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Endogenous technological change, innovation diffusion and transitional dynamics in a nonlinear growth model

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ABSTRACT

This paper addresses capital accumulation and capital productivity change in an economy with endogenous technological change and floors and ceilings in activity. The properties of the resulting two-variable nonlinear differential equation system are studied in some detail. The welfare implications are also considered. When discrete lags are introduced, wide-ranging behaviour emerges, which includes convergence to a steady-state, catastrophes, hysteresis, limit cycles and chaos. Simulations illustrate the results. It is found that external shocks, such as the diffusion of innovations from elsewhere, do not just change the level of the steady-state equilibrium but also the dynamical properties of the paths of output and productivity.

KEYWORDS:
Economic Growth; Nonlinear Dynamics; R&D; Bottlenecks.

I. Introduction

In recent years, there has been a renewed interest in the theory of economic growth.[1] Much of this literature is concerned with modelling the externalities associated with innovation and human capital formation. In such models, one can study the effects of formal education, learning by doing, innovation, or trade on the long-run equilibrium growth path of the economy.[2] While the presence of a positive feedback between income and technological change opens up the possibility of ever-increasing growth rates (as in e.g. Romer, 1986), there is also (among the developed countries at least) some evidence of convergence to a balanced growth path and "catching-up" (Baumol, 1986). Where convergence occurs, the capital-output ratio is constant over time and similar across countries, while the share of investment in GDP declines the higher the level of income (Romer, 1989, pp. 53-70). However, when less developed countries are also considered, the results are less conclusive. For example, post-war data compiled by Summers and Heston (1991) suggest that there is generally little correlation between the share of investment in GDP and the rate of growth in income per capita. Apparently, many economies experience varying capital-output ratios and are not on a balanced growth path. Therefore, it may be interesting to investigate more carefully under which conditions balanced development may emerge.

In this paper we consider the issue of variations in capital productivity (the reciprocal of the capital-output ratio) as a consequence of the existence of so-called structural floors and ceilings - as capacity constraints for activity - in the economy. More specifically, we are concerned with the impact of a Research and Development (R&D) sector on capital productivity in a situation where such a sector is only effective when economic activity is within certain lower and upper bounds. If the level of economic development in a region or country is low, channelling resources to the R&D sector may be counterproductive because of the lack of a "critical mass"-provoking synergy. In this case, overall productivity may decline rather than increase and the economy may benefit more from simply adopting - with some time delay - innovations generated by other economies. When the level of activity is much higher, the conditions may be favourable for the nurturing of an R&D sector, but the output of this sector will eventually face diminishing returns due to bottleneck phenomena such as inadequate infrastructure, congestion, high wage claims by workers, or the presence of a work force which is insufficiently trained to make the best of new technologies (a situation arising, among others, in Silicon Valley). The paper explores the impact on the path of output in the

[1] See, for example, Stern (1991) for an overview and assessment.

[2] Clear examples are the models developed by Lucas (1988), Romer (1990) and Grossman and Helpman (1991).

economy when conventional capital accumulation is combined with such endogenous productivity changes resulting from innovation diffusion and R&D activity.

The next section focusses on the impact of R&D investment and the acquisition of technology from elsewhere on capital productivity. Section III discusses the implications of the above mentioned bottlenecks which may thwart the efficiency of new technology. The implications for welfare are investigated in section IV. The additional consideration of an exogenously set floor for output to achieve productivity enhancement is taken up in section V.

The reformulation of these models in a framework with discrete lags in behavioural responses introduces many of the issues which are central to the burgeoning literature on nonlinear dynamics, specifically the conditions (i.e. the mathematical properties of the model) under which catastrophes and chaos emerge.[3] Simulations with our model of endogenous productivity highlight some of these issues in section VI. The final section puts the theoretical results of the paper in a broader perspective.

