y$. The values of the numerator and denominator variables for any firm depend upon the size of the firm, its history (both management decisions and the external environment), and its choices of accounting policy. Cross-sectionally in a population of firms,[2] there is thus a joint distribution of x and y, from which the ratio z is formed as a quotient.

In general, if variables x and y (with y being restricted to positive values) have a joint distribution with probability density $f(x,y)$, the density function of the quotient $z = x/y$ is[3]

$$f(z) = \int_0^\infty y\,f(zy,y)\,dy. \tag{1}$$

If y can take either sign, then the cases $y>0$ and $y<0$ must be treated separately and the results combined.

The distribution of the ratio z thus depends on the complete joint probability density function $f(x,y)$. To make progress, it must be recognized that $f(x,y)$ is nonzero only on a domain which depends on the nature of the algebraic restrictions on x and y. Five classes of ratio appear to be of importance for financial ratios; they are defined in Figure 1, which illustrates the domains and gives an example of a ratio in each class.

For ratios in classes 1, 3, and 5, it will be assumed that the function $f(x,y)$ can be expressed in the form

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[1] If a theory specifies a particular form of a ratio, then a redefined version may not be an acceptable proxy. That situation has been rare in practice.

[2] The nature of the population is unspecified. It might comprise one or more industries, one or more countries, and a certain time period. For any such definition, there will be a corresponding joint distribution function of x and y.

[3] Kendall and Stuart [1977, p. 282] give the result for a quotient of independent variables, but the extension of their proof to the general case is immediate.

$$f(x,y) = x^a y^b g(x,y) \tag{2}$$

where a and b are non-negative constants, $g(x,0)$ and $g(0,y)$ are nonzero, and $g(x,y)$ can be represented by Taylor series about $x = 0$ and about $y = 0$ whose convergence properties justify the manipulations in the Appendix.[4,5] These conditions are not very restrictive, although it is possible to construct functions which do not satisfy them. Typically, a probability density function with no point masses and no discontinuities will satisfy the conditions, unless $f(x,y)$ tends to zero as $x \to 0$ ($y \to 0$) faster than any power of x (y), or unless a is a function of y or b is a function of x.

Ratios in classes 2 and 4 can be transformed to class 1 by a suitable change of variables, so an equivalent assumption need not be stated for classes 2 and 4.

This assumption suffices to establish the following propositions, which are proved in the Appendix:

Proposition 1. The probability density and cumulative distribution functions of a ratio $z = x/y$ have tails which are asymptotically of the forms shown in Table 1 for the appropriate class of ratio.

Proposition 2. The values of the constants a and b which appear in Table 1 are associated with the numerator or denominator variable alone, where an association is shown in the Table. That is, if a particular variable (say, CA) appears in several ratios, the exponent associated with CA will have the same value in each of those ratios.

These propositions imply that all financial ratio distributions will have power-law tails (rather than exponential tails such as those of the normal or gamma distributions). The exponents for a particular ratio have accounting and economic content, and may vary from one ratio to another and from one population of firms to another. However, there are specific quantitative relationships between the exponents for different ratios.

Since the propositions are asymptotic ones, the rapidity of convergence will be of great importance. If convergence is slow, the propositions will be nearly true only for large values of the ratio. Correspondingly, very large samples may be required for the tails to show the predicted behavior. This fact, rather than the assumptions leading to equation (2), is likely to be the serious limitation on the model.

If the propositions are correct (that is, if the underlying assumptions are descriptively valid), they have significant methodological consequences. Before exploring these, it is desirable to submit the propositions themselves to empirical testing.

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4 The motivation for requiring equation (2) rather than assuming that $f(x,y)$ itself has a Taylor expansion is that, although $f(x,y)$ must approach zero on the domain boundaries if it is continuous there, it need not do so linearly. Thus, $f(x,y) \sim \sqrt{x}\ g(y)$ as $x \to 0$ is quite reasonable behavior for a Class 1 ratio, but such an $f(x,y)$ does not have a Taylor expansion about $x = 0$.

5 Since $x = 0$ is not part of the domain boundary for a Class 3 ratio, $f(0,y)$ need not be zero and so a should be zero. Similarly, both a and b should be zero for a Class 5 ratio.

3. Evidence from US Manufacturing Firms

A. Hill's Estimator

Standard tests for goodness of fit cannot easily be used for testing these propositions, because the propositions do not specify the complete distribution. However, a statistic due to Hill [1975] may be used to determine whether a distribution has power-law tails, and if so to find the length of the tails and to estimate the exponent.

Suppose that a sample of size n comes from a distribution for which

$$F(z) = 1 - k\, z^{-c} \tag{3}$$

for $z \geq D$; $F(z)$ is unspecified for $z < D$. Here k, c and D are positive constants and D is known. Sort the sample to give the order statistics $z_{(1)} \geq z_{(2)} \geq \ldots \geq z_{(n)}$ (this is the reverse of the usual ordering, but is convenient here). Define $V_i = i\, ln[z_{(i)}/z_{(i+1)}]$ for $i = 1, 2, \ldots, r$, conditional on $z_{(r+1)} \geq D$. Then Hill (1975) showed that the V_i are independent exponentially distributed random variables with mean $1/c$, and that the conditional maximum-likelihood estimator of c is

$$\hat{c} = r \Big/ \sum_{i=1}^{r} V_i \tag{4}$$

Hill's estimator $\hat{c}$ is known to be consistent and asymptotically normal (as $n \to \infty$ and $r \to \infty$ with $r/n \to 0$) in more general situations, notably when (3) is asymptotically true as $z \to \infty$ without being exactly true for any finite z[6].

In practice, D is unknown, but there is a rather vaguely defined tail region in which (3) may be a good approximation. Hill suggested exploiting the behavior of the V_i in this situation. If a goodness-of-fit test shows that all of the V_i for $i \leq r$ can have been drawn from a common exponential distribution, then one may accept that the $z_{(i)}$ have all been drawn from the tail region, i.e. that $z_{(r+1)} \geq D$.

Hill's procedure can thus be adapted to our problem. The behavior of the V_i values and the estimates $\hat{c}$ are examined for various values of r, to decide whether equation (3) appears valid for a reasonably long tail segment. While (3) holds, the estimates $\hat{c}$ will be independent of r and the V_i will be i.i.d. exponential. Dunmore [1993] found that it is commonly more powerful to examine a graph of $\hat{c}$ against r, to determine whether there exists a tail region over which $\hat{c}$ is approximately constant, rather than to test how many V_i appear to be i.i.d. exponential.[7]

Hill's procedure may be applied to each of the other forms of power-law tail shown in Table 1; all that is required is an appropriate redefinition of V_i.

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[6] See Csörgő et al. [1985], Goldie and Smith [1987], and Hsing [1991] for various extensions of Hill's result.

[7] This implies that there exists a formal test for the length of the tail which is more powerful than Hill's suggested procedure. However, the nature of that test is unknown.

B. Examples and Limitations of Hill's Estimator

Because Hill's estimator is unfamiliar, I present some examples of its behavior in known situations: a distribution given exactly by (3); the normal distribution, as a typical short-tailed distribution; the normal distribution contaminated by outliers; and the t distribution with two degrees of freedom, for which (3) is only asymptotically valid. These examples provide reference points for examining actual financial ratios.

Figure 2 presents Hill estimates for the left and right tails of the distribution

$$F(z) = \begin{cases} 0.6\, z^2 & 0 \leq z \leq 1 \\ 1 - 0.4\, z^{-3} & z \geq 1 \end{cases} \tag{5}$$

based on a sample of 3,000 points, which is similar to the sample sizes in the empirical work. The vertical axis displays the Hill estimate, which should be 2 for the left tail and 3 for the right tail. The bottom horizontal axis records the values of r and the corresponding values of z for the left tail; the scale for z is nonlinear. The top horizontal axis records r and z for the right tail, in reverse order so that the tip of the right tail is at the right edge of the graph. The solid line and the dashed line show the Hill estimates for the left and right tails respectively. The squares and crosses represent "local" Hill estimates based on 99-point moving averages, defined by

$$\hat{c}_r = 99 \Big/ \sum_{i=r-49}^{r+49} V_i \tag{6}$$

The local estimates indicate the behavior of the tail around the point shown, while the Hill estimate $\hat{c}$ captures the average behavior from the point r to the tip of the tail. Local estimates are plotted every 100 points, so that they use non-overlapping data. The local estimates give a more rapid indication when there is a change in trend, but have higher variance: the standard error of the Hill estimate is about $\hat{c}/\sqrt{r}$, while that of the local estimate is about 0.1 $\hat{c}_r$.

Figure 2 clearly shows that the Hill estimates have the appropriate values and are independent of r for the tail sections plotted, when the assumed model is exactly true. Figure 3, in contrast, shows the very different behavior when the underlying distribution is normal, i.e. with exponentially varying tails. Since an exponential-type function tends to zero faster than any finite power, the Hill estimate becomes larger and larger towards the tips of the tails. There is no region where it is even roughly flat.

The case of a short-tailed distribution containing outliers is presented in Figure 4. The distribution is the "10% 1/U" distribution used by Andrews et al [1972], with about 10% of the points coming from a much longer-tailed distribution. The data for the middle 90% of the sample is essentially normal, and the Hill estimate rises rapidly as in the normal case. This is particularly noticeable in the local estimates, which are less affected by what goes on in the tips of the tails. The last 150 or so points in each tail come mostly from the long-tailed component of the distribution; since smaller values of c in (3) correspond to longer tails, the Hill estimate falls sharply at the tips of the tails. A rising Hill estimate which abruptly falls near the tip suggests the presence of outliers in a shorter-tailed distribution, whereas a roughly flat graph suggests that equation (3) is a better model for describing the data. In practice, of course, there can be intermediate cases where it is unclear which description is better.

As a final example, Figure 5 is based on a t distribution with two degrees of freedom. The asymptotic distribution function is

$$F(t) \sim 1 - \tfrac{1}{2} t^{-2} + \tfrac{3}{4} t^{-4} - \ldots \qquad \text{as } t \to \infty \tag{7}$$

and so the Hill estimate should be 2. However, $\hat{c}$ is not constant in Figure 5; instead, it falls slowly below 2 with increasing r. The reason is that the t^{-4} term in (7) cannot be neglected unless $|t|$ is greater than about 4, and a sample of size 3,000 does not contain enough points with $|t| > 4$ to show a flat region in the graph. If equation (3) is only the leading term in an asymptotic expansion of F, then the Hill estimator is only asymptotically unbiased; a finite sample may be too small to reveal the asymptotic behavior.

These examples show that the behavior of Hill's estimate may signal either that a sample is drawn from a long-tailed distribution such as (3), or that the distribution is essentially short-tailed with a longer-tailed component giving rise to outliers. However, if the sample is not large enough to reveal the asymptotic behavior of the tail of the distribution, the interpretation may be ambiguous and Hill's estimator may be biased. Because of the need for judgment in interpreting the pattern shown by the graphs, I present below a complete set of empirical graphs for one year.

C. The Empirical Sample

I tested twelve ratios: eleven of these were the same as in the studies by Deakin [1976] and Frecka and Hopwood [1983], and I added Return on Equity to the set so as to include a ratio from Class 5.

The sample covered the years 1974-1992, and comprised all U.S. manufacturing firms (SIC codes 2000-3999) for which data was available on Compustat in either the current or research files. If I could not compute a ratio because of missing data, I kept the firm in the sample for other ratios. Very rarely, I had to drop a ratio from a sample because its value was zero or infinite, so that V_i could not be computed.[8] Table 2 lists the ratios and their definitions, and the largest and smallest sample sizes over the period. In a few cases, there is possible ambiguity about the definition of a ratio; since previous authors did not always describe the definitions that they used, there may be minor differences from those used here. The characteristics of ratios which I examine here should not be affected by the precise definition. Conventional basic descriptive statistics are not presented because they are of no relevance to this study; indeed, as noted in Section 1, moment-based statistics may be purely sample-specific, not estimating any meaningful parameters of the underlying population.

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8 In some years, one firm in the sample had a negative value of Net Sales. I deleted these cases for ratios involving Net Sales, and treated Net Sales as being a variable which cannot be negative (i.e. as being a close approximation to Gross Sales).

D. The Empirical Shapes of Ratio Tails

Graphs of the Hill estimates are presented for each of the twelve ratios for the year 1992, so that the reader may assess the reasonableness of my interpretations. The graphs are representative of the study period generally, with the minor exceptions noted below.

Current Assets/Sales (CA/S, Figure 6, $n = 2906$). In the left tail, $\hat{c}$ and $\hat{c}_r$ rise steadily to about 4 near the tip. The behavior is like that of Figure 5, and suggests that the sample is not quite large enough to show the asymptotic behavior. In other years, this tail typically has a flat region extending to about $r = 1000$. The right tail is clearly long-tailed; the local estimate $\hat{c}_r$ suggests that the best description is that of a tail with $\hat{c} \approx 0.9$ until about $r = 500$.

Quick Assets/Sales (QA/S, Figure 7, $n = 2934$). The left tail is flat with $\hat{c} \approx 2.8$ until about $r = 700$, after which the local estimate trends down. The behavior of the right tail is similar to that of CA/S.

Working Capital/Sales (WC/S, Figure 8, $n = 2902$). There are too few points in the left tail to allow any interpretation. The Hill estimate is only defined when $z_{(r)}$ does not change sign, and the underlying model (3) does not allow z to change sign. The left tail of this sample therefore includes only 302 firms, those having negative values of Working Capital in 1992. The right tail estimates show a lot of scatter; the extreme 300 or so points (the upper decile) appear to have $\hat{c} \approx 0.7$.

Current Ratio (CA/CL, Figure 9, $n = 2960$). The left tail, particularly as shown by the local estimates, seems to be short-tailed with outliers.[9] The right tail is nearly flat, with $\hat{c} \approx 1.5$, until about $r = 1500$. Thus, this ratio has a power-law right tail which extends virtually down to the median without much change in behavior. The sudden rise at the tip of the tail does not appear in other years and is presumably a sampling fluctuation.

Quick Ratio (QA/CL, Figure 10, $n = 2960$). The left tail has a long flat region with $\hat{c} \approx 1.9$, plus some outliers. The right tail is nearly flat with $\hat{c} \approx 1.3$; the rise at the tip of the tail does not appear in other years.

Current Assets/Total Assets (CA/TA, Figure 11, $n = 2960$). The left tail is nearly flat with $\hat{c} \approx 2.3$ until about the median. The right tail is nearly flat with $\hat{c} \approx 1.5$.

Quick Assets/Total Assets (QA/TA, Figure 12, $n = 2991$). Both tails seem to have a nearly linear trend, suggesting that the sample size is not large enough to expose the asymptotic behavior, but that the ratio does have power-law tails.

