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# Management of Effluent Discharges: A Dynamic Game Model*

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**Abstract**

This paper is concerned with the problem of the management of effluent dumped into a stream by identifiable polluters. The problem involves a *Regional Council* which imposes environmental levies on the polluters whose economic activity, otherwise beneficial for the region, results in pollution of the stream. The model for the problem of effluent management is formulated as a dynamic game between the *Regional Council* and the polluters. The game is "played" in discrete time. The players in the game are the polluters ("followers") and the Council (the "leader"). This formulation leads naturally to a Stackelberg concept of solution for the game at hand. Because of the obvious difficulties implied by this solution concept, an equilibrium will be sought through the use of an applicable Decision Support Tool wherever an analytical solution appears intractable.

The polluters are supposed to be *myopic* and *small*; and the Regional Council is interested in promoting production, collecting taxes, and in the clean environment. The model of spread of the pollution within the stream allows for advection and biodegradation.

## 1 Introduction

The problem of managing effluent is omnipresent. Practically every farm, factory and human settlement is producing liquid waste which is eventually dumped into a more or less distant river, lake or sea. These can cope with effluent quite well (*e.g.* by "neutralising" it by dilution) until *certain* critical concentration levels of environmentally unfriendly substances, present in the emissions, are exceeded. Concentrations above those levels can cause environmental damage like an epidemic, extinction of a species, destruction of recreational or spiritual value of an area *etc.* Keeping that damage minimal (in some sense) through managing effluent is therefore an acute necessity which has been recognised in literature; see [13], [4], [12], [14].

The cumulative effects

- of pollutants on the environment, and
- of human activities on production and pollution

imply the use of a dynamic model in the effluent management process. The dynamic aspect of pollution control has been studied, among others, in [14], [4], and [5].

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In general, the water pollution can be

◇ *point-source* where identifying effluent emission points is possible[1], (*cf e.g.* [4]), or

◇ *distributed-source* (*cf e.g.* [6]), in any other case[2].

Note that the above categorisation slightly differs from that which distinguishes between the point-source and the non-point-source pollution only (see [14], [12]). As effluent monitoring is costly, the non point-source approach might appear attractive even if monitoring *were* possible. However, the common conclusion which one can find in the literature, *cf* [14], [12], is that the minimum, or satisfactory, concentration of a pollutant can be achieved if "suspected" polluters are taxed *indiscriminately*[3] once the pollutant critical levels have been exceeded, apparently irrespective of the producers actual emission or waste abatement. Such a solution to the pollution problem implies "group responsibility" and can hardly be implemented in law conscious countries. In this paper, we concentrate on the point-source effluent management problem where the monitors' maintenance costs will be covered by an environmental levy, imposed on polluters for their using the purification capacity of the stream (so far, for free).

Another important distinction between the sources of effluent is whether it is an effect

*a.* of an economic activity of an agent, or

*b.* of a non economic activity of a community.

While the effluent from *b.* might, in some areas, be more intensive than a., it is difficult[4] to think that it would be controlled through a commercial mechanism. In this article we look for economic instruments and will deal with case a.

Individually monitored or not, indiscriminately levied or not, effluent producers are potentially in conflict with each other. The cause of the conflict can be economic competition, and/or global common constraints on the amount of effluent tolerated by the environment. On the other hand, the producers depend on some regional authority whose aim is to negotiate, and legislate, agreements on admissible pollution and abatement policies of each polluter. Features such as the conflicting interests and the possibility of negotiations naturally suggest a game theoretical approach towards solving the effluent management problem. Existing results in environment control, obtained through dynamic games, *cf* [10], [14], do not concern *point-source water* pollution. This paper is, to the author's knowledge, the first to try a dynamic game model for management of point-source water pollution.

The approach presented in this paper attempts to be consistent with the way in which negotiations concerning improvement of the water quality of a certain stream polluted by farmers are conducted. It assumes that if agreement about setting the emission levels is possible, it would be implemented by the Regional Council through the fees and charges levied on the polluters using the stream. The model for the effluent management problem is formulated as a (on-smooth) dynamic game between a leader and followers. The game is "played" in discrete time over a year but can be extended to an infinite time horizon. The players in the game are the polluters ("followers") and the Regional Council ("leader"). This formulation leads naturally to a Stackelberg concept of solution for the game at hand.

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[1] Which implies that monitoring of a single polluter is possible.

[2] Here, one can distinguish between the distributed-source pollution where it can be traced to a polluter, and where it cannot.

[3] Actually, for the stochastic case, [14] proposes to differentiate amongst polluters by making the levy amount dependent on the second derivative of a polluter value function. This does not seem to be easily computable, hence not applicable.

[4] Though not impossible. Think of penalties which a community would pay for not having installed a waste neutralisation plant.

In general, finding a Stackelberg equilibrium for a dynamic game is difficult (*cf* [1]). We will recommend the use of a Decision Support Tool (*DST*) similar to the one used in [8], should an analytical solution be unavailable.

The paper is organised as follows. First, in Section 2, the assumptions under which the model can function, are formulated. Next, in Section 3, the polluters' problem is discussed and modelled. In Section 4, The Regional Council problem is formulated. The possibility of solution to the resulting game problem is discussed in Section 5. In Section 6, a hierarchical optimisation problem of controlling a fictitious farm to environmentally acceptable standards is solved. Finally, the paper ends with conclusions and directions for future research.

## 2 Assumptions

Before we formulate the mathematical model of the problem at hand we draw up a list of assumptions about the physical situation which we model.

A *In the modelled area, the farms are "small" but "dangerous".*

It means that we expect a polluter to be able to abate his[5] pollution *entirely* *e.g.* by buying the manure removal services rather than investing in a physical abatement capital. On the other hand, an existing pollution sediment pond is almost full so that it will begin to leak to the river by the end of a production period if no abatement takes place.