II. Capital productivity and technological change

As in most models of economic growth, one of the primary sources of the expansion of output is the accumulation of productive and reproducible capital. It is assumed that the stock of fixed capital is subject to physical depreciation at a constant rate. Hence gross fixed capital formation is described by

$$\dot{K} = I - \delta K \tag{1}$$

where $\dot{K}$ denotes dK/dt, K is the capital stock, I is gross investment and δ the rate of depreciation. While the government budget and the balance of payments would affect macroeconomic savings, we shall for simplicity take for granted equilibrium between capital increase and savings. Hence we adopt the long-run investment function:

$$I = \sigma_1 Y \tag{2}$$

where σ_1 is the average propensity to save and Y is national income. The value of σ_1 is of course not arbitrary. If capital accumulation takes place in an economy with purely competitive markets and rational economic agents, the competitive equilibrium will be optimal in that agents choose σ_1 such that the present value of welfare is maximised (e.g. Romer, 1989). Moreover, it has been shown that under the conventional assumptions of

[3] The papers in Benhabib (1992) provide a broad overview, but see in the context of macroeconomic fluctuations also for example Hommes and Nusse (1989) and Nusse and Hommes (1990).

the standard neoclassical growth model the optimal growth path converges from any initial capital endowment to a steady-state one on which the propensity to save is constant. The steady-state level of σ_1 is determined by the production technology, the long-run rate of technical change, population growth and preferences (Cass, 1965 and Lucas, 1988). How σ_1 is adjusted off the steady-state growth path depends on the transitional dynamics of productivity. This issue will be addressed in section IV.

The link between production (or income) Y and capital K is, by definition, capital productivity ε. Hence

$$\varepsilon \equiv Y / K \tag{3}$$

Our main interest is in the driving forces of changes in capital productivity over time. In addition to conventional business cycle phenomena, the growth in labour supply and the substitution between capital and labour, capital productivity is affected by technological change. We shall focus solely on the latter aspect and consider other influences as exogenous to our model. We assume that the production efficiency can be enhanced to some extent by means of the adoption of innovations generated by R&D. Hence, if D represents R&D investments, we expect that $\dot{\varepsilon}$ will be a function of D. Assuming a linear relationship,

$$\dot{\varepsilon} = \nu D + \mu \tag{4}$$

where ν measures the impact on productivity of a unit of expenditure in R&D and μ is the residual.

Besides the exogenous factors noted above, μ will also be influenced by the adoption of externally generated innovations. Some of these are of a public good nature and provide an increase in the stock of knowledge and productivity without any outlay in the economy under consideration, but the implementation of others requires training and other claims on resources. It is therefore plausible that $\mu=\mu(E)$ with $\partial\mu/\partial E > 0$ for $0 \leq E < E^*$ and $\partial\mu/\partial E=0$ for $E \geq E^*$, where E is the expenditure on imported technology. Hence $\mu(E^*)$ is the maximum rate of productivity growth that can be obtained by importing technology and E^* is the minimum amount of expenditure required to achieve this.

Resources devoted to the national or regional R&D sector result from savings in a way analogous to (2):

$$D = \sigma_2 Y \tag{5}$$

As was the case for σ_1, σ_2 and E may both be chosen as to maximise welfare. Consumption in the economy is given by $C \equiv Y - I - D - E = (1 - \sigma_1 - \sigma_2)\, Y - E$. However, it was noted by Lucas (1988) that if knowledge, once created or obtained from elsewhere, can be made available to all at zero cost, the R&D sector generates an externality and the competitive equilibrium and optimal growth paths diverge. Whether or not policies are implemented that yield optimal savings behaviour, the rate of growth of output can be easily computed by observing that (3) implies that $\dot{Y}/Y = \dot{K}/K + \dot{\varepsilon}/\varepsilon$. Substituting (1) - (5), it can be seen that

$$\dot{Y}/Y = (\sigma_1 \varepsilon - \delta) + \nu\, \sigma_2 K + \frac{\mu(E)}{\varepsilon} \tag{6}$$

When $\sigma_2 = \mu(E) = 0$, there is no endogenous technological change. Real income grows in this case at the rate $\sigma_1 \varepsilon - \delta$, i.e. Harrod's well known "warranted" rate of growth (Harrod, 1948, p. 82), adjusted for physical depreciation. However, the positive feedback from income through R&D on capital productivity implies that in the general case the growth rate of income is itself growing. This growth can be decomposed into three parts, conditional on the propensity to save: first, higher capital productivity generating a higher rate of capital growth; second, the greater capital stock leading to accelerating technical change; and, third, the effect on productivity of the installation of technology generated abroad or in other regions. As the economy generates more and more product and process innovations itself and ε grows, equation (6) shows that the role of the adoption of externally generated innovations declines.

III. Constraints to productivity growth

Equation (6) provides a simple description of a growth process with endogenous technological change and increasing returns. Regions or countries which are identical in all respects except for the initial capital endowment exhibit in this model diverging growth. There are economies of scale in that the largest and best endowed region has permanently the highest growth rate. Growth models for regions or countries with endogenous technical change and divergence have received increasing attention in recent years. The model of long-run growth in competitive equilibrium with endogenous technological change formulated by Romer (1986) triggered much further work, reviewed in e.g. Romer (1989), van de Klundert and Smulders (1991) and Nijkamp and Poot (1991a). Romer (1989) gives some empirical evidence that since the eighteenth century the rate of growth in income per capita has shown an upward trend.