Working Capital/Total Assets (WC/TA, Figure 13, $n = 2963$). The left tail has too few points for interpretation. The right tail is flat, with $\hat{c} \approx 2$.

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[9] The word "outlier" may seem inappropriate here, for the situation in which there are more companies than one would expect with current ratios between 0 and 0.1. Barnett and Lewis [1978, p. 4] define outliers as points which appear to be inconsistent with the remainder of the data. These companies are outliers in that sense, although their presence should not cause any statistical difficulties. If it had been conventional to use the current ratio in the equivalent form CL/CA, these companies would have had ratios greater than 10, and would have caused statistical problems.

Cash Flow/Total Debt (CF/TD, Figure 14, n = 2659). The left tail has too few points for interpretation. The right tail is flat until about $r = 1000$, with $\hat{c} \approx 0.7$.

Net Income/Total Assets (NI/TA, Figure 15, n = 3002). The left tail falls off linearly, suggesting that the sample size is not large enough to expose the asymptotic behavior, but that the ratio does have power-law tails. The right tail appears to be short-tailed with outliers.

Total Debt/Total Assets (TD/TA, Figure 16, n = 2676). The left tail is flat, with $\hat{c} \approx 0.8$. The right tail appears to be short-tailed with outliers.

Return on Equity (NI/EQ, Figure 17, n = 3002). The left tail is flat, with $\hat{c} \approx 1$, until about $r = 400$. The right tail is flat, with $\hat{c} \approx 1$, until about $r = 300$.

Of the 24 tails from these 12 ratios, then, 14 exhibit a tail region of at least 300 points which seems consistent with equation (3), and four more show a linear trend towards the tip, suggesting power-law behavior which is not fully displayed because the sample size is too small. In three cases, the best description appears to that the distribution is fundamentally short-tailed but with outliers. The remaining three cases have too little data to allow any conclusion to be drawn. The results for other years are similar, and so it appears that the prediction that ratios have power-law tails is borne out for most, but not all, financial ratios.

E. The Values of the Exponents of Power-Law Tails

Restricting attention now to the 18 ratio tails for which a power-law description seems appropriate, Figure 18 summarizes the estimates for the 19 years of the study. For each tail, the values plotted are the Hill estimates $\hat{c}$ for r = 100, 200, 300, 500, and 1,000. When the values are very close together, the graph of $\hat{c}$ against r is nearly flat until r = 1,000, indicating that the ratio has a power-law tail at least this long. When the values spread out in order, there is a linear trend, and the best (but possibly biased) estimate of the tail exponent is the value for $r = 100$, closest to the tip.

Several features are apparent in Figure 18. First, the values for a given ratio vary only slowly from year to year. Second, the values differ between ratios, so that the values of a and b in equation (2) are not the same for all accounting variables. Third, there is a striking shift towards longer-tailedness for several ratios, which occurred over a period of a few years around 1980. The shift is not exactly contemporaneous for different ratios.

Apparently, some economic or accounting change affected the financial structure of U.S. manufacturing firms at about that time, but it is not clear what kind of change would cause a shift to greater long-tailedness in several ratio distributions. This is much subtler than a change in the location or dispersion of a distribution. The shift in tail exponents is not an artifact caused by any changes in Compustat's sample selection procedure: I found similar shifts in the exponents when I repeated the calculations for CA/S and CA/CL using only firms which appeared in the Compustat file for every year from 1974 to 1992.

Proposition 2 predicts certain equalities between the exponents of different ratio tails. These are listed in Table 3 for the ratios under study, omitting those for which the power-law description seems invalid or there is insufficient data. It is obvious from Figure 18 that the exponents associated with CA are unequal, especially after 1980, and that those associated with QA are unequal. No statistical test is necessary to establish that the prediction fails for these ratios. However, the exponents associated with TD appear nearly equal, those associated with S are all close together, which is especially striking because their values change substantially over time, and the two tails of NI/EQ appear to have exponents of about 1.

As formal tests of the hypotheses that the exponents of the right tail of CF/TD and the left tail of TD/TA are equal and that the exponent of each tail of NI/EQ is equal to 1, I performed paired t tests,[10] whose results are presented in Table 4. The values of $\hat{c}$ used in the test were for $r = 100$, since these should be least affected by bias if the sample size is not large enough. The only hypothesis rejected by the tests is that the exponent of the right tail of NI/EQ is equal to 1. Even in this case, the magnitude of the mean difference is less than 0.1.

To test the hypothesis that the exponents associated with S are all equal, I performed a two-way analysis of variance, again using the Hill estimates for $r = 100$. The results are presented in Table 5. The hypothesis of equality is rejected at the 0.002 level; but the mean pairwise differences between the exponents in different tails are not more than 0.16.

F. Summary of the Empirical Evidence

Two kinds of prediction are made by the theory: that ratio tails follow power-law distributions, and that there are certain numerical equalities between the exponents. Of the 24 ratio tails examined, 18 appear to follow power-law distributions, three appear to follow shorter-tailed distributions but contain a fairly small number of outliers with unknown distributional properties, and three have too little data to admit of any conclusion. For the tails which do appear to follow power-law distributions, the estimated exponents are generally between about 1 and 3, or a little more for certain ratios, corresponding to values of a or b between about 0 and 2. The exponents do not change sharply from year to year, but several ratios showed a long-term change to smaller exponents (longer-tailed distributions) occurring around 1980. The equalities predicted by Proposition 2 can be rejected in most cases, but the departures from prediction are not large except for the tails associated with the variables CA and QA.

It appears, then, that the theory has some substantial explanatory power, but that there is still much more to be understood about the nature of cross-sectional ratio distributions. However, the results seem to justify making methodological inferences from the theory. The next section of the paper draws these out.

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[10] Under the null hypothesis, the Hill estimator is asymptotically normal. Thus a nonparametric test is not required.

4. Methodological Implications

A. Population and Sample Moments of Financial Ratios

If the distributions of financial ratios have the asymptotic behavior shown in Table 1, with values of a and b which are less than about 2, there are important consequences for the use and interpretation of ratios in Classes 1, 3, 4 and 5. These consequences can be sufficiently illustrated for a ratio in Class 1, and so the discussion will concentrate on that case.

The population mean of the ratio z is given by

$$E(z) = \int_0^\infty z f(z)\, dz \tag{8}$$

For $z \to \infty$, the asymptotic behavior of the integrand is $B_0 z^{-(b+1)}$, so the integral converges only if $b > 0$. If $b = 0$, the population mean is infinite. Assuming that the mean exists, the population variance is given by

$$\text{var } z = \int_0^\infty (z - E(z))^2 f(z)\, dz \tag{9}$$

and this integral converges only if $b > 1$. If $b \leq 1$, the population variance is infinite. Similarly, the population has infinite skewness or kurtosis if $b \leq 2$ or $b \leq 3$ respectively.

Of course, the mean and all higher moments for any finite sample from such a population must be finite. When the population parameter is infinite or undefined, the corresponding sample statistics behave inconsistently: the expected value and its variance both increase as the sample size increases. This explains the findings of Frecka and Hopwood [1983, Table 2] that the sample moments of ratio distributions show great instability from year to year.

If the population mean does not exist, it is obviously impossible to draw inferences about its value. For example, Weetman and Gray [1991] define an index of 'conservatism' for a country's accounting practices relative to the US. Their index is a Class 5 ratio, so that its mean presumably does not exist. Thus, hypotheses concerning whether the mean index is greater than or less than 1 (the US value) are not appropriate. Hypotheses concerning the median index are appropriate, however.

If $b \leq 1$, the population mean exists, but the Central Limit Theorem for the sampling distribution of the mean does not apply, because it requires that the population variance be finite. If b is not much greater than 1, the Central Limit Theorem applies, but convergence to the asymptotic limit will be very slow: sample-based estimates of the standard error of the mean will be greatly understated except in very large samples.

B. Multivariate Normality and Multiple Discriminant Analysis

Multiple discriminant analysis has been largely replaced by probit and logit methods for financial distress prediction, but is still used in other types of classification studies. It requires two distributional assumptions: that the predictor variables are multivariate normal, and that their covariance matrices are equal for the various populations being classified. When the predictor variables are financial ratios with infinite variances, the covariance matrix contains infinite values; indeed, it may consist entirely of infinities. Thus, the assumptions of MDA are not matters that the empirical researcher should verify for his or her sample; they represent a fundamentally unreasonable description of the situation.

The power of goodness-of-fit tests depends strongly on the sample size and the actual population distribution. Thus, moderate-sized samples of financial ratios may wrongly pass tests for univariate or multivariate normality.[11] This is still more likely if several transformations are performed to see which transformation best improves the normality of the sample, or if the sample is trimmed or apparent outliers are deleted before use.[12] Whether the sample passes a test for normality, either before or after such treatment, is irrelevant to the issue: MDA requires that the *populations* have the prescribed properties. The theoretical model advanced here indicates that the populations do not have the required properties, and the properties of a particular sample do not alter that conclusion.

C. Prediction Using Linear Combinations of Financial Ratios

Linear combinations of financial ratios are often used to predict the classification of firms into two groups, such as bankrupt and nonbankrupt. A linear combination of variables minimizes the total cost of misclassification if the variables follow a multivariate normal distribution with the same covariance matrix for the two groups, even if the groups are of unequal size and the costs of Type I and Type II errors are unequal [Kendall et al. 1983, section 44.6]. If the variables do not follow a multivariate normal distribution, the optimal boundary separating the two groups need not be linear.[13] If the boundary is nonlinear in regions where the probability density is high, then linear combinations of the predictor variables are necessarily suboptimal predictors of group membership.

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[11] Deakin [1976], Lee [1985] and Ezzamel and Mar-Molinero [1990] reported that normality was rejected less frequently in samples from single industries than in multi-industry samples. The theory presented here suggests that controlling for industry effects should have no effect on the general nature of the distribution, although it may affect the parameters. These authors did not control for sample size, and that by itself could account for their results.

[12] See Taffler [1982], Mutchler [1985], Hopwood et al. [1988], and Watson [1990] for examples of such procedures.

[13] The formula in the general case is given by equation (44.7) in Kendall et al. [1983].

As a model for prediction using two Class 1 financial ratios, I computed the optimal boundaries between groups using two predictors of group membership which followed independent F distributions. Depending upon the distributional parameters and the relative cost of Type I and Type II errors, the optimal boundaries had a variety of shapes which were typically highly nonlinear. Thus it appears possible that linear combinations of financial ratios may be highly suboptimal predictors. This problem applies to logit and probit models as well as to discriminant analysis.

D. Correlations and Factor Patterns Among Financial Ratios

Much research with financial ratios has involved factor analysis, either to identify possibly important patterns in financial data or as a data reduction step so that identified factors may be used in subsequent analysis. Factor analysis normally begins with the Pearson correlation matrix (in some versions the covariance matrix is used). However, for a set of Class 1 financial ratios with $b \leq 1$, the population covariance matrix contains infinities down the diagonal and perhaps elsewhere, and the correlation matrix does not exist. In any sample from such a population, of course, the covariance matrix is finite and the correlation matrix can be computed; thus, the data can be subjected to factor analysis.

It is unclear how to interpret the results, however; they do not have the interpretation placed upon them in the factor-analysis literature. Further, it is not valid to assume that the factors so produced may be used as proxies for the variables which load on them. That assumption is usually justified by supposing that the factors represent latent or unobservable variables, which generate the observed variables as linear combinations. The factors may be inferred from a particular sample and, since these factors generate the observed variables, they may be used as data reduction tools in future studies with other samples. That view underlies all of the theory behind factor analysis.[14] This paper emphasises that financial ratios must be thought of as being generated by division, and this view is not compatible with the latent-variable model. Thus, the theoretical justification for factor analysis is not applicable to financial ratios.

There are, of course, many possible definitions of the correlation between two variables. It may be that some other correlation concept could be applied to financial ratios, and that factor analysis would give interpretable results using that correlation concept. However, the body of theory which would be necessary for that to happen does not now exist.

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14 If factors are being identified purely as a data reduction procedure within a single sample, as in Taffler [1982], no such view is required. However, if the population correlation matrix does not exist, there is no apparent reason for supposing that the sample correlation matrix is the right tool for identifying and removing redundant variables.

E. The Interpretation of Outliers

There is a well-established link between the existence of extreme values in samples of financial ratios and the fact that those samples fail tests for normality. It has been inferred that non-normality is often the result of outliers [Frecka and Hopwood, 1983]. The theory presented here indicates that the causality runs the other way: the existence of outliers is one manifestation of a very deep-seated non-normality. For example, for each of the Class 1 ratios QA/S, CA/CL, or QA/CL, the power-law tail is a good description of virtually the entire upper half of the distribution, not merely of a few extreme values.

If a sample contains outliers which are unreasonable on the basis of some assumed probability model, there are two ways to proceed. One is to maintain the assumed model, and remove the outliers from the analysis on the grounds that they are not representative of the population or phenomenon that one is studying. The other is to decide that one's assumption rather than the data is defective, and adopt a more realistic model; this may perhaps lead to a decision that outlier deletion will form part of an analysis which will be valid under the more realistic model. Although each approach may involve outlier deletion, it is important to distinguish carefully between the two motivations, because they require different types of evidence if inferences from outlier-deleted data are to be accepted.

If outlying values of financial ratios were caused by errors in the underlying data, there would be a clear case for correcting the errors if possible, and otherwise for deleting them. Also, if the outliers represented some identifiably different subpopulation, the entire subpopulation (not just the outliers) should be removed and treated separately[15] in order to improve the efficiency of the analysis. On the other hand, if the apparent outliers are consistent with, and emerge continuously from, the bulk of the distribution, then the assumed model should be abandoned, and statistical procedures adopted which perform satisfactorily under a more realistic model. These procedures might include trimming or outlier deletion as a first step, but the researcher must know the sampling distribution of the entire procedure under the realistic distributional model. In general, this is different from the sampling distribution under the originally assumed model; thus, results which are valid for samples from normal populations cannot be assumed to be valid in trimmed samples from non-normal populations.

The theory of this paper leads to viewing "outlying" values as forming a natural part of a very long-tailed distribution, suggesting that maintaining the normality model and deleting or accommodating the outliers is not an appropriate response. Frecka and Hopwood [1983] point out that outliers arise from companies which have an unusually small denominator. However, small denominators merge imperceptibly into not-quite-so-small denominators and on into medium-sized denominators. There is no natural cutoff point; the entire ratio distribution has a severely non-normal shape. To make safe inferences from outlier-deleted data, it will be necessary to know the true population distribution and to study the robustness of outlier deletion theoretically and case by case.