B *In the modelled area, the polluters are "myopic".*

It means that we expect a polluter to maximise his yearly profit rather than a sum of discounted profits over an optimisation horizon. That behaviour can be perceived as practical in a situation when the Regional Council adjusts the levies each year and the polluters are left in uncertainty concerning the amount of future levies (for some results on how uncertainty impacts on *repetitive control* see [9]).

C *The Regional Council is interested in economic prosperity of its region.*
D *The Regional Council taxes the pollution emission "proportionally".*

Here, "proportionally" means that a polluter will be levied according to how much he contributes to pollution at a critical spot, rather than to how much effluent he injects into the environment.

E *The stream flow is fast so that the pollution diffusion process can be neglected.*

F *The Regional Council is interested in maintaining an average pollutant's concentration level within acceptable bounds.*

It means that the Regional Council's environmental concern is to avoid *prolonged* exposure of the environment to the pollution.

In the subsequent sections, we will build a model subject to the above assumptions.

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[5] Without prejudice against either gender an anonymous polluter will be referred as *he* and the Regional Council as *they*.

# 3 The polluters' problem

In this section we model the polluters' economic response to a given level of an environmental levy.

## 3.1 Pollution production

We suppose that each polluter makes decisions that control his emission levels (as well as the market output, abatement *etc.*). Assume that $i$-th polluter produces a pollutant within a production period (a month, say) $t$, $t = 1, 2, \dots$, in the amount $\alpha\pi_i$ where $\pi_i$ is the polluter's stock (or capital) and where $\alpha > 0$ can be interpreted as a "technological" coefficient, *e.g.* $n_1$ kg of manure *per* $n_2$ kg of stock *per* month.

The pollution in the sediment pond will change from one period to the other according to the following equation:

$$Y_i^{(t+1)} = Y_i^{(t)} + \alpha\pi_i^{(t)} - K_i^{(t)} - Q_i^{(t)}, \qquad Y_i^{(0)} \; - \text{ given} \tag{1}$$

where $K_i^{(t)} \geq 0$ is abatement "effort" (*e.g.* manure removal), $Q_i^{(t)}$ is effluent and $Y_i^{(0)}$ is the original contents of the pond.

In general, *effluent* $Q_i^{(t)}$ (here, the leak of the manure into the stream from the $i$-th farm), calculated for the same period as $\pi_i^{(t)}$, is a function of pollution $Y_i^{(t)}$, abatement $K^{(t)}$ and the size of the pond $\bar{Y}$. We will suppose that this function is of the following aggregate form:

$$Q_i^{(t)} = \max(0, Y_i^{(t)} + \alpha\pi_i^{(t)} - K_i^{(t)} - \bar{Y}_i). \tag{2}$$

The abatement effort $K_i^{(t)}$, $0 \leq K_i^{(t)} \leq \min(\bar{Y}_i, Y_i^{(t)})$ will be how much pollution a polluter decides to neutralise. Note that equation (2) and the above constraint make the pollution production model non smooth.

## 3.2 Pollution transport

Suppose that we are interested in keeping track of the *Ammonia-N*[6] concentration at a given section of the stream. Let

$$q_i^A(\pi_i, \bar{Y}_i; \tau) = \alpha_A \, q_i(\pi_i, \bar{Y}_i; \tau)$$

be concentration of the *Ammonia-N* emissions released to the stream by the $i$-th polluter at time $\tau$, $\tau \in [\tau_0, \tau_1]$ and $\alpha_A$ is the *Ammonia-N* contents in the effluent intensity $q_i(\pi_i, \bar{Y}, \tau)$.

The level of *Ammonia-N* concentration $C_i(\tau, x)$ at time[7] $\tau$, $\tau \in [\tau_0, \tau_1]$, and at the chosen section which is $x$ meters from where the $i$-th polluter is dumping his waste waters,

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[6] In this paper we will implicitly assume that curbing *Ammonia-N* emission is satisfactory. Actually, there are a dozen or so water quality measures like ***Suspended Solids, Conductivity, Enterococci, Biological Oxygen Demand etc.***, concentration of *Ammonia-N* being just one of them. Bearing in mind that all the measures are correlated quite strongly, choosing one of them should not be regarded as limiting the results obtained in the sequel. In particular, *Ammonia-N* usually dominates the total dissolved inorganic nitrogen so it appears to be a highly significant measure of water quality, see [7].

[7] *I.e.*, in a "micro" time scale as opposed to a "macro" time scale $t = 0, 1, 2 \dots$ More generally, the variables $q_i, c_i, Q_i$ *etc.* refer to one (production) period $t$, $t = 0, 1, 2, \dots$ For notational simplicity we drop the index $t$ in this section. The variables will be indexed with $t$ when we will consider the intertemporal character of the effluent management problem.

can be described by the following partial differential equation (*cf* [3]; compare [11]):

$$\frac{\partial C_i(\tau,x)}{\partial x} = -\frac{\lambda}{v}C_i(\tau,x) - \frac{1}{v}\frac{\partial C_i(\tau,x)}{\partial \tau} \tag{3}$$

where $\lambda$ is biodegradation rate and $v$ is the stream velocity. Here we assume that $v$ is large relative to the diffusion coefficient and neglect diffusion, *cf* [3].