However, such ever-increasing growth rates must be considered within a shorter window of time with some caution. Empirical evidence for industrialised nations provides some rather convincing evidence of a "catching up" of the standard of living to the level of the initially wealthiest nation (e.g. Baumol, 1986). Because industrialised economies with relatively low initial incomes tend to grow faster than the wealthier ones, there appears to be a convergence process with growth rates decelerating to a fairly low long-run fundamental rate of growth as incomes increase. The remainder of this paper is concerned with endogenising the role of R&D in this convergence process.

The convergence hypothesis suggests that unlike the linear relationship (4), the effect of R&D expenditure on productivity growth would depend on the state of the economy. There are a number of ways through which the impact of endogenous technological change on the growth in real income can be checked eventually and a constant long-run growth rate can emerge. If ε is considered endogenous by introducing a neoclassical production function (with constant returns to scale and declining marginal products of the inputs) and a technical change function (which transforms R&D into labour augmenting technical change), a stable long-run growth rate again emerges. This has been shown by Nijkamp and Poot (1991a) in a simple extension of the standard neoclassical growth model formulated by Solow (1956), but qualitatively similar results follow in Lucas' (1988) model of human capital accumulation and in a model of product innovations, patents and R&D formulated by Romer (1990).

An alternative negative feedback from technological change to productivity can result when the marginal efficiency of R&D expenditure declines when output grows (Dosi, 1988). Under a given "technological regime" ultimately a saturation level of output may exist at which further R&D expenditure has no longer an impact on productivity. Such a saturation level may arise from capacity limits (technological, economic and social) and reflects a "limits to growth" phenomenon stemming from congestion or lack of natural resources and other reproducible capital. Evidence on the decreasing productivity of R&D in case of more mature economic conditions can be found, among others, in Ayres (1987) and Metcalfe (1981). More generally, the productivity slowdown in recent decades of countries with historically high incomes has been well documented (see Williamson (1991) for a review).

A third explanation for decreasing productivity of R&D expenditure has been recently proposed by Baumol and Wolff (1992). These authors note that the cost-disease model (in which sectors with low productivity growth, such as services, account for growing shares of aggregate expenditure) is also applicable to the R&D sector. Thus, an expansion of the R&D sector raises its relative price, which in turn reduces demand for R&D. Ultimately this has a negative impact on productivity growth in manufacturing.

We capture such negative feedback effects by the following relationship

$$v = \lambda \left(1 - \frac{Y}{Y^C} \right) \tag{7}$$

where Y^C represents a capacity constraint on economic activity. As there are likely to be feedback processes which will relax such capacity constraints in the long-run, Y^C may grow over time. For simplicity it is assumed that Y^C grows at the exogenous rate g. A complete description of the dynamics of productivity and accumulation is then obtained by the following two equations of motion for K and ε respectively:

$$\dot{K} = \sigma_1 \varepsilon K - \delta K \tag{8}$$

$$\dot{\varepsilon} = \lambda \sigma_2 \left(1 - \frac{\varepsilon K}{Y^C} \right) \varepsilon K + \mu(E) \tag{9}$$

combined with $\dot{Y^C} = g Y^C$. The differential equation system (8) and (9) is mathematically a non-linear predator-prey system. Such systems have been studied extensively for biological populations (see e.g. Pimm, 1982). Here we can interpret K as a predator (which increases with ε) and ε as a prey (which decreases with K). To study the dynamical properties of this process, it is convenient to define the variable $\xi = K / Y^C$ which, like ε, is independent of the monetary unit of measurement. The differential equation system (8) and (9) can then be rewritten as

$$\dot{\xi} = (\sigma_1 \varepsilon - \delta - g) \xi \tag{10}$$

$$\dot{\varepsilon} = \lambda \sigma_2 (1 - \varepsilon \xi) \varepsilon \xi Y^C + \mu(E) \tag{11}$$