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[15] If the existence of a subpopulation is suspected but its members cannot be identified except by their appearance as outliers, then deleting the outliers approximates the effect of deleting the subpopulation. The approximation may or may not be a good one.

There may, of course, be outlying values even from a distribution as long-tailed as that given by equation (3). A very few points (less than 0.1%) were deleted from the samples in the present study because the ratio value was infinite, which is not consistent with model (3). The issue must therefore be faced of whether to delete these firms or to alter the probability model again. I will suggest (below) a way of redefining the ratios that will allow safer analysis of samples which include even these infinite outliers.

F. Robustness

Outlier deletion may be regarded as an adaptive trimming procedure,[16] in that the extent of trimming in each tail is sample-dependent. Since typical rates of outliers found in practice are less than 5% and are unequal in the two tails, outlier deletion represents a light and asymmetric trim. Studies of the robustness of trimming procedures have usually been based on estimators of location, and the findings are not reassuring. It has long been known that lightly trimmed means are inefficient estimators of location from long-tailed distributions, and are biased when the distribution is skewed [Andrews et al. 1972, Exhibits 5–19 and 5–20; see also Huber 1981, pp. 72 and 104-106]. Thus, the robustness of using an outlier deletion procedure on a Class 1 financial ratio with $b \leq 2$, which has infinite variance and/or skewness, is highly questionable. Huber [1981, p. 4] gives more general reasons for preferring other robust procedures to outlier rejection followed by classical procedures.

The performance of a proposed procedure cannot be determined empirically. Typically, the performance is related to the frequency of Type I and Type II errors or to the bias in estimating a parameter, and these cannot be established empirically since the true answer is unknown. An apparent exception is in classification studies, where Lachenbruch or other resampling procedures have commonly been used to estimate the misclassification rate. However, such estimates of performance are always misleading if the performance of a procedure is affected by the moments of the population distribution, because resampling procedures start from a sample whose moments must all be finite; if the population distribution has infinite moments, resampling procedures cannot fairly reflect the true sample space. Thus, it is unknown whether the findings of such studies reflect the actual performance of the techniques, even approximately.[17]

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16 Similar comments apply to an adaptive Winsorizing procedure, in which the detected outliers are not removed from the sample but are replaced with the largest non-outlying value.

17 The method gives a good estimate of error rates with multivariate normal predictors [Lachenbruch and Mickey 1968].

There have been simulation studies of the effect of certain types of non-normality in classification studies [Lachenbruch et al. 1973, Richardson and Davidson 1983, Stone and Rasp 1991]. However, these have either started from a multivariate normal distribution and added a moderate amount of skewness and kurtosis, or have assumed distributional forms (such as the lognormal) which, while not normal, are not nearly long-tailed enough to be applicable to financial ratios.[18] Even so, the results were not generally robust to non-normality; these findings can be expected to apply *a fortiori* to financial ratios.

A different robustness question arises when the regression or other model being used is mis-specified. If the model is correctly specified, outlying values of the independent variables are highly desirable, because they fix the parameters of the model very accurately. If the model is mis-specified, its weaknesses are badly exposed by extreme values of the independent variables. The only really satisfactory treatment for this problem is to correct the model specification. However, since a wrong model may sometimes give a reasonable approximation to the correct model over a limited domain, restricting the range of the independent variables may sometimes allow an invalid model to be used. Any procedure which restricts the variables sufficiently is suitable; outlier deletion is only one possible way to achieve this. The performance of any procedure will be highly specific to the particular situation, including the nature of the model mis-specification. Kennedy et al. [1992] evaluate various procedures in several situations involving prediction by a linear model.

G. Transformations

Transformations are sometimes used in an attempt to improve the distributional properties of financial ratios. The most common transformations are the square-root, cube-root, and logarithmic transformations, which are special cases of the general Box-Cox transformation [Box and Cox 1964]:

$$u = \begin{cases} (z^{\lambda} - 1)/\lambda & \lambda \neq 0 \\ ln\, z & \lambda = 0 \end{cases} \tag{10}$$

Applying this variable transformation to the limiting behavior of a Class 1 ratio, which is

$$f(z) \sim B_0 z^{-b-2} \qquad \text{as } z \to \infty$$

shows that for $\lambda > 0$ the limiting behavior is

$$f(u) \sim B_0 (\lambda u)^{-b'-2} \qquad \text{as } u \to \infty, \text{ where } b' = (b+1)/\lambda - 1$$

and that for $\lambda = 0$ the limiting behavior is

$$f(u) \sim B_0 e^{-(b+1)u} \qquad \text{as } u \to \infty.$$

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[18] The F distribution with a small number of denominator degrees of freedom would be a reasonable model distribution for the type of non-normality exhibited by a Class 1 financial ratio. This is not to say that an F distribution would necessarily give a good fit to ratio distributions. However, both tails of the $F(\nu_1,\nu_2)$ distribution have the power-law form, and for suitable ν_2 the mean or any higher moments can be made infinite. Thus the distribution has the correct properties to act as a known but realistic model distribution.

Thus, for $\lambda > 0$, the Box-Cox transformation produces a variable with inverse-power-law tails, but with a larger exponent (e.g. for the square-root transformation, where $\lambda = \frac{1}{2}$, the transformed variable has exponent $2b+1$). Thus, the long-tailed nature of the distribution will be less severe after transformation, but the problems associated with the power-law tails are not completely cured. The logarithmic transformation produces a short-tailed variable with exponential tails; it is still not normally distributed, but the nature of the non-normality is not likely to have such severe methodological consequences.

Previous work [Deakin 1976; Frecka and Hopwood 1983] has found that neither the square-root nor the logarithmic transformation produces normality; which transformation comes closer varies from sample to sample. These findings may now be understood, since neither transformation in fact produces normality. The logarithmic transformation is always preferable for Class 1 ratios because it produces a short-tailed population, even if a square-root or cube-root transformation appears to work better on a particular sample.

The Box-Cox transformation cannot be applied to Class 3, 4 or 5 ratios. A sample-dependent version, in which a constant is first added to all of the values in the sample so that all values are positive, cannot be readily analyzed because it is not a transformation of the underlying variable. In principle, it might be possible to find the sampling distribution of various statistics resulting from the use of such a transformation, to see if these statistics are suitably robust.

Statistics based on rank transformations lead to many well-known hypothesis tests which are nearly distribution-free and which may be used with financial ratios. Tests based on normal scores are less common, but are often more powerful. However, when estimates of some parameter are required in terms of original variables, the rank or normal score must be transformed back, and the results are no longer distribution-free.

If $F(x)$ is any continuous cumulative distribution function, then $z' = F(z)$ is a monotonic transformation which ensures that z' lies between 0 and 1. All moments of z' are finite, and many of the previous difficulties do not apply to inferences about z'. However, there are interpretive difficulties in using z', and also a statistical difficulty: if the chosen distribution F is short-tailed (as in logit or probit procedures), then the probability density of z' is infinite at $z' = 1$ and perhaps also at $z' = 0$. Thus, z' is bimodal or trimodal, and its distribution is so far from being normal that conventional statistical inferences may be seriously invalid. (However, if a procedure is known to be robust when applied to dichotomous data, it may be applied to z' with some confidence.) If a transformation of this kind is to be used, the safest one may be the arc-tan transformation, $z' = \tan^{-1} z$, which bounds z between $-\pi/2$ and $\pi/2$. This transformation, based on the long-tailed Cauchy distribution, ensures that the density of z' approaches zero at the bounds in a smooth power-law manner similar to that of a Class 2 ratio.

H. Redefining Financial Ratios

The problems described above for Class 1 ratios exist also for ratios in every class except Class 2. A Class 2 ratio has values which are bounded between 0 and 1; hence, all of the population moments are finite. The distribution must be non-normal, but the non-normality does not take the form of infinitely long tails. The tail behavior shown in Table 1 is consistent with a beta distribution, although the central part of the true ratio distribution might depart from the beta form. There will still often be outliers in any sample, as measured from a working hypothesis of normality, but the outliers will not usually have methodologically serious consequences.

By a simple redefinition, any financial ratio in Class 1, 3, or 4 can be brought to a Class 2 form. Since the current conventional definitions are arbitrary, the redefined versions of the ratios are in no way inferior: they contain the same information, are monotonic functions of the conventional form, and are just as easy to interpret. They have the considerable advantage, however, of having distributions which differ from normality only in relatively harmless ways.

If the current ratio were defined, not as CA/CL, but as CA/(CA+CL), it would be a Class 2 ratio. Companies with good short-term liquidity would have values close to 1, and companies with liquidity problems would have values close to 0. This definition would have the benefits already described, and has an obvious interpretation, so that back transforming to express results in terms of CA/CL is unnecessary.[19] For any Class 1 ratio z, the ratio $z' = z/(1 + z)$ is an equivalent Class 2 form; that is, the Class 1 ratio z should be deflated by the factor $1 + z$.

A Class 3 ratio may be viewed as the difference between two Class 1 ratios and transformed accordingly. For example, if NI = R – E, where revenues R and expenses E are both positive, the ratio $z = \mathrm{NI/TA} = \mathrm{R/TA} - \mathrm{E/TA}$ may be replaced by

$$z' = \frac{\mathrm{R}}{\mathrm{R+TA}} - \frac{\mathrm{E}}{\mathrm{E+TA}} = \frac{\mathrm{R{\bullet}TA - E{\bullet}TA}}{\mathrm{(R+TA)(E+TA)}} = \frac{\mathrm{NI{\bullet}TA}}{\mathrm{(R+TA)(E+TA)}}$$

$$= \frac{\mathrm{NI}}{\mathrm{TA}}\frac{1}{\mathrm{(1+R/TA)(1+E/TA)}} \tag{11}$$

where the last form shows that this redefinition simply deflates the ratio appropriately. For an arbitrary Class 3 ratio $z = (x_1 - x_2)/y$, the deflated form is

$$z' = \frac{z}{(1 + x_1/y)(1 + x_2/y)} \tag{12}$$

which takes values between -1 and 1. The form $(1 + z')/2$ takes values between 0 and 1, but there is no statistical gain from that refinement.

A Class 4 ratio may be deflated in the same way as a Class 3 ratio, or it may be viewed as the difference between a Class 2 and a Class 1 ratio. The latter view, however, gives a more complicated form for the redefined ratio. For a Class 4 ratio, deflating according to equation (12) leads to values z' between -1 and ½.

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[19] Conventions can change, and textbook writers and teachers may be in a position to encourage the gradual replacement of statistically undesirable conventional ratio definitions.

No similar simple redefinition exists for a Class 5 ratio such as Return on Equity. The use of this ratio in a model implies that a firm which earns $20 million on a net worth of $200 million is in the same position, *ceteris paribus*, as one which loses $20 million on a net worth of negative $200 million. Unless this is expected to be true on theoretical grounds, the researcher should consider whether the use of Return on Equity comprises a mis-specification of the model. If so, the cure is to substitute a more appropriate ratio, not to seek a transformation.

The redefinitions of financial ratios recommended here can deal easily with infinite values of financial ratios. Infinite values caused by a zero denominator cannot be reconciled with the model of equation (3) or with the results of Table 1. However, after redefinition, these values are equal to one of the bounds of the redefined ratio.[20] Thus, they can remain in the sample and require no special handling. The limiting behavior of the redefined variable cannot be exactly as shown for a Class 2 ratio in Table 1, because the density cannot be quite zero at the bounds. However, since infinite values are very rare, the density is very nearly zero there.

5. Directions for Further Research

This paper has addressed the problem of describing the distribution of financial ratios by pushing it back one stage, to the joint distribution of the numerator and denominator variables. Since the predictions of the theory are largely but not entirely borne out, the assumption made in Section 2 about the form of the joint distribution may not be entirely correct, or more terms must be considered in the asymptotic expansion for some distributions. Better understanding of the distributions of financial ratios will therefore require a better understanding of the joint distribution of accounting variables. The economic arguments which could assist that understanding are by no means evident; but both theoretical and empirical work are likely to be sharpened and clarified if they focus on the accounting variables, without adding the complexity caused by the division operation which produces a financial ratio. At the same time, since it is much easier to examine a univariate distribution than a multivariate one, and since the distribution of a ratio depends on the entire joint distribution of numerator and denominator, financial ratios may provide useful probes for studying certain hypotheses about the joint distributions.

Although I have pointed out that the distribution of financial ratios makes many common practices statistically unsound, and have suggested that redefinitions of the ratios may mitigate the worst problems, research is still needed to determine the robustness of using the redefined ratios in typical research contexts. This research cannot be empirical; it must be either analytical or based on simulation studies. In either case, an appropriate assumption must be made as to the distributional form. For Class 2 ratios (or other ratios which have been redefined), the most appropriate model distribution seems to be the beta distribution, which is bounded and has tails of the correct power-law form.

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20 Care may sometimes be needed to decide which is the correct bound.

Finally, if the joint distribution of accounting variables is in fact more fundamental than the distribution of financial ratios, it may be that many studies which have traditionally been performed with ratios would work better using accounting variables. Beaver [1968] noted that failed firms had 65% as much mean current assets and 65% as much mean sales as nonfailed firms, but this difference cancels out in the ratio CA/S. Some multivariate failure prediction models have included a size variable to supplement the use of ratios, but this is not the same as including all of the financial variables in raw form. It may be that bivariate plots of the probability density of current assets and sales would show a clear distinction between the regions occupied by bankrupt and nonbankrupt companies. If that turns out to be so, then the multivariate generalization would be worth exploring in the same way.

Appendix. Proof of Propositions 1 and 2

This Appendix establishes the results given in Table 1 for the various classes of ratio. A given ratio $z = x/y$ falls into only one class, and so only one of the following arguments applies. Accordingly, the notation has been kept simple by using the same symbol for analogous constants in different classes. The appropriate definition of the constants is given, or is immediately obvious, in each class.

The first two terms of each series are computed below, but only the first term has been carried into Table 1, because this establishes the asymptotic behavior.