The boundary conditions for equation (3) are: $C_i(0,x_i) = 0$, $C_i(\tau,0) = q_i^A(\pi_i,\bar{Y}_i;\tau)$ (and $C_i(\tau,\infty) = 0$). The constants $\lambda$ and $v$ can depend on the season. The solution to (3) is:

$$C_i(\tau,x_i) = e^{-\frac{\lambda}{v}x_i} q_i^A\left(\pi_i,\bar{Y}_i;\tau - \frac{x_i}{v}\right). \tag{4}$$

As we have $N$ polluters, each $x_i$ meters distant from the chosen section of the stream, the total concentration of the pollutant $C(\tau)$ at the section will be

$$C(\tau) = \sum_{i=1}^{N} C_i(\tau,x_i). \tag{5}$$

As said in *Assumptions* we are interested in an average (*per* production period) level of pollution. Assuming that $v$ is large enough to make $\frac{x_i}{v}$ small when compared with $T$ we can easily compute the average concentration $c_i(Q_i,x_i)$ of pollution originated from the $i$-th polluter within a production period $[\tau_0^t,\tau_1^t]$ as :

$$c_i(Q_i,x_i) = \frac{e^{-\frac{\lambda}{v}x_i}\int_{\tau_0}^{\tau_1} q_i^A\left(\pi_i,\bar{Y}_i;\tau - \frac{x_i}{v}\right)d\tau}{\tau_1 - \tau_0} = \alpha_A\, e^{-\frac{\lambda}{v}x_i}\frac{Q_i}{T}. \tag{6}$$

Also, the overall average concentration at a given stream section can be assessed:

$$c(Q_1,x_1,...,Q_i,x_i,...,Q_N,x_N) = \sum_{i=1}^{N} c_i(Q_i,x_i). \tag{7}$$

## 3.3 Profit maximisation

A one-period polluter's profit can be defined as

$$g_i^{(t)}(\mu;u_i^{(t)},K_i^{(t)}) = p\cdot u_i^{(t)} - f_i(\pi_i^{(t)}) - d\cdot K_i^{(t)} - \mu(c^{(t)})\,c_i(Q^{(t)},x_i) \tag{8}$$

where $u_i^{(t)}$ is the "other" control[8] of the $i$-th polluter, which can be interpreted as sales (at the end of period $t$) and is related to the average stock $\pi_i$ through the state equation:

$$\pi_i^{(t+1)} = \nu\,\pi_i^{(t)} - u_i^{(t)}, \quad \pi_i^{(0)} \text{ given.} \tag{9}$$

We expect the sales to be positive; however, as long as $\pi_i^{(t)}$ remain non-negative, a negative $u_i^{(t)}$ could be tolerated and interpreted as the cost to the polluter of purchasing reproductive material[9]. The (exogenous) production price is $p$, $f_i(\cdot)$ is the $i$-th polluter's cost function, $d$ is the cost of a unit of abatement, $\nu$ is stock reproduction/decay rate[10] and $\mu(c)$ is the

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[8] The first is $K_i^{(t)}$.

[9] Here, we assume that the sale price equals the purchase price.

[10] Obviously related to $\xi$ as $\nu = e^{\xi T}$.

*leader*'s decision rule on how much tax on pollution has to be applied in order to keep the river environmentally sound.

If we assume that a production period $[\tau_0, \tau_1]$ corresponds to a month, the $i$-th polluter's annual problem consists of maximisation of:

$$\sum_{t=0}^{11} g_i^{(t)}(\mu; u_i^{(t)}, K_i^{(t)}) + W(\pi_i^{(12)}) \tag{10}$$

with respect to $u_i^{(0)}, ..., u_i^{(11)}$ and $K_i^{(0)}, ..., K_i^{(11)}$, where $W(\pi_i^{(12)})$ is a final state function.

The choice of $W(\pi_i^{(12)})$ depends on the policy which the Regional Council wants to implement through the environmental levies. The Regional Council does not expect farmers to dramatically change their fixed capital, or liquidate their stock unless the farm's production resulted extremely environmentally unfriendly. Therefore we will model $W(\pi_i^{(12)})$ as

$$W(\pi_i^{(12)}) = -\rho(\pi^{(12)} - \pi^{(0)})^2 - \psi\mu(c^{(12)})\, c_i(Q^{(12)}, x_i) \tag{11}$$

$\rho > 0, \psi > 0$. The first term models a polluter's preference for not changing his "fixed" capital (*e.g.* buildings). The second term reflects the fact that, should a farmer feel obliged to consider winding down his business, he would have to allow for the next-year first-month's environmental tax in the current year's budget. Parameter $\psi$ is the Regional Council policy instrument. Through setting $\psi < 1$, the Regional Council will encourage farmers to stay in business; having legislated $\psi > 1$ would make diminishing stock more attractive.

For notational compactness we introduce a new symbol $\Pi_i$ to denote all $i$-th polluter's decision variables as

$$\Pi_i \equiv \begin{bmatrix} \mathbf{u}_i \\ \mathbf{K}_i \end{bmatrix} \tag{12}$$

where $\mathbf{u}_i \equiv [u_i^{(0)}, ..., u_i^{(11)}]'$ and $\mathbf{K}_i \equiv [K_i^{(0)}, ..., K_i^{(11)}]'$. Note now that through the levy rule $\mu(c)$, polluter $j$-th's decision $\Pi_j$ has an impact on $i$-th polluter's action $\Pi_i$. We will allow for that dependence in the $i$-th polluter's profit by noting it as

$$G_i(\mu; \Pi_1, ...\Pi_i, ..\Pi_N) \equiv \sum_{t=0}^{11} g_i^{(t)}(\mu; u_i^{(t)}, K_i^{(t)}) + W(\pi_i^{(12)}). \tag{13}$$

Let us assume that $f_i(\pi_i)$ and $\mu(c)$ were chosen such that the following Nash equilibrium exists[11]

$$\hat{\Pi}_i = \arg\max G_i(\mu; \hat{\Pi}_1, \hat{\Pi}_2, ..\Pi_i, ..\hat{\Pi}_N). \tag{14}$$

If agents behave rationally they will choose $\hat{\Pi}_i$, given $\mu$ and other exogenous parameters. In the result, the maximising sales and abatement effort become functions of $\mu$: $\hat{\mathbf{u}}_i(\mu)$ and $\hat{\mathbf{K}}_i(\mu)$. Subsequently, the levels of stock $\hat{\pi}_i^{(t)}(\mu)$, pollution "production" $\hat{Y}_i^{(t)}(\mu)$ and effluent $\hat{Q}_i^{(t)}(\mu)$ can be computed for $t = 0, 1...11$. This, through (6) and (7), allows us to establish the relationship between the polluter's decision variables and the pollution. This relationship is the $i$-th polluter's *reaction function* to the levy rule $\mu$.