Solving (10) and (11), the equilibrium values of ξ and ε are

$$\bar{\xi} = \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4\mu(E)}{\lambda \sigma_2 Y^C}} \right) \frac{\sigma_1}{g + \delta} \tag{12}$$

and

$$\bar{\varepsilon} = \frac{g + \delta}{\sigma_1} \tag{13}$$

These equilibrium values can only exist in two circumstances, either when $\mu(E) = 0$, or when Y^C = constant (i.e. g = 0). In the general case, $\bar{\xi}$ is not constant but becomes smaller over time, due to the growth in Y^C. However, asymptotically this case is equivalent to the case that $\mu(E) = 0$. The economic interpretation is that when Y^C is very large, the economy which approaches this capacity constraint obtains most of its productivity growth through the internal R&D sector (which is then also very large) and the role of imported technical change is consequently reduced. The equilibrium capital stock $\bar{K} = \bar{\xi}\, Y^C$.

Hence, in the long-run K grows at the same rate as Y^C. As noted earlier, the growth rate of Y^C is considered exogenous and refers to the fundamental growth rate of the mature economy, which is a function of natural increase, human capital accumulation and other engines of growth. Naturally, R&D may have some impact on Y^C but this process is expected to be much slower than its effect on actual output Y. Thus, the model describes how R&D expenditure can aid the process of "catching up" or "falling back" to a balanced growth path. For example, it can be seen from (12) that an intensified adoption of externally generated innovations (an increase in E) increases the level of $\bar{\xi}$ (and, hence, $\bar{Y} = \bar{\varepsilon}\; \bar{\xi}\; Y^C$) up to a certain point (until $E \geq E^*$), but it does not affect the long-run equilibrium growth rate, which is determined by the growth in Y^C.

Moreover, the long-run steady-state is locally stable. This can be demonstrated in the usual way by considering the Jacobian matrix which results from taking the partial derivatives of (10) and (11) evaluated at $(\bar{\xi}, \bar{\varepsilon})$. This matrix is given by

$$J = \begin{pmatrix} 0 & \sigma_1 \bar{\xi} \\ -\frac{\mu}{\bar{\xi}} - \lambda\sigma_2\, \bar{\varepsilon}^2\, \bar{\xi} Y^C & -\frac{\mu}{\bar{\varepsilon}} - \lambda\sigma_2\, \bar{\varepsilon}\, \bar{\xi}^2 Y^C \end{pmatrix} \tag{14}$$

The determinant of J is positive, while the trace is negative. Hence $(\bar{\xi}, \bar{\varepsilon})$ is a stable fixed point (see e.g. George 1988, p.97).[4]

It is instructive to illustrate the stability of long-run growth in this model by means of simulation of (10) and (11) for given parameter values. It is desirable to pick parameter values which would be empirically plausible. As a matter of convenience, we use observations on long-run growth in the New Zealand economy, but the choice of this economy is not crucial to studying the R&D effects considered in this paper.[5] Over the

[4] Topologically, the phase portrait of the system can be a stable spiral, star node or proper node, dependent on the values of the parameters.

[5] Data sources are the data base of the Research Project on Economic Planning at Victoria University and Sissons et al. (1990). The simulations were carried out with the STELLA program on a Macintosh SE/30 computer (Richmond et al. 1987).

period 1952-91, gross fixed capital formation was on average 24 percent of GDP (σ_1 = 0.24). Capital and output grew both at the rate of about 2.6 percent per annum (g = 0.026). The rate of depreciation was 3.5 percent per annum (δ = 0.035). The long-run capital-output ratio was about 4 and had no discernable trend. Hence long-run capital productivity $\bar{\varepsilon}$ was about 0.25, as can be checked by substituting the parameters in (12), which gives a value of 0.254. R&D expenditure accounted for 1 percent of GDP (σ_2 = 0.01). Constraints on output expansion were the strongest in the early 1970s, particularly 1973/74 when the Capacity Utilisation Index from a business opinion survey reached a record level.[6] Using this information we take Y^C = 26,000 at the time.[7] No empirical information about the parameters λ and μ exist, but we shall take λ = 0.001 and μ = 0 to generate magnitudes of change in ε, which would appear empirically feasible and guarantee convergence to the steady-state.