Class 1

Expand $g(x,y)$ in a Taylor series about $x = 0$:

$$g(x,y) = a_0(y) + a_1(y)\,x + \ldots \tag{A.1}$$

where the coefficients $a_n(y)$ could be expressed in terms of partial derivatives evaluated at $x = 0$. Then, using equations (1) and (2),

$$f(z) = \int_0^\infty y\,(zy)^a\,y^b\,g(zy,y)\,dy$$

$$= z^a \int_0^\infty y^{a+b+1}\,a_0(y)\,dy \;+ z^{a+1} \int_0^\infty y^{a+b+2}\,a_1(y)\,dy \;+ \ldots$$

$$= A_0\,z^a + A_1\,z^{a+1} + \ldots \tag{A.2}$$

where the constants are given by

$$A_n = \int_0^\infty y^{a+b+n+1}\,a_n(y)\,dy \tag{A.3}$$

By assumption, $g(0,y) > 0$, so that $a_0(y) > 0$ and hence $A_0 > 0$. Integrating (A.2) gives the cumulative distribution function of z:

$$F(z) = \int_0^z f(z)\,dz = \frac{A_0}{a+1}\,z^{a+1} + \frac{A_1}{a+2}\,z^{a+2} + \ldots \tag{A.4}$$

Now expand $g(x,y)$ about $y = 0$:

$$g(x,y) = b_0(x) + b_1(x)\,y + \ldots \tag{A.5}$$

so that

$$f(z) = \int_0^\infty y\,(zy)^a\,y^b\,g(zy,y)\,dy$$

$$= z^a \int_0^\infty y^{a+b+1}\, b_0(zy)\, dy + z^a \int_0^\infty y^{a+b+2}\, b_1(zy)\, dy + \ldots \quad \text{(A.6)}$$

Make the substitution $w = zy$ so that $y = w/z$ (noting that z cannot be zero, so that the resulting series will be an asymptotic series as $z \to \infty$, and not valid for all z); then

$$f(z) = z^{-b-2} \int_0^\infty w^{a+b+1}\, b_0(w)\, dw + z^{-b-3} \int_0^\infty w^{a+b+2}\, b_1(w)\, dw + \ldots$$

$$= B_0\, z^{-b-2} + B_1\, z^{-b-3} + \ldots \quad \text{(A.7)}$$

with the obvious definitions of the constants. Hence

$$F(z) = 1 - \int_z^\infty f(z)\, dz = 1 - \frac{B_0}{b+1} z^{-b-1} - \frac{B_1}{b+2} z^{-b-2} - \ldots \quad \text{(A.8)}$$

This establishes Proposition 1 for Class 1 ratios. For Proposition 2, note that equation (2) implies that the marginal distribution of x is

$$\int_0^\infty f(x,y)\, dy = x^a \int_0^\infty y^b\, g(x,y)\, dy$$

$$= x^a \int_0^\infty y^b\, a_0(y)\, dy + x^{a+1} \int_0^\infty y^b\, a_1(y)\, dy + \ldots \quad \text{(A.9)}$$

which behaves like x^a as $x \to 0$. Similarly, the marginal distribution of y is

$$\int_0^\infty f(x,y)\, dx = y^b \int_0^\infty x^a\, g(x,y)\, dx$$

$$= y^b \int_0^\infty x^a\, b_0(x)\, dx + y^{b+1} \int_0^\infty x^a\, b_1(x)\, dx + \ldots \quad \text{(A.10)}$$

which behaves like y^b as $y \to 0$. The exponents a and b which appear in equations (A.2), (A.4), (A.7) and (A.8) are therefore characteristic of the marginal distributions of x and y respectively, and do not depend on the other variable in the ratio. Hence they must be the same in any Class 1 ratio involving the particular variable.

Class 2

Make the variable transformation $x' = y - x,\ y' = x,\ z' = x'/y' = (1-z)/z$. Then the Class 2 conditions $x \geq 0, y \geq x$ are equivalent to $x' \geq 0, y' \geq 0$ so that the ratio z' belongs to Class 1. Thus (A.2) and (A.7) apply to z'; writing A_0' and A_1' for the constants in (A.2) and transforming the result back to a function of z, (A.2) becomes

$$f(z) \sim \{A_0'(z')^a + A_1'(z')^{a+1} + \ldots\} \left|\frac{dz'}{dz}\right| \qquad \text{as } z' \to 0^+$$

$$\sim \left\{A_0'\left(\frac{1-z}{z}\right)^a + A_1'\left(\frac{1-z}{z}\right)^{a+1} + \ldots\right\} \frac{1}{z^2} \qquad \text{as } z \to 1^-$$

$$\sim A_0'(1-z)^a + \{(a+2)A_0' + A_1'\}(1-z)^{a+1} + \ldots$$

$$\sim A_0(1-z)^a + A_1(1-z)^{a+1} + \ldots \qquad \text{as } z \to 1^- \qquad \text{(A.11)}$$

with the obvious definitions of A_0, A_1, etc. Then

$$F(z) = 1 - \int_z^1 f(z)\,dz$$

$$\sim 1 - \frac{A_0}{a+1}(1-z)^{a+1} - \frac{A_1}{a+2}(1-z)^{a+2} - \ldots \qquad \text{as } z \to 1^- \qquad \text{(A.12)}$$

Similarly, from (A.7),

$$f(z) \sim \{B_0'(z')^{-b-2} + B_1'(z')^{-b-3} + \ldots\} \left|\frac{dz'}{dz}\right| \qquad \text{as } z' \to \infty$$

$$\sim \left\{B_0'\left(\frac{1-z}{z}\right)^{-b-2} + B_1'\left(\frac{1-z}{z}\right)^{-b-3} + \ldots\right\} \frac{1}{z^2} \qquad \text{as } z \to 0^+$$

$$\sim B_0'(1-z)^{-b-2} z^b + B_1'(1-z)^{-b-3} z^{b+1} + \ldots$$

$$\sim B_0 z^b + B_1 z^{b+1} + \ldots \qquad \text{(A.13)}$$

so $$F(z) = \int_0^z f(z)\,dz$$

$$\sim \frac{B_0}{b+1} z^{b+1} + \frac{B_1}{b+2} z^{b+2} + \ldots \qquad \text{as } z \to 0^+ \qquad \text{(A.14)}$$

For a Class 2 ratio, the variables a and b characterize the marginal distributions of x' and y' respectively, in the same sense as before. Since $y' = x$, the variable b is characteristic of x, while a is not characteristic of either x or y.

Class 3

Since the line $x = 0$ is not on the domain boundary, $f(0,y)$ is nonzero and so a must be zero in equation (2). With that restriction, (A.5) still holds and (A.6) becomes

$$f(z) = \int_0^\infty y^{b+1}\, b_0(z\,y)\, dy + \int_0^\infty y^{b+2}\, b_1(z\,y)\, dy + \ldots \qquad \text{(A.15)}$$

If $z > 0$, the substitution $w = zy$ leads to (A.7) as before. If $z < 0$, write $w = -zy = |z|y$; then (A.15) becomes

$$f(z) = |z|^{-b-2} \int_0^\infty w^{b+1}\, b_0(-w)\, dw + |z|^{-b-3} \int_0^\infty w^{b+2}\, b_1(-w)\, dw + \ldots$$

$$= B_0\, |z|^{-b-2} + B_1\, |z|^{-b-3} + \ldots \qquad \text{as } z \to -\infty \qquad \text{(A.16)}$$

and $\quad F(z) = \int_{-\infty}^{z} f(z)\, dz$

$$\sim \frac{B_0}{b+1} |z|^{-b-1} + \frac{B_1}{b+2} |z|^{-b-2} + \ldots \qquad \text{as } z \to -\infty \qquad \text{(A.17)}$$

Since $a = 0$, the marginal distribution of y is

$$\int_{-\infty}^{\infty} f(x,y)\, dx = y^b \int_{-\infty}^{\infty} g(x,y)\, dx$$

$$= y^b \left\{ \int_{-\infty}^{0} g(x,y)\, dx + \int_0^\infty g(x,y)\, dx \right\} = y^b \int_0^\infty (g(x,y) + g(-x,y))\, dx$$

$$= y^b \int_0^\infty (b_0(x) + b_0(-x))\, dx + y^{b+1} \int_0^\infty (b_1(x) + b_1(-x))\, dx + \ldots \qquad \text{(A.18)}$$

which behaves like y^b as $y \to 0$. Thus b is characteristic of the variable y.

Class 4

Make the variable transformation $x' = y - x,\ y' = y,\ z' = x'/y' = 1 - z$. Then the Class 4 conditions $x \le y,\ y \ge 0$ are equivalent to $x' \ge 0,\ y' \ge 0$ so that the ratio z' belongs to Class 1. Transforming the result back to a function of z, (A.2) becomes

$$f(z) \sim A_0\, (1 - z)^a + A_1\, (1 - z)^{a+1} + \ldots \qquad \text{as } z \to 1^- \qquad \text{(A.19)}$$

and (A.7) becomes

$$f(z) \sim B_0' \, (1-z)^{-b-2} + B_1' \, (1-z)^{-b-3} + \dots$$

$$\sim B_0 \, |z|^{-b-2} + B_1 \, |z|^{-b-3} + \dots \qquad \text{as } z \to -\infty \qquad \text{(A.20)}$$

and the expressions for $F(z)$ follow immediately.

For Proposition 2, note that b is characteristic of the variable $y' = y$.

Class 5

When y can take either sign, a more general form of (1) must be used (compare Kendall and Stuart, 1977, p. 285):

$$f(z) = \int_0^\infty y \, f(zy,y) \, dy + \int_0^\infty y \, f(-zy,-y) \, dy \qquad \text{(A.21)}$$

Since the lines $x = 0$ and $y = 0$ are inside the domain, both of the constants a and b must be zero. The expansion (A.5) is valid, and so

$$f(z) = \int_0^\infty y \, b_0(z\,y) \, dy + \int_0^\infty y^2 \, b_1(z\,y) \, dy + \dots$$

$$+ \int_0^\infty y \, b_0(-z\,y) \, dy - \int_0^\infty y^2 \, b_1(-z\,y) \, dy + \dots$$

$$= \int_0^\infty y \, [b_0(z\,y) + b_0(-z\,y)] \, dy + \int_0^\infty y^2 \, [b_1(z\,y) - b_1(-z\,y)] \, \mathrm{dy} + \dots \qquad \text{(A.22)}$$

If $z > 0$, put $w = zy$ so that $w > 0$. Then (A.22) becomes

$$f(z) = z^{-2} \int_0^\infty w \, [b_0(w) + b_0(-w)] \, dw + z^{-3} \int_0^\infty w^2 \, [b_1(w) - b_1(-w)] \, dw + \dots \qquad \text{(A.23)}$$

$$= B_0 \, z^{-2} + B_1 \, z^{-3} + \dots \qquad \text{(A.24)}$$

If $z < 0$, put $w = -zy = |z|y$ so that $w > 0$. Then (A.22) becomes

$$f(z) = |z|^{-2} \int_0^\infty w \, [b_0(-w) + b_0(w)] \, dw + |z|^{-3} \int_0^\infty w^2 \, [b_1(-w) - b_1(w)] \, dw + \dots \qquad \text{(A.25)}$$

$$= B_0 \, |z|^{-2} - B_1 \, |z|^{-3} + \dots \qquad \text{(A.26)}$$

The constants $B_0, B_1, \dots$ are the same in equations (A.24) and (A.26).

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TABLE 1. Theoretical asymptotic behavior of the probability density function $f(z)$ and the cumulative distribution function $F(z)$ in the left and right tails, for various classes of the ratio $z = x/y$. The notation $z \to 0^+(1^-)$ indicates the one-sided limit in which z approaches 0 from above (1 from below). The constants a, b, A_0, and B_0 take different values for each class; however, if a variable occurs in more than one ratio, the values of a or b with which that variable is associated will be the same in each ratio.

Class	Left Tail Behavior		Right Tail Behavior		Variable Associated with	
					a	b
1	$f(z) \sim A_0 z^a$ $F(z) \sim \frac{A_0}{a+1} z^{a+1}$	$z \to 0^+$	$f(z) \sim B_0 z^{-b-2}$ $F(z) \sim 1 - \frac{B_0}{b+1} z^{-b-1}$	$z \to \infty$	x	y
2	$f(z) \sim B_0 z^b$ $F(z) \sim \frac{B_0}{b+1} z^{b+1}$	$z \to 0^+$	$f(z) \sim A_0 (1-z)^a$ $F(z) \sim 1 - \frac{A_0}{a+1} (1-z)^{a+1}$	$z \to 1^-$	–	x
3	$f(z) \sim B_0 \lvert z \rvert^{-b-2}$ $F(z) \sim 1 - \frac{B_0}{b+1} \lvert z \rvert^{-b-1}$	$z \to -\infty$	$f(z) \sim B_0 z^{-b-2}$ $F(z) \sim 1 - \frac{B_0}{b+1} z^{-b-1}$	$z \to \infty$	–	y
4	$f(z) \sim B_0 \lvert z \rvert^{-b-2}$ $F(z) \sim 1 - \frac{B_0}{b+1} \lvert z \rvert^{-b-1}$	$z \to -\infty$	$f(z) \sim A_0 (1-z)^a$ $F(z) \sim 1 - \frac{A_0}{a+1} (1-z)^{a+1}$	$z \to 1^-$	–	y
5	$f(z) \sim B_0 \lvert z \rvert^{-2}$ $F(z) \sim 1 - B_0 \lvert z \rvert^{-1}$	$z \to -\infty$	$f(z) \sim B_0 z^{-2}$ $F(z) \sim 1 - B_0 z^{-1}$	$z \to \infty$	–	–

TABLE 2. Ratios Used in the Study. The variable names used in the numerator and denominator are those of Standard and Poor's Compustat.

Abbreviation	Numerator	Denominator	Sample Size Range, 1974-92
CA/S	Current assets - Total	Sales (net)	2675-3094
QA/S	Cash and Equivalents, *plus* Receivables - Total	Sales (net)	2675-3090
WC/S	Current Assets - Total, *minus* Current Liabilities - Total	Sales (net)	2673-3094
CA/CL	Current assets - Total	Current Liabilities - Total	2682-3175
QA/CL	Cash and Equivalents, *plus* Receivables - Total	Current Liabilities - Total	2681-3168
CA/TA	Current Assets - Total	Assets - Total	2683-3174
QA/TA	Cash and Equivalents, *plus* Receivables - Total	Assets - Total	2684-3170
WC/TA	Current Assets - Total, *minus* Current Liabilities - Total	Assets - Total	2682-3176
CF/TD	Income before Extraordinary Items, *plus* Depreciation and Amortization	Debt in Current Liabilities, *plus* Long-Term Debt - Total	2551-2920
NI/TA	Net Income (Loss)	Assets - Total	2706-3185
TD/TA	Debt in Current Liabilities, *plus* Long-Term Debt - Total	Assets - Total	2558-2943
NI/EQ	Net Income (Loss)	Stockholders' Equity	2705-3185

TABLE 3. Predicted Equalities Between Exponents of Different Ratio Tails.