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[11] For an $N$-person non-zero sum infinite game to have a Nash equilibrium in pure strategies, strict convexity of $G_i$ in $\Pi_i$ for every $\Pi_j \quad j = 1, 2, ..., i-1, i+1, ..., N$ is required, and $\Pi_i$ has to be from a compact set, *cf* [2].

Note that, even for one polluter, the problem (14) is a rather difficult optimisation problem as $g_i^{(t)}$ are non smooth functions of $K_i$. Moreover, because of the lack of the discount factor, the actions taken in month four, say, will probably be interchangeable with those of month five. This will make the maximum of $G_i$ "flat" and difficult to compute.

# 4 The Regional Council's problem

The Regional Council is interested in the clean environment, in promoting production, in collecting revenue;and they have also to maintain the pollution monitors. This objective, for one production period (for the time being, we omit $t$), can be modelled by a function[12]

$$h(c; \Pi_1, \Pi_2 \ldots \Pi_N; \mu) = -\phi c^2 + \sum_{i=1}^{N} (\zeta p u_i + \mu_i c_i). \tag{15}$$

As previously, $\Pi_i$ is the $i$-th follower's decision, and $\mu$ is the decision rule of the Regional Council; coefficient $\zeta$ represents a "local" tax rate (could be part of GST[13]). Expression $\phi c^2$ is (obviously) non negative and represents the cost (within a production period) which the Regional Council has to incur in order to "clean the mess" (or "face" it), resulting from the average pollution level $c$. It also captures the fact that small concentrations are less costly to deal with than large concentrations. Note that, because concentration levels cannot be negative, the adopted cost function will not punish the Regional Council for over-abating the pollution.[14]

Now, suppose that the followers behave rationally *i.e.* they use a solution to (14). Substituting it in (15) defines

$$\bar{h}(c, \mu) \equiv h(c; \hat{\Pi}_1(\mu), \hat{\Pi}_2(\mu), \ldots \hat{\Pi}_N(\mu); \mu). \tag{16}$$

Assume that the Regional Council is engaged in long-term planning and that $\mu$ is their *stationary* levy rule[15]:

$$\mu_i(c^{(t)}) \tag{17}$$

which means that the Regional Council sets the levy for period $t$ depending on the average pollution level. (The pollution level, in turn, is a function of the polluters' state and decision variables (see (14), (7), (6)).

The Regional Council's objective function can be defined as

$$J(c^0, \mu) = \sum_{t=0}^{\infty} \delta^t \bar{h}(c_t, \mu) \tag{18}$$

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[12] Which represents certain revenue of the Regional Council. The revenue should also cover the monitors' maintenance cost $mN$ where $m$ is a monitor maintenance cost which is independent of the problem's decision variables. In fact, the Regional Council's objective is multicriterial and could be modelled as in [8]; in that context, (15) can be interpreted as an attempt of scalarisation of the "true" multidimensional objective.

[13] Goods and Services Tax

[14] However, if a critical level $\bar{c}$ was given, below which the pollutant's concentration levels were considered harmless the penalty function should have the following form

$$\phi(c - \bar{c})_+^2$$

where $(z)_+$ means $\max\{0, z\}$.

[15] Because the strategy is stationary *i.e.*, same for each $t$, we do not need to distinguish between a decision rule at $t$, and the whole sequence of such rules.

where $0 < \delta < 1$ is a discount factor; $c^0$ and $c^t$ are initial and at time $t$ average concentrations of the pollutant, respectively. Hence the Regional Council optimisation problem can be defined as follows:

$$\hat{\mu} = \arg\max J(c^0; \mu). \tag{19}$$

# 5 The Decision Support Tool

The effluent management problem consists of the Regional Council finding an optimal levy strategy while the polluters are maximising their profits. Using the notation and formulae from the previous section the problem defines a game as follows:

$$\left.\begin{array}{rcl} \hat{\mu} & = & \arg\max J(c^0; \mu) \\ \hat{\Pi}_i(\mu) & = & \arg\max G_i(\mu, \hat{\Pi}_1, \ldots \Pi_i, \ldots \hat{\Pi}_N), \quad i = 1..N \end{array}\right\} \begin{array}{l} (a) \\ (b) \end{array} \tag{20}$$

Note that (20) is a hierarchical game problem. One can look for a solution to this game under a Stackelberg solution concept (*cf* [2]).

An analytical solution to (20) will exist not very often[16]. We recommend a *satisfactory solution* to this game which will be computed through a Decision Support Tool (*cf* [8]), as one which will be easier to interpret and apply by the Regional Council decision makers than other solutions to this problem. The *DST* will function in the following way:

I. The Regional Council sets up an environmental levy rule $\mu^I$ (see (17)) which places a price on the emission levels. Typically, as this price increases production output decreases as well as the use -or abuse- of the environment.

II. The Regional Council solves the followers' problem (20b) which simulates a polluter's reaction to the levies imposed.

    The Regional Council examines the (simulated) results. In particular, indices $\bar{h}(c, \mu)$, and $J(c^0; \mu^I)$, are to be computed. As the indices are only aggregated measures of the Regional Council objectives, answers to the following questions have to be considered: has concentration been confined to acceptable limits? by how much has the output decreased? will this have an impact on the region's employment situation? *etc.*

    If answers to the above questions are not satisfactory, *i.e.* the Regional Council cannot accept the trade-off between the economic activity and conservation implied by the levy rule, its form is modified[17] and the Regional Council returns to step I.

III. Otherwise the levy rule is saved as an element of a set $\mathcal{T}$; the corresponding values of index $\bar{h}$, and $J$, are also saved.

    The Regional Council returns to step I and repeats the steps I - III until the set $\mathcal{T}$ contains a few elements.