The model shows how the economy, when perturbed, returns to a steady-state through investment behaviour and the innovative activities of entrepreneurs. Figure 1 shows the phase portrait for two simulations, one in which ($\xi(0)$, $\varepsilon(0)$) = (3.5, 0.2) and the other in which ($\xi(0)$, $\varepsilon(0)$) = (4.5, 0.3). In the former case, the economy operates well below its capacity and R&D expenditure is highly effective in raising capital productivity. However, soon a point is reached at which bottleneck effects reduce productivity although a momentum has built up through which capital accumulates faster than at the long-run growth rate g. Hence from that point on ξ increases while ε decreases until convergence at ($\bar{\xi}$, $\bar{\varepsilon}$) = (3.937, 0.254). As usual, points for which $\dot{\xi}$ = 0 and $\dot{\varepsilon}$ = 0 show where the direction of change in the phase portrait is vertical or horizontal respectively and these points are given by the horizontal line $\varepsilon = (\delta + g) / \sigma_1$ = 0.254 and the hyperbola $\varepsilon \xi = 1$. The stable equilibrium is at the intersection of these two points. In the second simulation, economic activity is initially too high and productivity declines until a trough is reached. From then on R&D expenditure raises productivity again, but the abundance of capital leads to a low level of investment and a declining value of ξ. As in the first simulation, the economy converges again rapidly to ($\bar{\xi}$, $\bar{\varepsilon}$) = (3.937, 0.254).

[6] The data source was the CUBO index from the New Zealand Institute of Economic Research *Quarterly Survey of Business Opinion.*
[7] The CUBO index was 104 in 1973/74 and GDP was $27,756 million in 1981/82 prices.

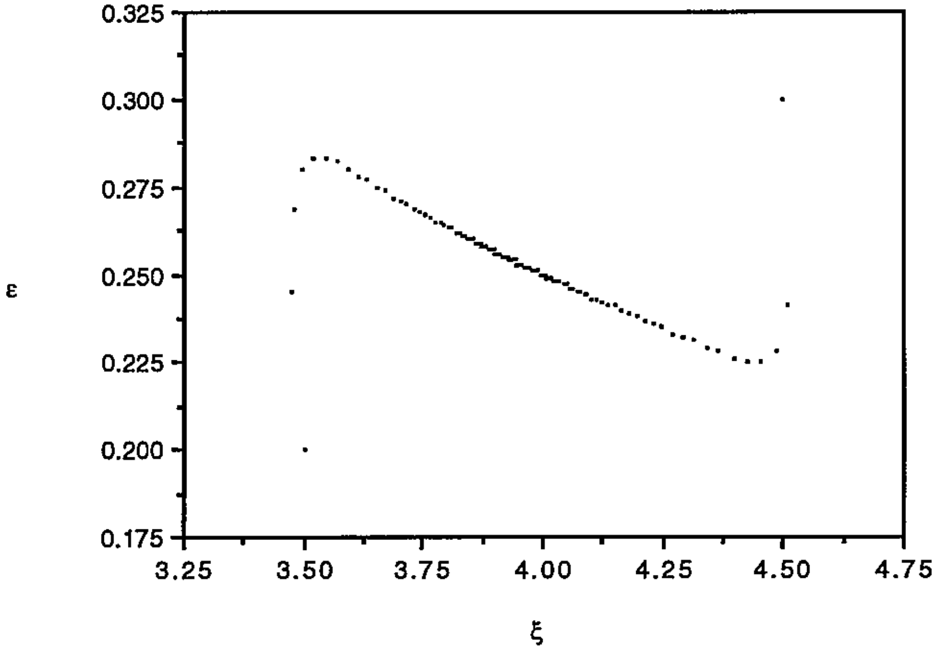

Figure 1

Capital productivity in a model with endogenous technological change and capacity constraints.

IV. Optimal growth

It was mentioned earlier that σ_1, σ_2 and E are not fixed parameters, but will be chosen by optimising economic agents such that the present value of welfare is maximised. It is informative to describe the resulting optimal control problem. As is commonly assumed, welfare is measured by the function $U(C) = (C^{1-\tau} - 1) / (1 - \tau)$ where marginal welfare (or utility) has a constant elasticity $-\tau$ with respect to C, with τ being the coefficient of relative risk aversion (and $1/\tau$ the intertemporal elasticity of substitution). We noted earlier that $C \equiv Y - I - D - E$. Since $K = \xi\, Y^C$, and using (2) and (5), $C = (1 - \sigma_1 - \sigma_2)\, \varepsilon\, \xi\, Y^C - E$. Discounting future welfare at the discount rate ρ, the optimal control problem is to maximise

$$W = \int_0^\infty \left([(1 - \sigma_1 - \sigma_2)\, \varepsilon\, \xi\, Y^C - E]^{\tau-1} - 1 \right) (1 - \tau)^{-1}\, e^{-\rho t}\, dt \quad (15)$$