Associated Variable	Exponents Predicted to be Equal		
CA	CA/S left	CA/TA left	
QA	QA/S left	QA/CL left	QA/TA left
TD	CF/TD right	TD/TA left	
S	CA/S right	QA/S right	WC/S right
(Class 5)	NI/EQ left (= 1)	NI/EQ right (= 1)	

TABLE 4. Paired t Tests of Relations Between Exponents. All tests are based on the Hill estimate $\hat{c}$ for $r = 100$.

Comparison	Mean Difference	Standard Deviation	t (18 d.f.)	p (2-tailed)
CF/TD(right) = TD/TA(left)	0.010	0.209	0.21	n.s.
NI/EQ(left) = 1	0.005	0.097	0.21	n.s.
NI/EQ(right) = 1	-0.070	0.090	-3.40	.005

TABLE 5. Two-Way Analysis of Variance to Test Equality of Exponents Associated with the Sales Variable. Based on the Hill estimate $\hat{c}$ for $r = 100$.

Source of Variation	Sum of Squares	d.f.	Mean Square	F	p
Between years	21.96	18	1.220	77.9	<.001
Between ratios	0.25	2	0.125	8.0	.002
Error	0.56	36	0.016		

Figure 1. The five classes of financial ratios $z = x/y$, showing the domains of the joint distributions of the numerators and denominators and the limit which the ratio takes along various lines, and giving an example of each class. CA = current assets; CL = current liabilities; TA = total assets; NI = net income; EQ = shareholders' equity; WC = working capital.

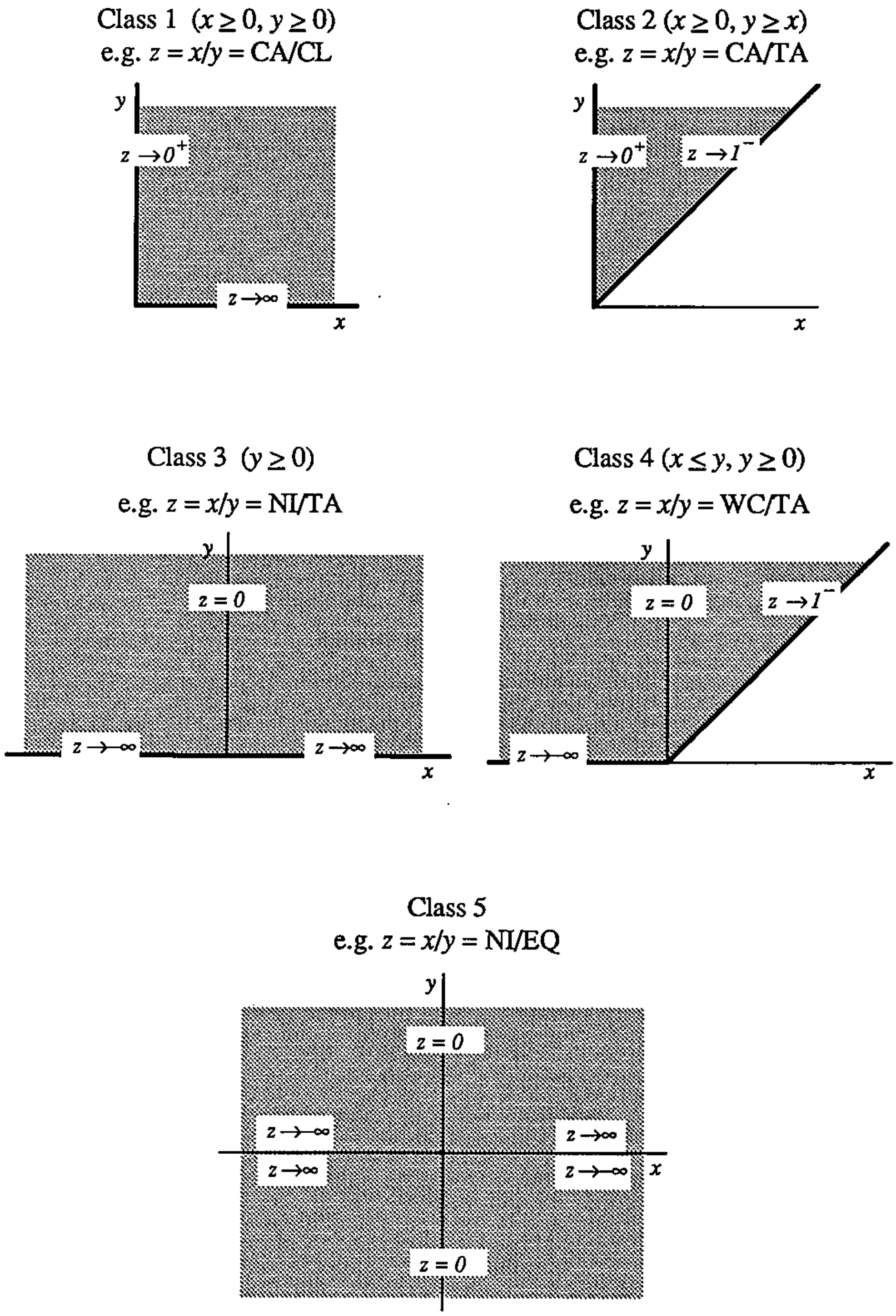


Figure 2. Hill's estimator for a sample of 3,000 values from the distribution of equation (5). Values of r and z for the left tail are marked along the bottom axis, and values for the right tail along the top axis in reverse order; thus the tip of the tail is at the left edge of the graph for the left tail and at the right edge for the right tail. —— Hill estimate for left tail; – – – for right tail. ■ ■ ■ Local 99-point moving average estimate for left tail; + + + for right tail.

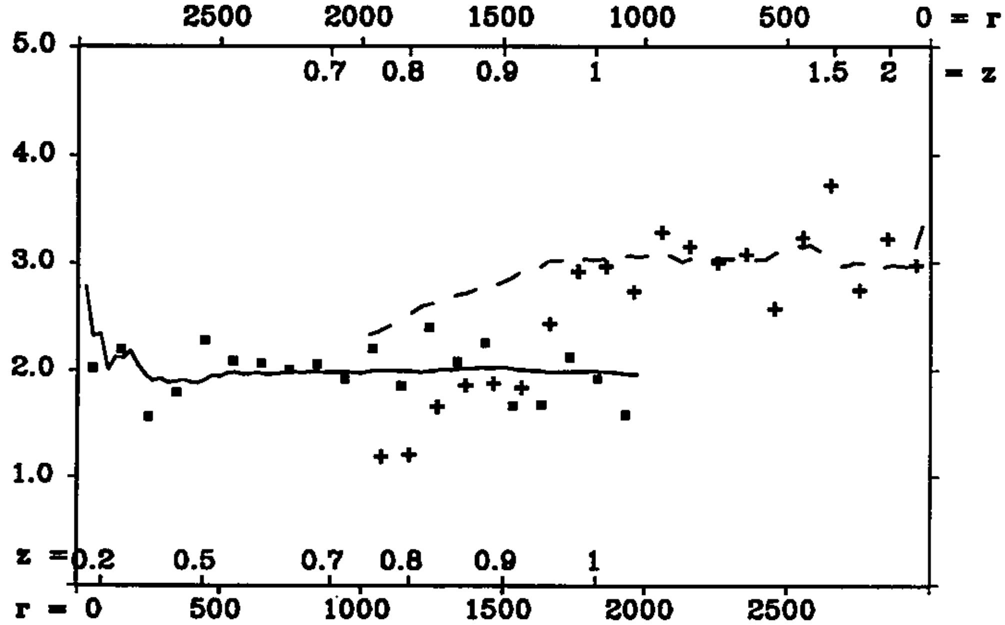


Figure 3. Hill's estimator for a sample of 3,000 values from the standard normal distribution. Description as for Figure 2.

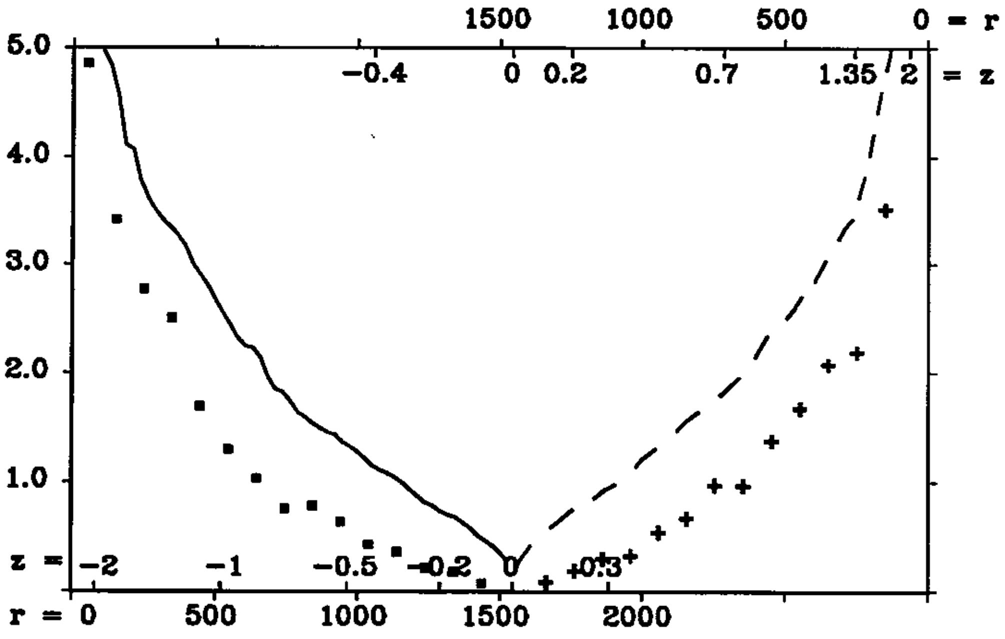


Figure 4. Hill's estimator for a sample of 3,000 values from the "10% 1/U" distribution, which is largely normal but with 10% contamination by a long-tailed distribution. Description as for Figure 2.

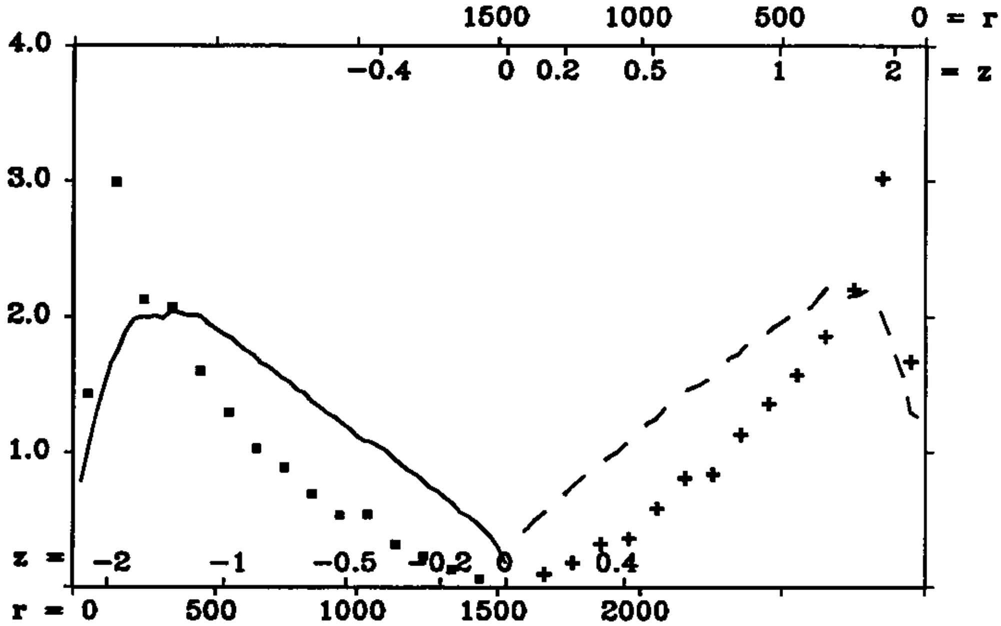


Figure 5. Hill's estimator for a sample of 3,000 values from the *t* distribution with 2 degrees of freedom. Description as for Figure 2.

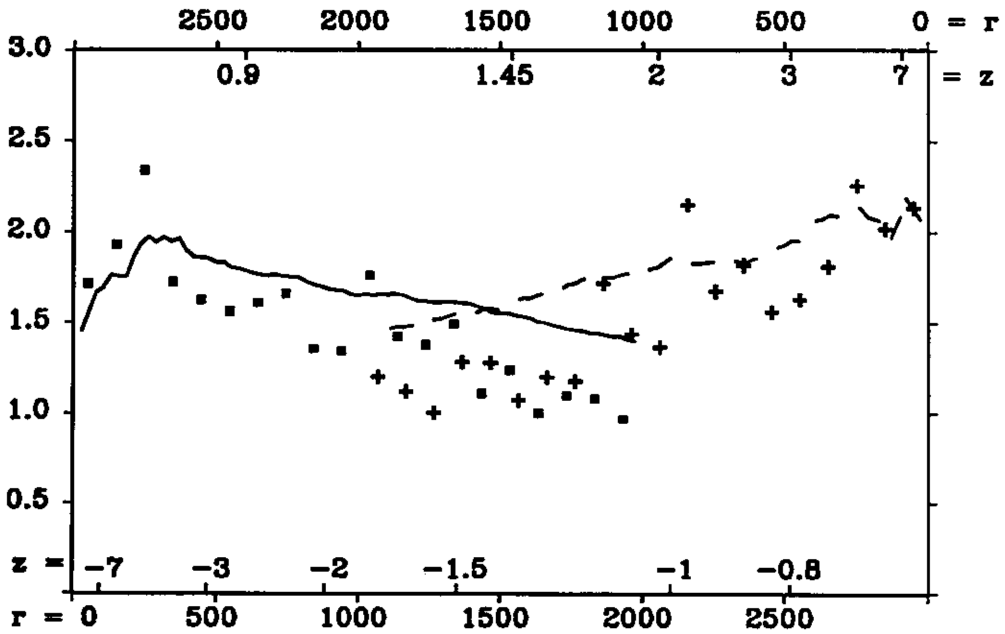


Figure 6. Hill's estimator for the ratio CA/S for manufacturing firms in 1992.
Description as for Figure 2.