IV. The Pareto set $\mathcal{P}$ is created from $\mathcal{T}$ by eliminating $\mu$ for which the corresponding $J$s are *dominated.*

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[16] Even if a solution to (20) exists it might not be . unique. In particular, if (14) admits more than one equilibrium the leader's strategies will range between so called ***pessimistic*** and ***optimistic*** Stackelberg solutions, *cf* [1].

[17] If the rule (17) was chosen linear (or affine) the slope (and/or the intercept) would be modified.

V. The Regional Council selects one levy rule from $\mathcal{P}$ which will be enforced.

We suppose that between two extreme situations:
*i.* no levies, maximal production, uncontrolled pollution, and
*ii.* high levies, no production, minimal pollution
there will be at least one which will be acceptable for the Regional Council; hence the above "algorithm" should have an element to converge to. In general, many satisfactory solutions may exist each with a different trade-off between economic activity and the use of the environment. The solution which will be enforced will typically be chosen form the Pareto set using some voting procedure or, simply, common sense.

A solution which is arrived at through the Decision Support Tool will be called *satisfying*.

# 6 The numerical example

We will illustrate how the *DST* introduced in Section 5 should be used for solving a problem of type (20).

In this section, we assume that the "lower" optimisation level is constituted by one follower[18] only. In other words, we restrict our interest to the hierarchical component of game (20).

## 6.1 Meanwhile "back on the farm" ...

We are concerned with the effluent discharge from a farm and in this case have initially chosen a pig farm to model the process. The operation of this farm is assumed to occur over a period of $12T$ *i.e.* twelve months, with monthly sales, checks or controls. In order to illustrate the process, early figures were based on statistical averages for pig farms.

We assume that the farmer holds pigs up to an average weight of around 60 kg and has an alternative at the end of a month either to maintain the stock or to sell. Suppose that this farm begins with approximately 330 pigs (for sale). Most farms have about 10-12 sows per 100 pigs so this farm would have about 30-35, on which we base the reproduction rate $\nu$. Each sow can wean (in ideal conditions) around 20 pigs a year, in this case 600-700 pigs (36 000-42 000 kg). So if we start[19] with $\pi^{(0)}$ =20000kg, and maintain a constant number of pigs through sales each month, we should have produced close to 40000 kg over the year. The rate $\nu = 1.167$ allows this.

We run a few scenarios of sales $\{u^{(t)}\}_{t=0}^{11}$ to examine what income can be associated with a farm this size, still without an environmental levy. Figure 1 shows an "optimal" and a plausible sales scenario; the corresponding stock quantities are in Figure 2.

As introduced in Section 3, $Y^{(t)}$ represents the production of the manure each month. With a technological coefficient of $\alpha = .5$ each pig is discharging half of its weight in effluent each month. The effluent $Q^{(t)}$ depends on the abatement effort $K^{(t)}$; let $d = \$.1/kg$. The average pollution contribution of each farmer over a month is simply the amount emitted ($Q$) multiplied by the exponential dispersion rate ($e^{-\frac{\lambda}{v}x}$) and divided by the time frame $T$ = 30 days (see (6)). A distance of $x$ = 1000 metres away from the pollution source was chosen for the test section of the river which has a linear velocity of $v$=1 m/s. From

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[18] Single, or a *multifollower* which could be an aggregate.

[19] Since now $N = 1$ we drop the index $i$ from all variables.

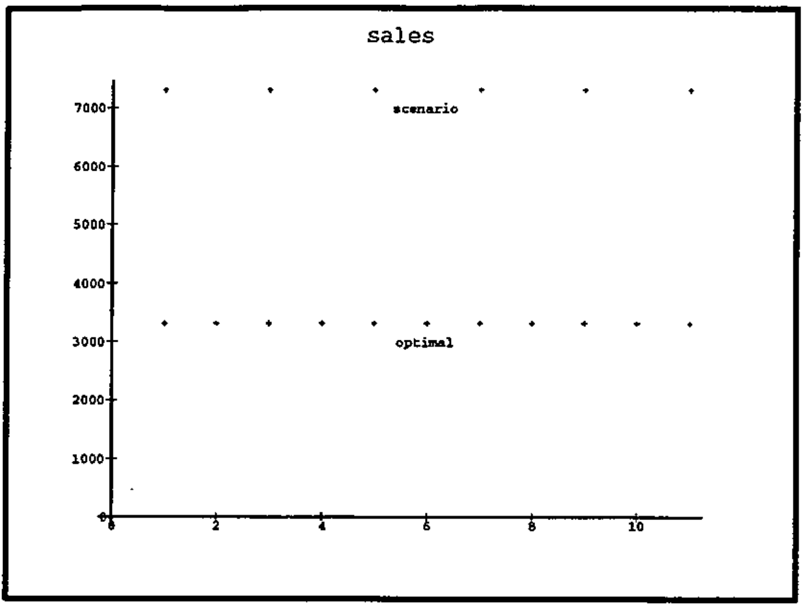

Figure 1: An "optimal" sales and a plausible sales scenario ($u^{(t)}$ in kg *per* month).

these parameters a biodegradation rate of $\lambda = .0001$ seemed reasonable, based on resulting pollution figures. Subject to these parameter values, the size of the manure sediment pond $\bar{y} = 300\,000$ kg and its initial contents $Y_0 = 295\,000$ kg the pollution concentration levels corresponding to the "optimal" and a "plausible" sales scenarios were computed, see Figure 3.

The farmer's revenue is assumed to come only from the sale of stock, that is, the price ($p$) multiplied by the $u$s (sales). The farmer's cost function is assumed to be quadratic[20] - the usual convex shape, and a function of the stock only - in the form

$$f(\pi) = a_1 \pi^2 + a_2 \pi + a_3. \tag{21}$$

The parameters $a_1 = .6667 \cdot 10^{-5}$, $a_2 = -.26667$, $a_3 = 14\,000$ ($\rho = .002$ and $\psi = .5$ for the final state function, see (10)) were chosen so as to obtain realistic costs each month for this size farm). The incomes generated were: \$62 647 and \$41 404 for the two sales scenarios, respectively.