subject to equations (10) and (11). Here ε and ξ are state variables, while σ_1, σ_2 and E are controls. The Hamiltonian for this problem is the function (see e.g. George (1988, pp.120-122)):

$$H(t, \varepsilon, \xi, \sigma_1, \sigma_2, E, p_1, p_2) =$$

$$([(1 - \sigma_1 - \sigma_2)\, \varepsilon\, \xi\, Y^C - E]^{1-\tau} - 1)\,(1-\tau)^{-1} e^{-\rho t}$$

$$+ p_1 [\,(\sigma_1 \varepsilon - \delta - g)\ \xi\,] + p_2 [\,\lambda\, \sigma_2 (1 - \varepsilon\xi)\, \varepsilon\, \xi\, Y^C + \mu(E)\,] \tag{16}$$

with p_1 and p_2 the co-state variables.

In principle, the solution to this problem can be studied by setting $\partial H/\partial\xi = -\dot{p}_1$, $\partial H/\partial\varepsilon = -\dot{p}_2$, $\partial H/\partial\sigma_1 = 0$, $\partial H/\partial\sigma_2 = 0$ and $\partial H/\partial E = 0$ and checking the transversality conditions. However, the resulting differential equations for this particular system are too cumbersome to provide any clear insights. Fortunately, an intuitive approach is available. We can use the fact that each of the three controls has the same function: a reduction in present consumption to create more wealth, and thus consumption, in the future. Therefore we need only to compare how a unit of expenditure on each of the three forms of investment I, D and E contributes to the growth in output. Substituting (7) in (6) provides a clear indication of what the initial change in Y will be when starting from a given level of income $Y(0) = \varepsilon(0)\, K(0)$:

$$\dot{Y} = (\sigma_1\, \varepsilon(0) - \delta)\, Y(0) + \lambda\,(1 - \frac{Y(0)}{Y^C})\, \sigma_2\, K(0)\, Y(0) + \mu(E)\, K(0) \tag{17}$$

Differentiating (17) with respect to expenditures $\sigma_1 Y(0)$, $\sigma_2 Y(0)$ and E yields

$$\partial \dot{Y} / \partial(\sigma_1 Y(0)) = \varepsilon(0) \tag{18}$$

$$\partial \dot{Y} / \partial(\sigma_2 Y(0)) = \lambda\,(1 - \frac{Y(0)}{Y^C})\, K(0) \tag{19}$$

and

$$\partial \dot{Y} / \partial E = \mu'(E)\, K(0) \tag{20}$$

respectively. Since one unit of expenditure on any of these three reduces consumption initially by the same amount and since they do not enter multiplicatively in the utility function, it will be optimal at any point in time to set two of the three controls equal to zero. The only control variable not equal to zero will be the one which yields the greatest effect on future income. Which of the three controls takes this role depends on the values of the parameters. However, inspection of (18) to (20) suggests a plausible process of development:

1. In an initial stage of development, the capital stock and capital productivity are expected to be relatively low. Moreover, there may be an abundance of unutilized resources, suggesting that $Y(0)$ is much less than Y^C. In this case, we expect that the greatest effect on output expansion is obtained by devoting resources to importing new technology. Mathematically, $\varepsilon(0) < \lambda K(0) < \mu'(E) K(0)$. Of the three effects, $\partial \dot{Y} / \partial E$ is then the largest. Hence the optimal decision is to set $\sigma_1 = \sigma_2 = 0$ and $0 < E \leq E^*$ (recall that $\mu'(E) > 0$ for $0 < E < E^*$ and $\mu'(E) = 0$ for $E \geq E^*$). The optimal value of E will depend on the discount rate ρ in the usual way. Note that at this stage there is an absence of physical capital accumulation: the new technology is applied by means of the existing and depreciating capital stock. The technological change must therefore be disembodied and in practice this is only possible to a limited extent (see e.g. the critique by Scott (1989) on the neoclassical production function approach).
2. In the second stage of development, $\partial \dot{Y} / \partial (\sigma_2 Y(0))$ is the largest contributing factor to growth. In this case, the economy switches from importing technology to generating new technology by devoting resources to its own R&D sector. This strategy remains efficient as long as $Y<Y^C$. When $Y \geq Y^C$, $\partial \dot{Y} / \partial (\sigma_2 Y(0)) \leq 0$ and it is then optimal to set $\sigma_2 = 0$.
3. In the third stage, the economy has moved onto the balanced growth path where $Y=Y^C$ and therefore grows at the constant rate g. In this case $\sigma_2=E=0$ and we saw earlier that this equilibrium growth path is stable. The optimal value of σ_1 can be easily determined by noting that the economy grows in this case in the same way as on the balanced growth path of the neoclassical growth model without endogenous technological change (since ε is constant when $\sigma_2=E=0$). Hence g is here the "natural" growth rate (including exogenously determined long-run technological change) and the optimal control problem (15) reduces to the standard problem of optimal capital accumulation solved by Cass (1965). Using our notation, it can be straightforwardly established that the optimal propensity to save on the balanced growth path is in this case