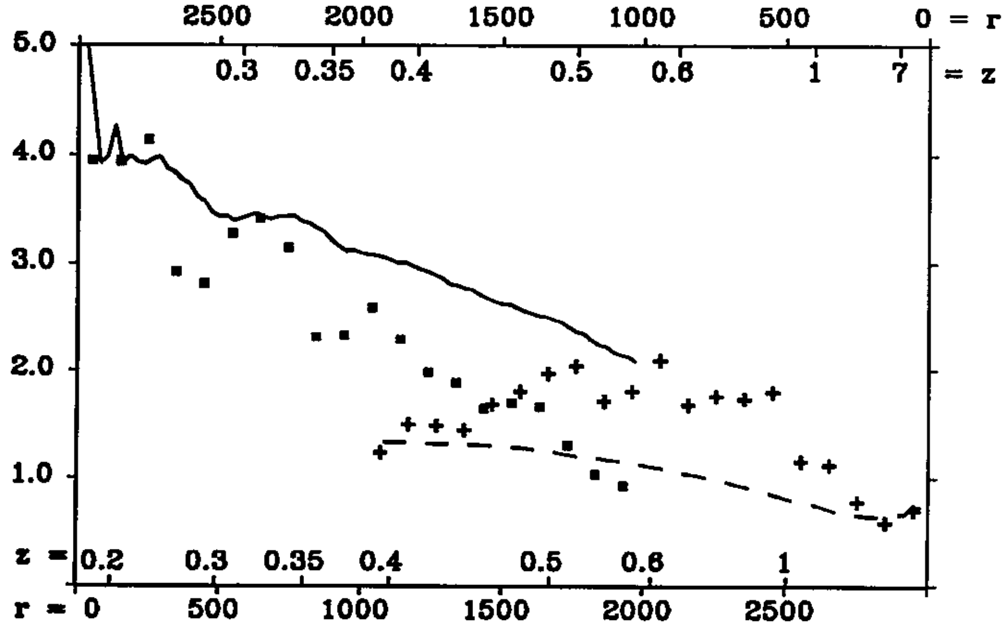


Figure 7. Hill's estimator for the ratio QA/S for manufacturing firms in 1992.
Description as for Figure 2.

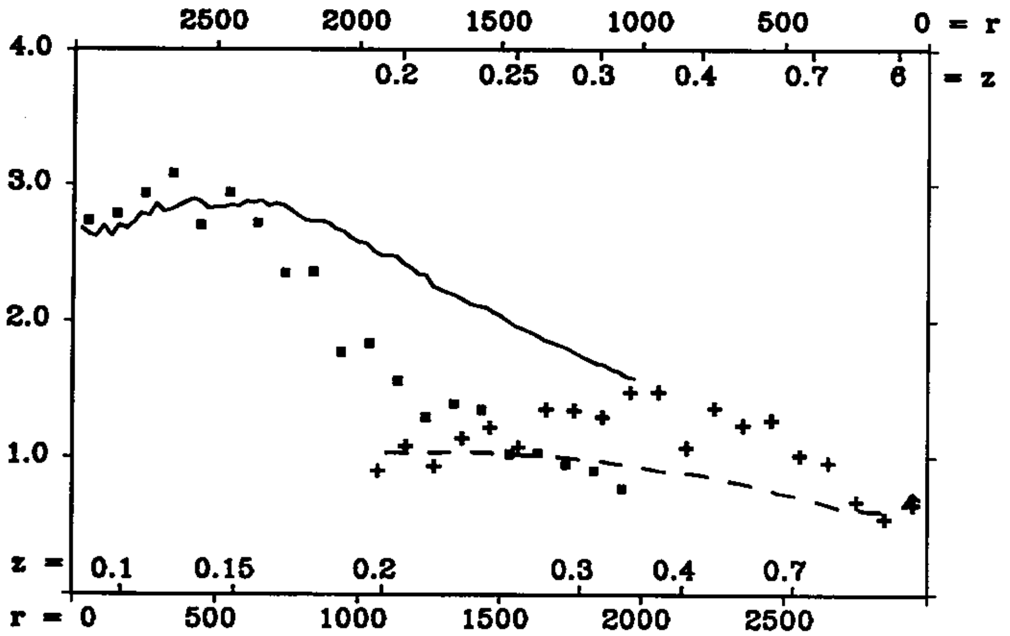


Figure 8. Hill's estimator for the ratio WC/S for manufacturing firms in 1992. Description as for Figure 2.

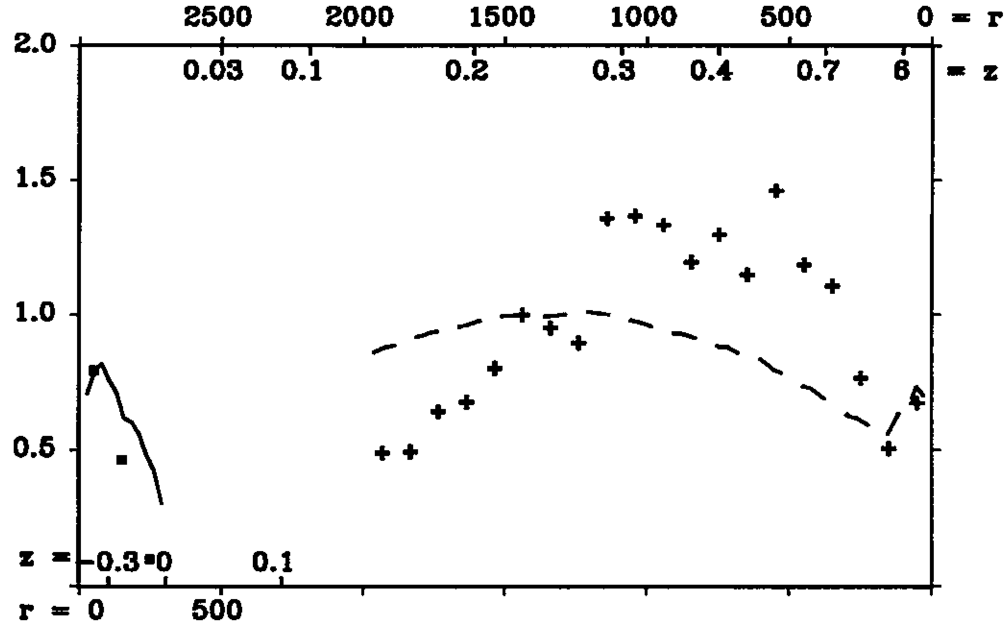


Figure 9. Hill's estimator for the ratio CA/CL for manufacturing firms in 1992. Description as for Figure 2.

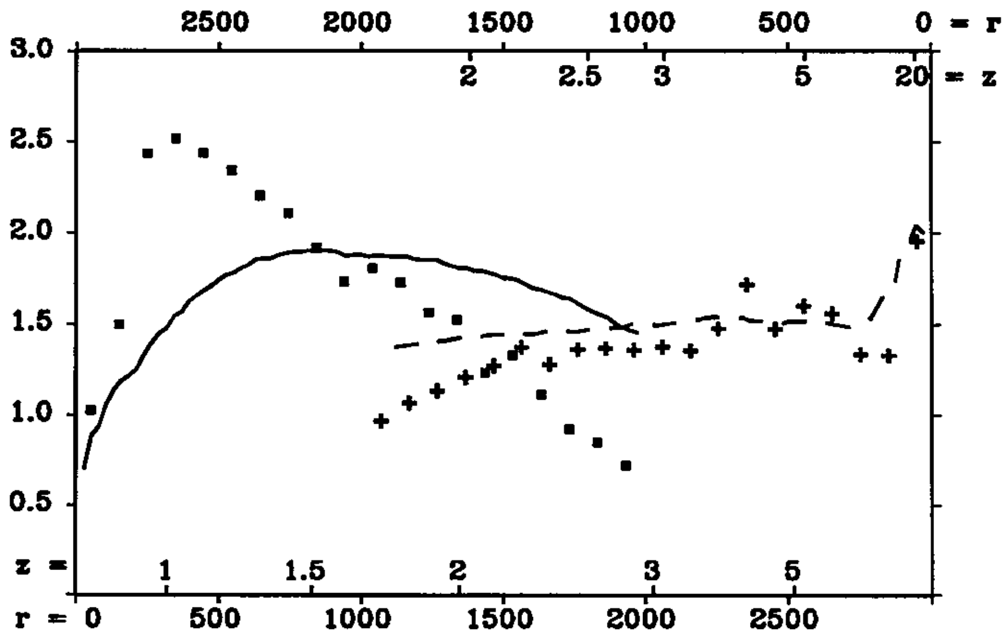


Figure 10. Hill's estimator for the ratio QA/CL for manufacturing firms in 1992. Description as for Figure 2.

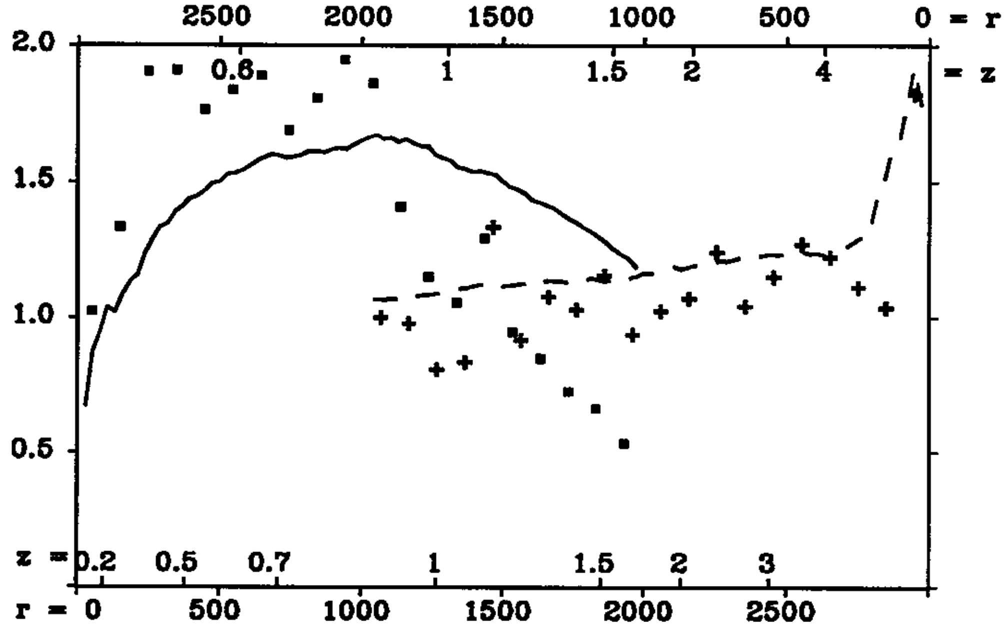


Figure 11. Hill's estimator for the ratio CA/TA for manufacturing firms in 1992. Description as for Figure 2.

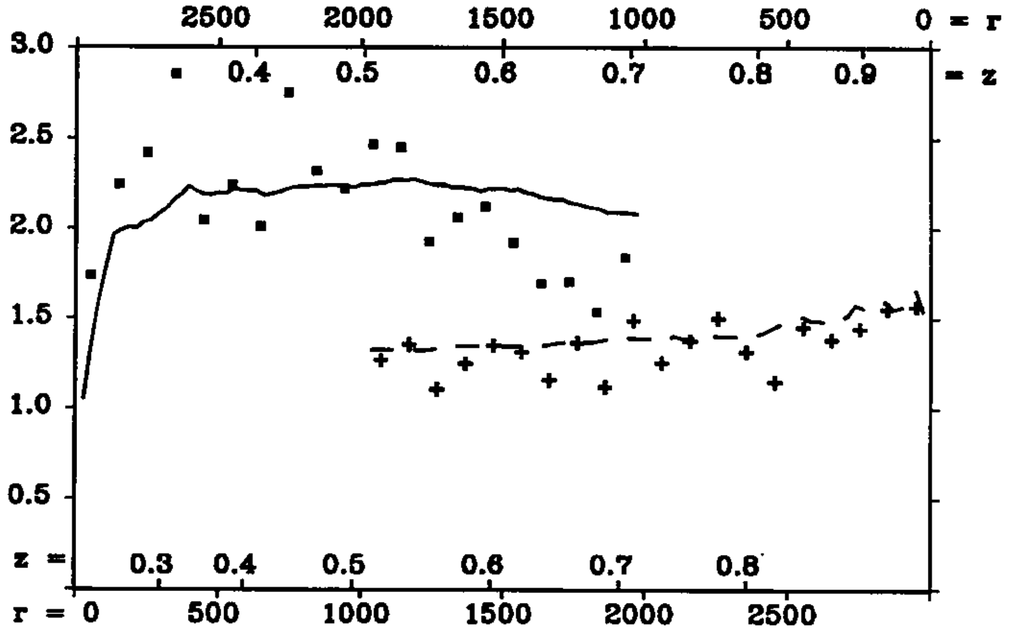


Figure 12. Hill's estimator for the ratio QA/TA for manufacturing firms in 1992. Description as for Figure 2.

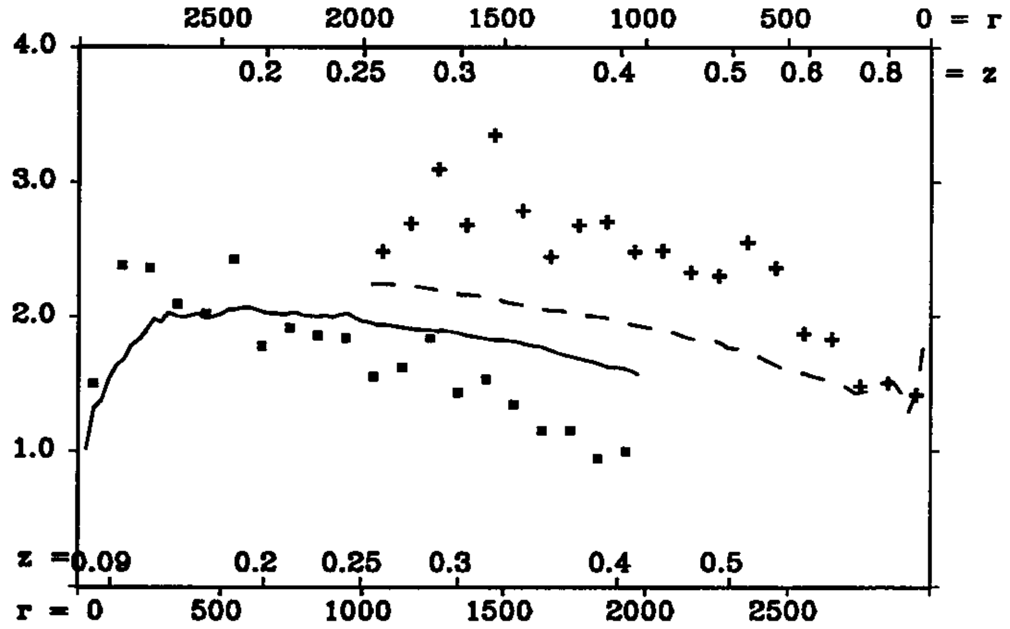


Figure 13. Hill's estimator for the ratio WC/TA for manufacturing firms in 1992. Description as for Figure 2.