Assume now that the farmer, for altruistic reasons, decides to abate the pollution in the amount of \$500 *per* month. This will result in diminishing his income by

$$12 \text{ months } \times 500\$/\text{month } = \$6\,000.$$

The resulting pollution concentration is shown in Figure 3 (together with concentrations resulting from other abatement efforts). We assume that neutralisation of one unit of $Y$ costs $d$ =\$.1 therefore the \$500 spent on abatement *per* month can correspond to the removal of 5 000 kg of the manure (*per* month). The \$6 000 (*per* year) polluter's effort toward the pollution abatement, which diminishes his profit by the same amount, is a guide-line for us of how to design the environmental rule $\mu$. If we accept the pollution levels which result from the \$500/month abatement, for the rule to be an incentive to abate the pollution, this farmer's environmental tax should exceed \$6 000 in the event of no abatement.

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[20] Compare footnote (11).

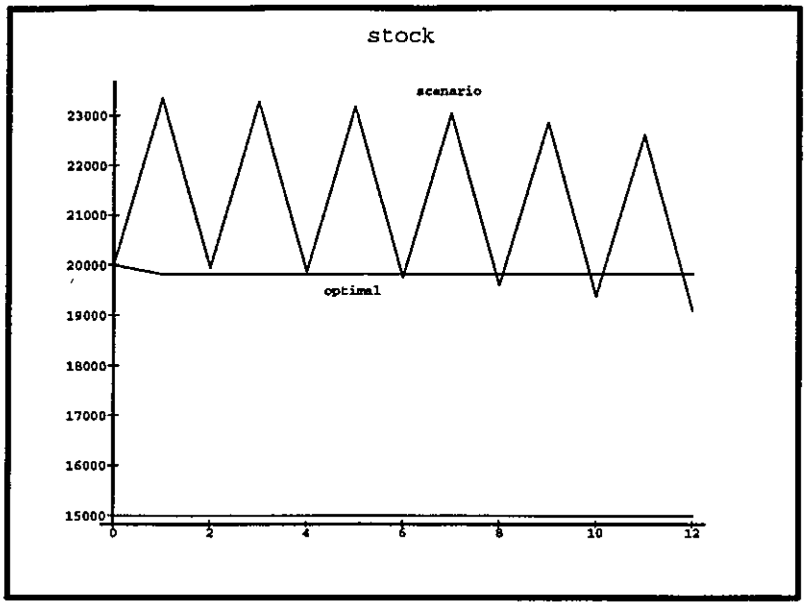

Figure 2: An "optimal" stock and a plausible-sales-scenario stock ($\pi^{(t)}$ in kg *per* month).

## 6.2 ... and in the Regional Council

In the event of no abatement by the farmers the Regional Council is faced with the necessity of "cleaning the mess" in order to not forgo tourist incomes, avert an epidemic disaster *etc.* If abatement of one unit of pollution ($Y$) costs $d$ to the farmer we will assume that the cost to the Regional Council of neutralising one unit of the "pure" pollutant $c$ has to exceed

$$D \equiv \frac{d \cdot T e^{\frac{\lambda}{v} x}}{\alpha_A}$$

for environmentally significant levels of $c$. We will arbitrarily assume that the pollutant's level $\underline{c}$=.25 kg/month is critical for the environment and calibrate $\phi c^2$ in the following manner:

$$\phi \cdot \underline{c}^2 = D \tag{22}$$

which results in

$$\phi = 16D.$$

Furthermore, assume that $\zeta = .075$ (7.5 %).

In this numerical example, we will not compute the long-term Regional Council objective function $J$. The polluter is expected to keep the twelfth month's stock close to the zeroth month's one ($\rho > 0$) therefore the next year's solution should not differ too much from the current one's. Consequently, $\bar{h}(c, \mu)$ (or $\bar{h}(c, \mu)/(1 - \delta)$, $\delta$ - discount factor), can also be a measure of the Regional Council's long term objective.

Having set all the problem parameters we can proceed with the application of the Decision Support Tool.

In Step I, we have to propose a particular form of the levy rule. Let the pollution tax

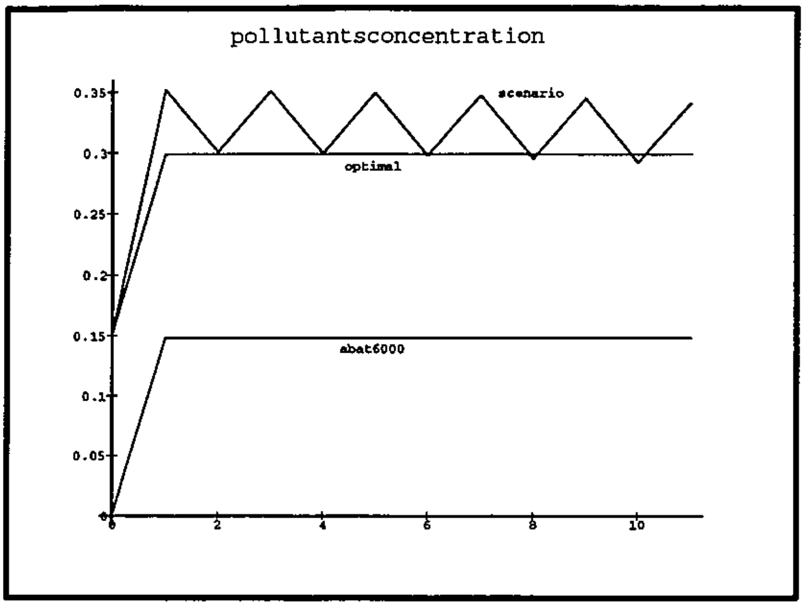

Figure 3: Average non abated pollutant's concentration corresponding to the "optimal" stock, to a stock scenario; and the partially abated pollution ($c^{(t)}$ in $\frac{kg}{m^3}$ *per* month).

be *proportional*[21] to the amount of pollutant

$$\mu(c) = M \cdot c \tag{23}$$

where $M$ (in $\$/\frac{kg}{m^3}$) is the the Regional Council's decision variable. Now, we can compute the value of a one-year Regional Council objective function $\bar{h}(c, \mu)$, and the follower's profit, for a given value of $M$.