$$\sigma_1 = \frac{\alpha (g + \delta)}{\rho + \delta + \tau g} \tag{21}$$

(see also Nijkamp and Poot, 1991a, p.4), where α represents the share of capital in aggregate income.

V. Multiple equilibria

In addition to a limited effectiveness of R&D expenditure when the economy is constrained by bottleneck phenomena, such investment may also be unproductive when the economy has not reached a certain floor of activity (cf. Myrdal, 1957). Unless there is social infrastructure and human capital available, the innovations generated by the R&D sector may have little impact on productivity. During a transitional phase of implementation of new production techniques or the setting up of new product lines, aggregate productivity can even decline. Floor and ceilings in economic activity have of course a long tradition in the literature.[8] The implications of floors and ceilings for capital accumulation and productivity can be assessed by replacing equation (7) by

$$\nu = \lambda (Y - Y^F) (Y^C - Y) Y \tag{22}$$

where Y^F and Y^C are the exogenously given floor and ceiling levels of output respectively. Substituting this in the model of section II, the dynamics of the resulting growth model are fully described by the following two non-linear differential equations:

$$\dot{K} = \sigma_1 \varepsilon K - \delta K \tag{23}$$

$$\dot{\varepsilon} = \lambda \sigma_2 (\varepsilon K - Y^F) (Y^C - \varepsilon K) \varepsilon K + \mu(E) \tag{24}$$

For simplicity, we do not consider changes in these levels themselves. Hence any equilibrium, if it exists, has a constant capital stock and a constant level of capital productivity. The equilibrium level of capital productivity can be computed directly from (23): $\dot{K} = 0$, when $\varepsilon = \delta / \sigma_1$, hence $\bar{\varepsilon} = \delta / \sigma_1$. The equilibrium capital stock itself depends on the adoption of externally generated innovations, $\mu(E)$, through setting $\dot{\varepsilon} = 0$ in equation (24). For a given $\mu(E)$, there are one, two or three different equilibria $\bar{K}$, since (24) is a cubic equation in K. The characterisation of these equilibria is depicted in Figure 2.[9]

[8] See e.g. Hicks (1950); and McKenzie and Zamagni (1991) on related issues.

[9] See also Isard and Liosattos (1979) for a similar model. However, theirs is a single variable model rather than the two-variable model described by (23) and (24).

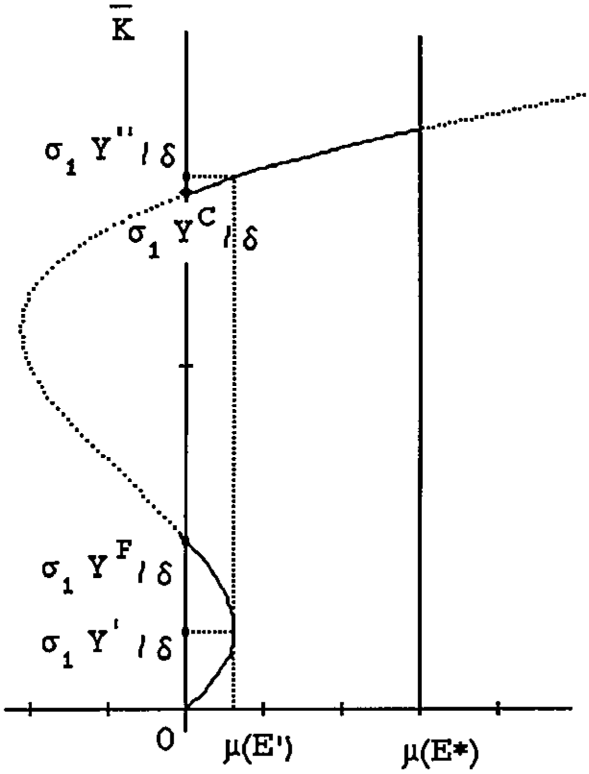

Figure 2
Innovation diffusion, multiple equilibria and discontinuities.