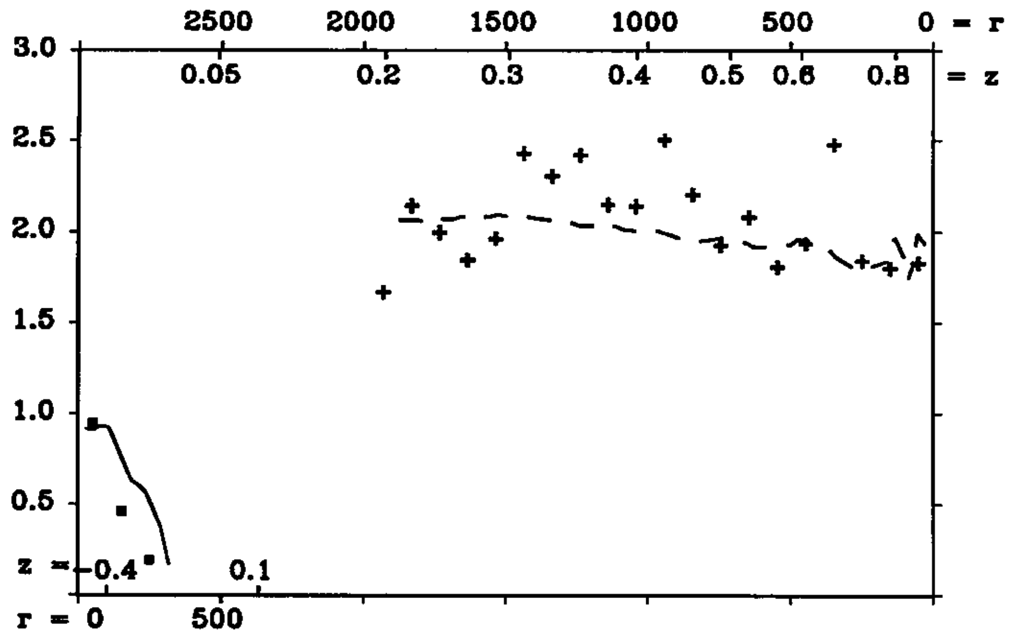


Figure 14. Hill's estimator for the ratio CF/TD for manufacturing firms in 1992. Description as for Figure 2.

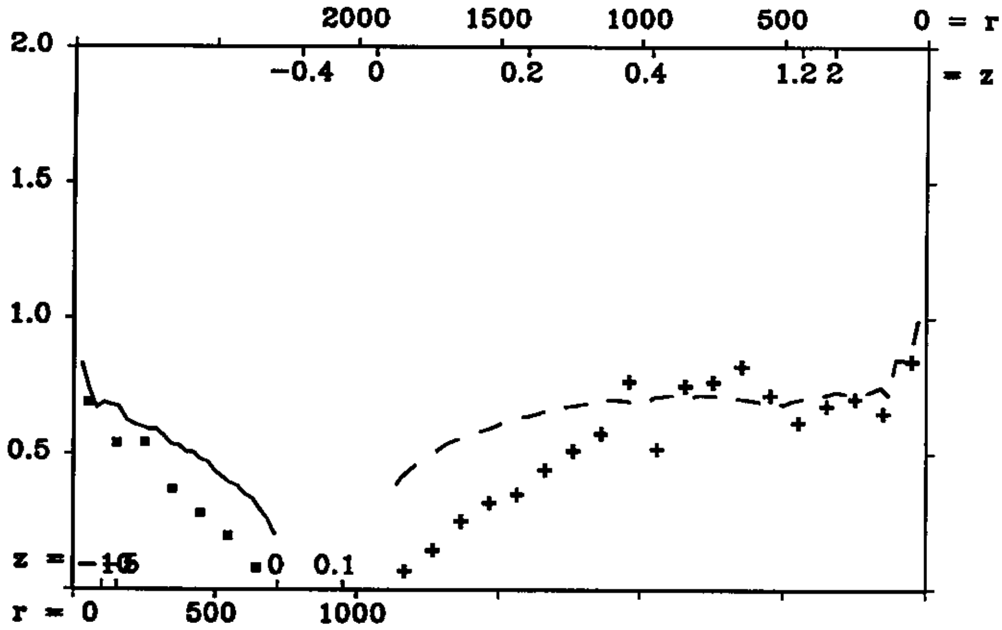


Figure 15. Hill's estimator for the ratio NI/TA for manufacturing firms in 1992. Description as for Figure 2.

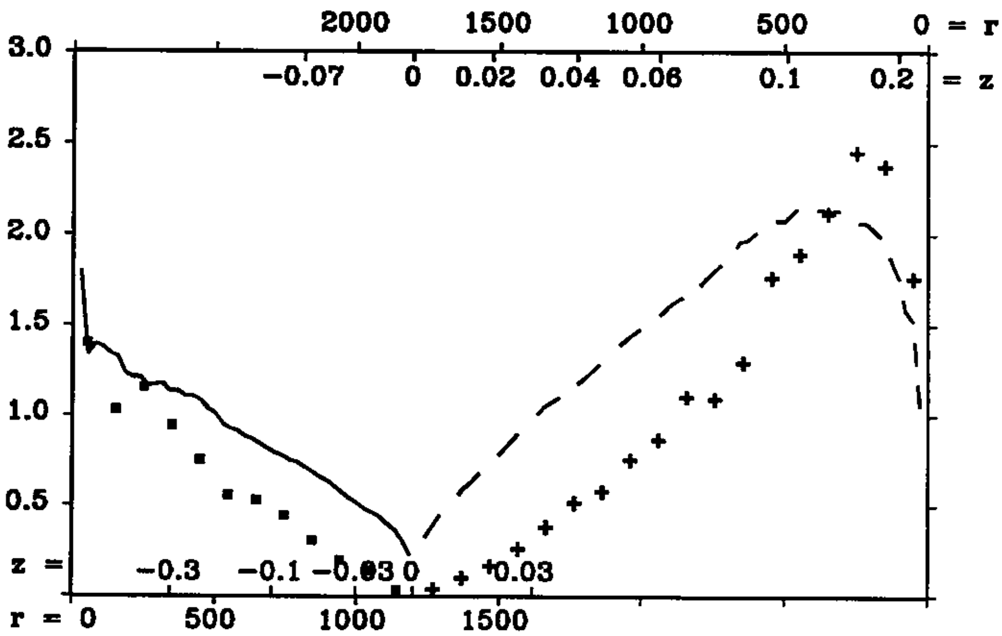


Figure 16. Hill's estimator for the ratio TD/TA for manufacturing firms in 1992. Description as for Figure 2.

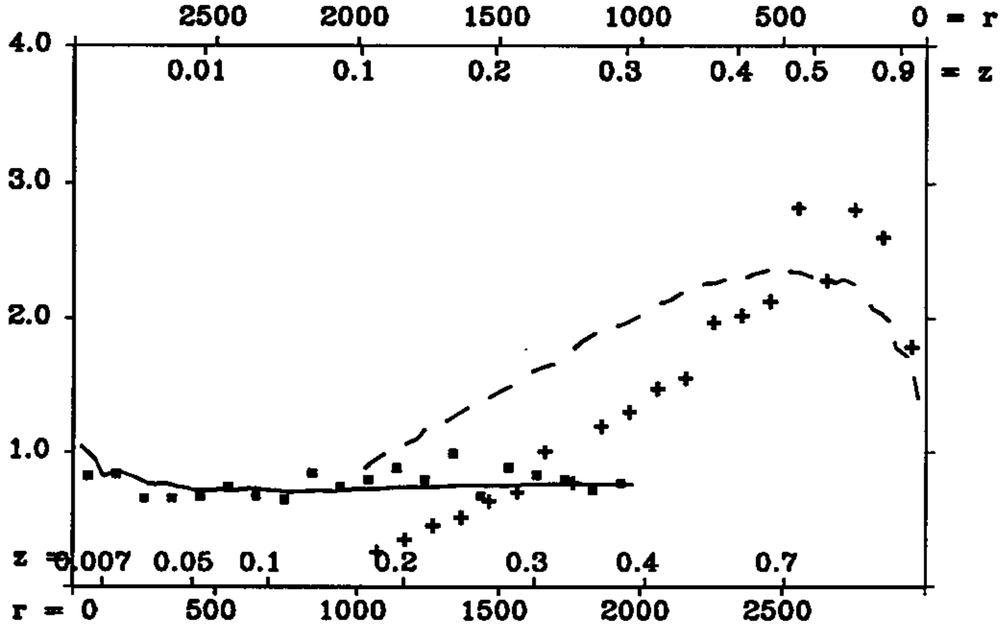


Figure 17. Hill's estimator for the ratio NI/EQ for manufacturing firms in 1992. Description as for Figure 2.

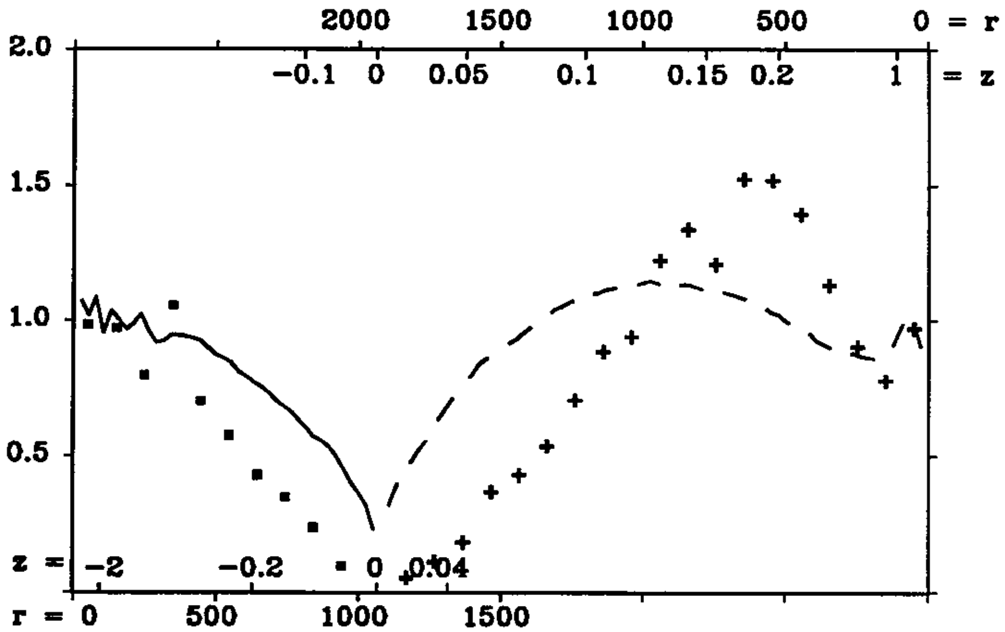
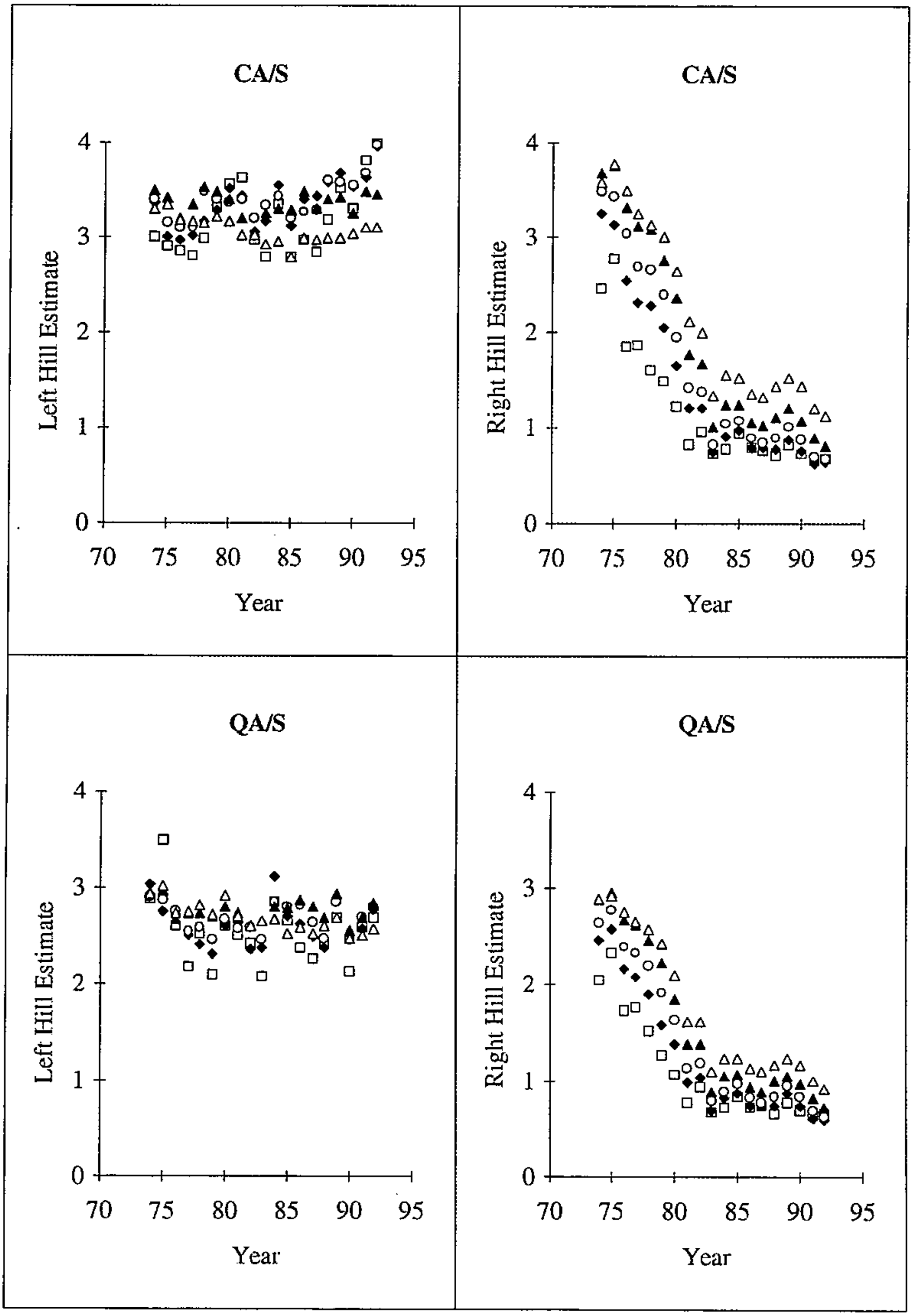
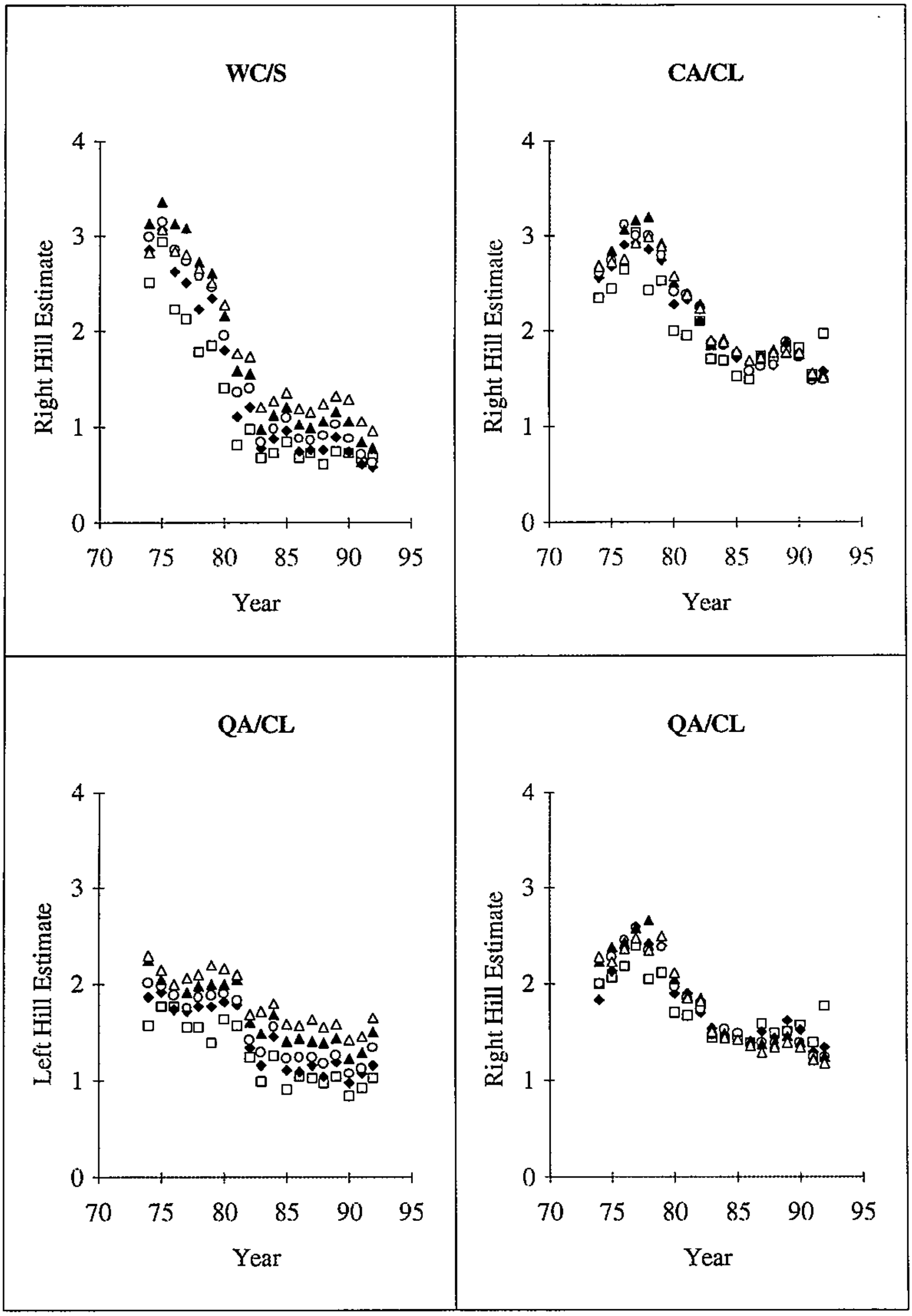