The $\bar{h}$'s values corresponding to the farmer's "optimal" (still without abatement) and "plausible" sales scenarios, for $M$ chosen at the level of 1000, are given in Table 1. This table shows, in general terms, how the levy mechanism could work: threatened by the tax, the follower will choose to abate, which will improve his, and the leader's, objective function values.

| abatement: | \$0.0 | \$6 000 |
|---|---|---|
| $M = 0$ | (-50 736; 62 647) | (-10 214; 56 647) |
| $M = 10\,000$ | ( -40 919; 52 149) | (-7 806; 53 787) |

Table 1: Leader's and follower's objective values.

In Step II the follower is expected to solve his problem (14) optimally. Table 2 shows the follower's profits computed as the results of his optimal responses to various $M$, and the corresponding leader's objective values. The annual abatement cost is now determined by the product sum:

$$\sum_{t=0}^{11} d \cdot \hat{K}_M^{(t)}$$

---

[21] Other decision rules can and should be discussed. Note that the constant tax rule (*i.e.* just $\mu(c) = M$) would lead to the polluter's profit (8) linear in $K$ with all consequences of this fact like the bang-bang control; which, on the other hand cannot be excluded *a priori*.

where $\hat{K}_M^{(t)}$ is the optimal abatement given $M$. The optimal abatement time profiles shows Figure 4.

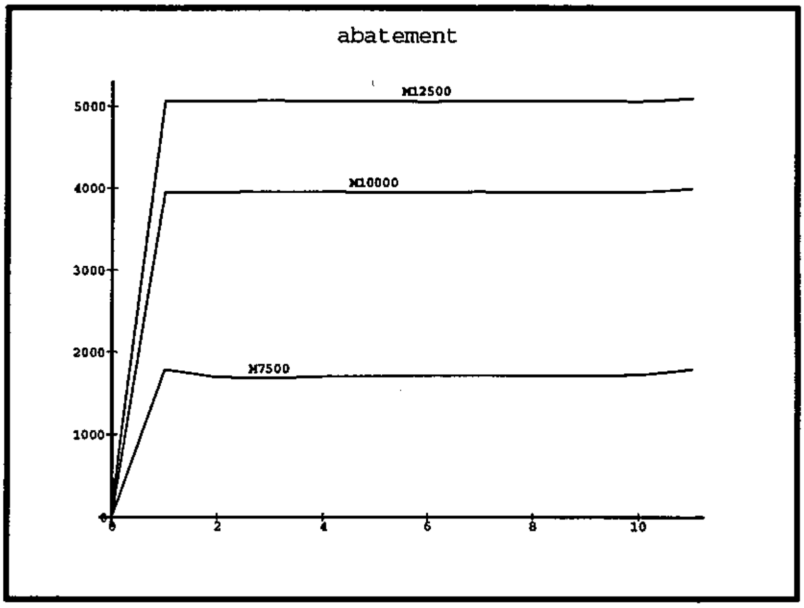

Figure 4: Optimal abatement ($K^{(t)}$ in kg *per* month).

| abatement: | \$0.0 | \$0.0 | \$0.0 | \$1 |
|---|---|---|---|---|
| $M = 0$ | (-50 736; 62 647) | – | – | – |
| $M = 500$ | – | (-50 182; 62 123) | – | – |
| $M = 2\,000$ | – | – | (-47 556; 60 569) | – |
| $M = 5\,000$ | – | – | – | (-43 016; 57 532) |

| abatement: | \$1 888 | \$4 345 | \$5 579 |
|---|---|---|---|
| $M = 7\,500$ | (-26 332; 55 369) | – | – |
| $M = 10\,000$ | – | (-12 123; 54 262) | – |
| $M = 12\,500$ | – | – | (-6 702; 53 492) |

Table 2: Leader's and optimal follower's objective values.

As said in footnote (12), and in step II of the *DST*, the Regional Council's problem is multicriterial. Hence $h(c, \mu)$ is an aggregate measure of the plethora of indices. Consequently, the decision maker will usually also want to know what the follower's *reaction profiles* corresponding to each $M$ are. (The comparison of a few sets of the profiles with the corresponding $h(c, \mu)$s can teach the decision maker how to interpret the different values of $h(c, \mu)$.) In that sense, Figures 5, 6 and 7 complement Table 2 by showing the pollutant's concentration, and the sales and stock, which are the consumer's optimal replies to the leader's decisions on $M$.

The set $\mathcal{T}$ of Step III is defined through Table 2 and Figures 4 - 7 . As it contains the non dominated solutions only, it is identical with $\mathcal{P}$ (Step IV).