Figure 18. Hill estimates for the years 1974-92 for those ratio tails which exhibit power-law behavior. Values of the estimate $\hat{c}$ are plotted for various values of r. □ $r = 100$; ◆ $r = 200$; ◊ $r = 300$; ▲ $r = 500$; Δ $r = 1000$.




WC/S
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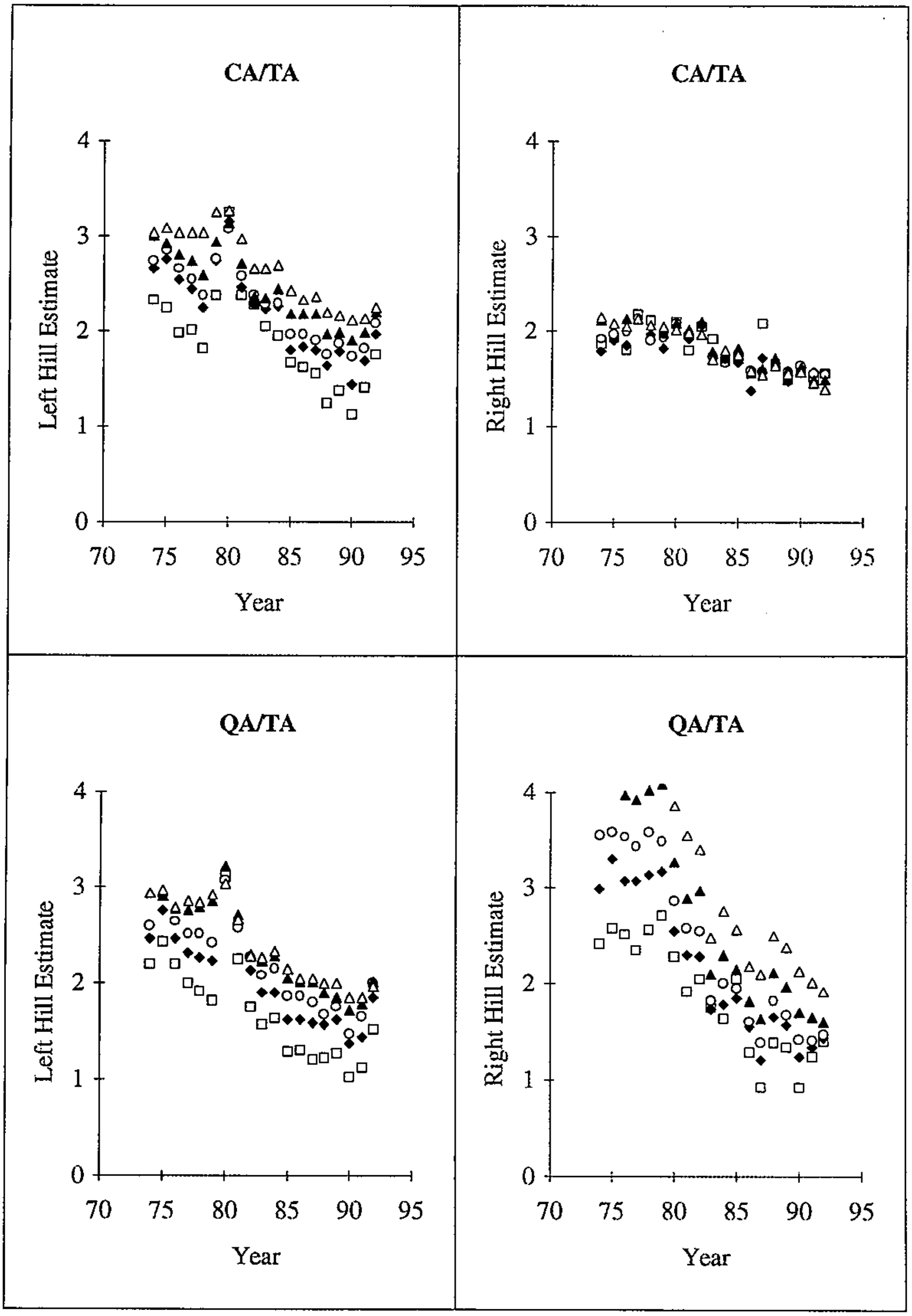
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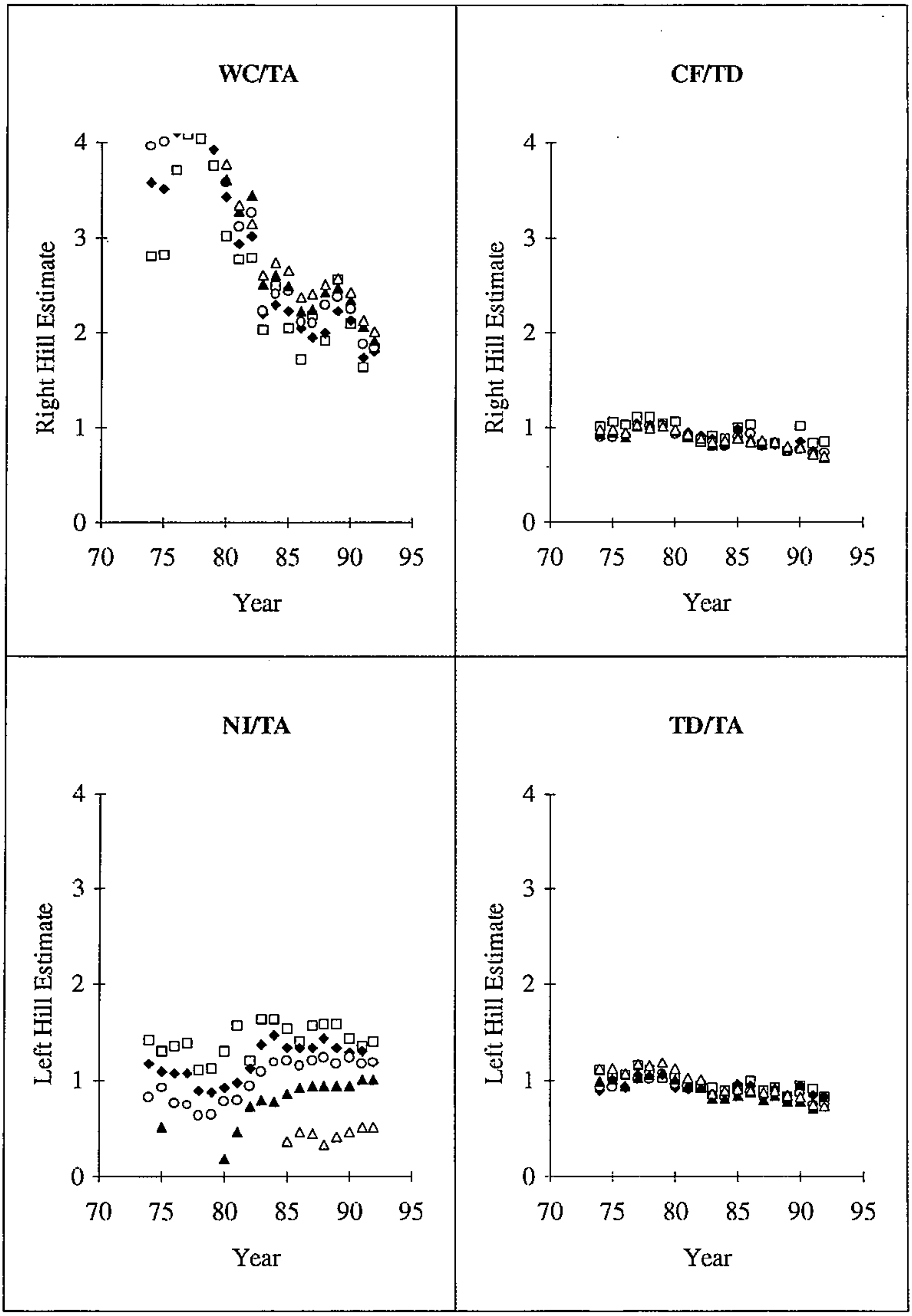
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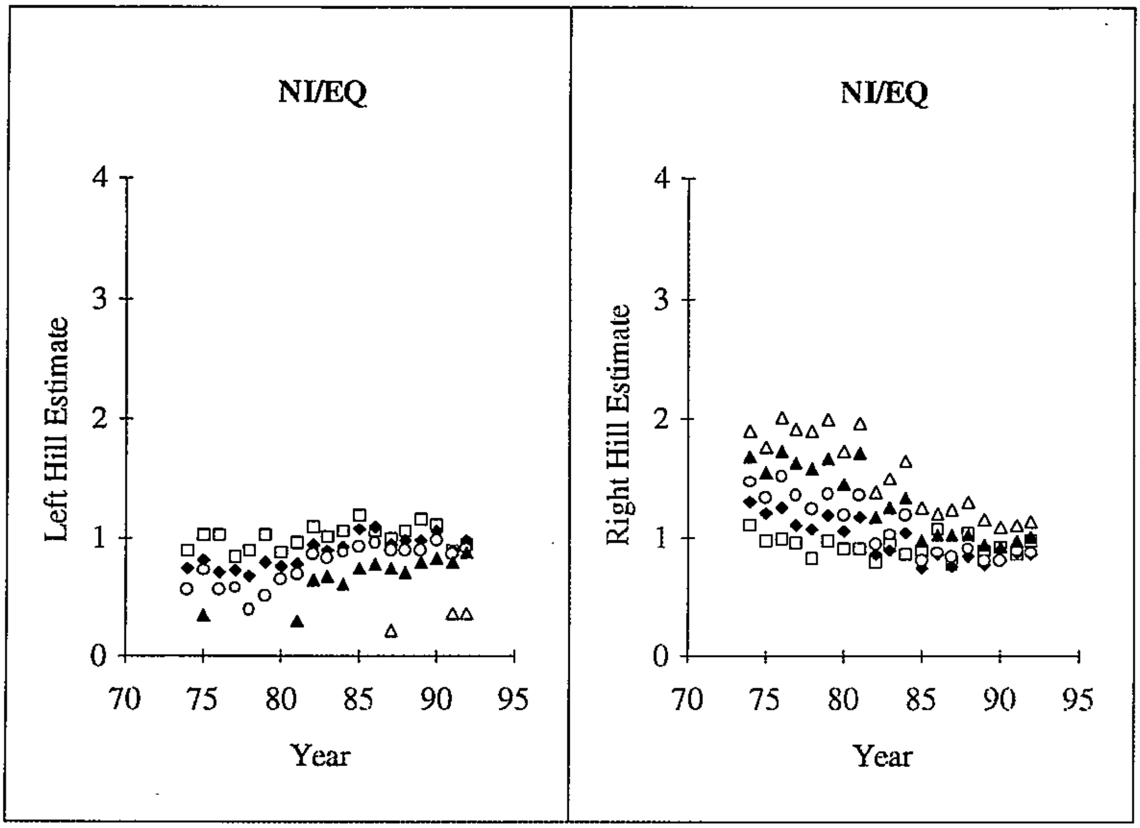
NI/EQ
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