It is interesting to see how the polluter's behaviour will be modified depending on the environmental tax. For small $M$, the reduction in pollution (see Figure 5) is not large

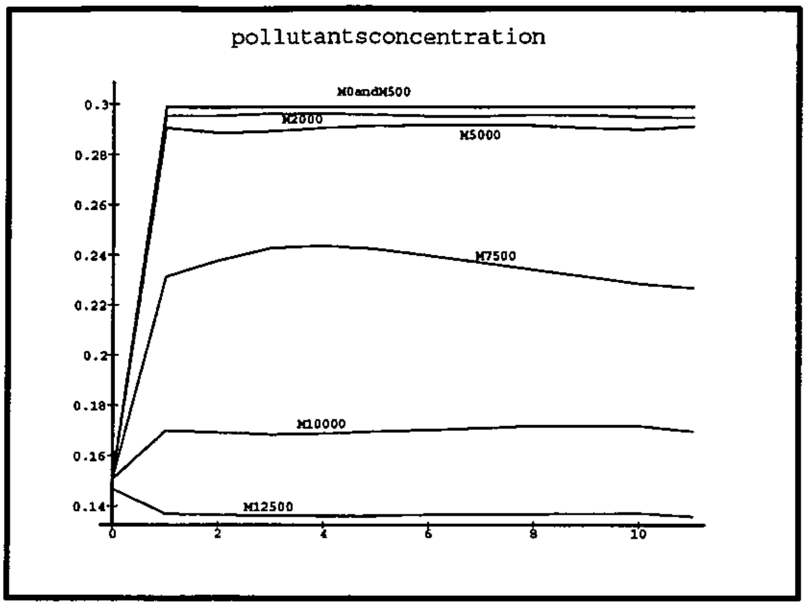

Figure 5: Average pollutant's concentration ($c^{(t)}$ in $\frac{kg}{m^3}$ *per* month).

and achieved only through reducing the stock (see Figure 7). For $500 < M \leq 5\,000$ the pollution diminishes substantially, but still by the reduction of the stock. And finally, $M > 5\,000$, the abatement is becoming intensive whereas the stock is kept approximately constant.

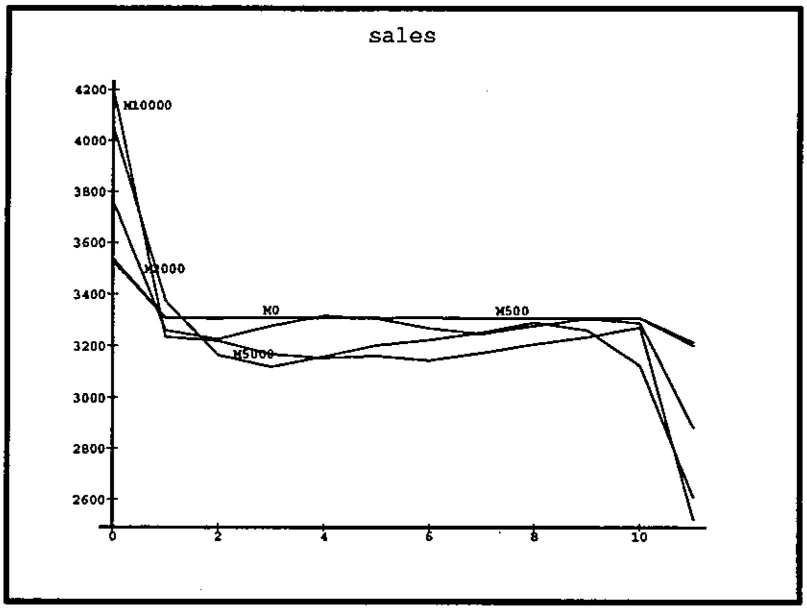

Figure 6: Optimal sales ($u^{(t)}$ in kg *per* month).

The apparently irregular pattern of optimal sales and stock for $M \geq 2\,000$ (see Figures 6 and 7) requires comments. The follower's objective function is "flat" in $u^{(t)}$ for $t$s from the middle of the year; in other words, months in the middle of the year are indistinguishable for the farmer, and for the optimisation routine. Therefore, which value of $\hat{u}^{(t)}$ the routine was picking up as optimal, depended to a great extent on the starting point.

Finally, we know the solution that the Regional Council may want to enforce in Step V. The $M = 7\,500$ solution seems to be a secure candidate. It gives the Council a "good" index and also guarantees that the pollutant's concentration is kept below the limit of .25 kg/month (Figure 5). In (22), this level was assumed to be critical. In this sense, the $M = 7\,500$ is a *satisfying* solution.

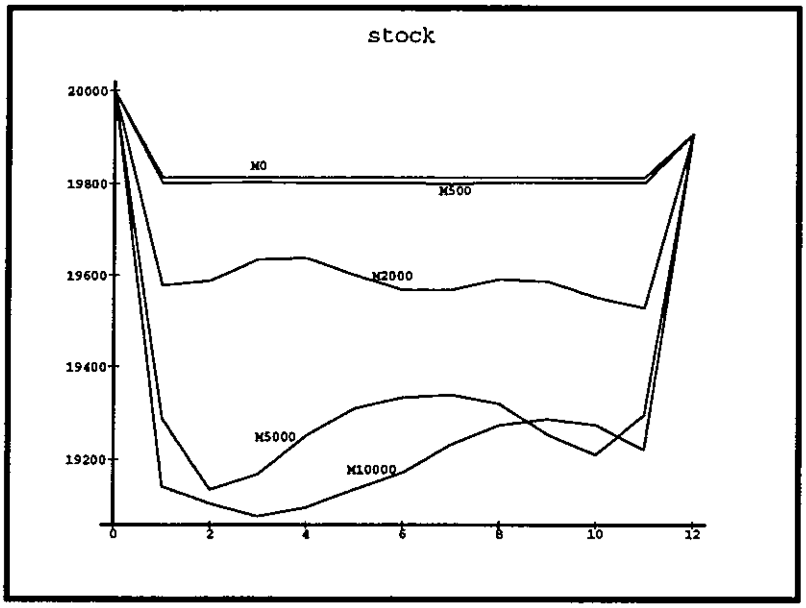

Figure 7: Optimal stock ($\pi^{(t)}$ in kg *per* month).

# Concluding remarks

We presented in this paper a comprehensive model of effluent management which resulted in a hierarchical game with a Nash equilibrium on the lower level. In the numerical example, we concentrated on the hierarchical aspect of the game and examined the interactions between the leader and a follower. We showed that this game can be solved by arriving at a satisfactory solution obtained through the use of a Decision Support Tool.

Further research should include a study of a numerical procedure which would handle more than one follower and relaxation of some of the Assumptions.

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