When $\mu(E) = 0$, there are three equilibria: $\bar{K}_1 = 0$, $\bar{K}_2 = \sigma_1 Y^F/\delta$ and $\bar{K}_3 = \sigma_1 Y^C/\delta$. Proceeding along similar lines as in section III, it can be established that $\bar{K}_1$ and $\bar{K}_3$ are stable equilibria, while $\bar{K}_2$ is unstable. For $0 < \mu(E) < \mu(E')$ there continue to be three equilibria, of which the middle one is unstable. However, when E=E', the two "low level" equilibria become one. This occurs when the function $K=K(\mu)$ has a local maximum. In Figure 2 this is the point at which $\bar{K} = \sigma_1 Y'/\delta$. Y' can be expressed in terms of Y^F and Y^C by

$$Y' = \frac{1}{3}\left(Y^F + Y^C\right) - \frac{1}{3}\sqrt{\left(Y^F - Y^C\right)^2 + Y^F Y^C} \tag{25}$$

as can be established straightforwardly with calculus. There is no simple expression for the "high level" equilibrium $\bar{K} = \sigma_1 Y''/\delta$. When $\mu(E) > \mu(E')$, one stable "high level" equilibrium remains. It can be seen from (24) and (25) that while Y' is not affected by the proportion of resources devoted to the R&D sector (σ_2), $\mu(E')$ is. Recall that we defined in section II $\mu(E^*)$ as the maximum rate of productivity growth that can be obtained by importing technology and E^* is the minimum amount of expenditure required to achieve this.

The model makes interesting predictions regarding the process of development. We saw in the previous section that at low levels of capital productivity, the acquisition and adoption of "blueprints" for production from external sources was the most effective way of generating growth in income and welfare. Here we see that an economy endowed with little capital remains captured in a "trap" of low development as long as $E<E'$. An increase in E beyond E' causes the economy to accumulate capital rapidly and benefit from productivity growth to the extent that it converges to a high level of income. This type of jump, referred to mathematically as a catastrophe, has now been studied extensively in economics.[10]

It was argued in the previous section that at high income levels in large scale economies, an optimal growth strategy may at some stage involve a switch from importing technology to an expansion of the own R&D sector. The reallocation of resources (a decrease in E and increase in D) will lead to a reduction in μ, eventually below $\mu(E')$. However, at this stage the economy does not return to a low-level equilibrium, but instead moves towards $\bar{K} = \sigma_1 Y^C / \delta$. This is often referred to as a hysteresis phenomenon. Eventually, the economy reaches a steady state on which $Y=Y^C$, capital productivity is constant and the capital stock grows at the same rate as Y^C. As in the previous section, it is then optimal to set $\sigma_2=E=0$. In this mature economy which has converged to a balanced growth path, the optimal propensity to save is therefore the same as in the model of the previous section (and given by equation (21)). It must of course be stressed that through changes in the parameters or shocks in Y^C, the economy may move again to a lower level of capital productivity at which an outward orientation with positive E and a revitalisation of the R&D sector becomes beneficial. Nonetheless, a thriving R&D sector requires in our model the removal of bottleneck phenomena.

VI. Gestation lags in capital accumulation

Until now we have assumed smooth adjustments to the capital stock and productivity, which were encapsulated in first order differential equations. It may be considered more appropriate to allow for discrete lags and, hence, adopt a difference equation approach. In this section we shall see that the latter approach gives the models additionally interesting dynamical properties. A traditional argument in favour of discrete lags is that the production and implementation of physical capital requires time. Although it may be

[10] See e.g. George (1988) for an introduction.

argued that such lumpiness of new investment should be smoothened out by aggregation, observations on the economy (which take place over discrete time intervals in any case) often exhibit "thick market effects", i.e. economic activity appears more efficient when concentrated over space or over time. Spatial and temporal agglomeration effects provide a rationale for this (see Hall, 1991). Moreover, the evolutionary nature of creation of new technologies and products and the nature of the adoption process (following the well established s-shape curve) would also tend to lead to jumps in productivity in time and space rather than a smooth change.[11]

Replacing the model of section III with the corresponding difference equations, we obtain for given Y^C

$$K_{t+1} - K_t = (\sigma_1 \varepsilon_t - \delta) K_t \tag{26}$$

$$\varepsilon_{t+1} - \varepsilon_t = \lambda \sigma_2 \left(1 - \frac{\varepsilon_t K_t}{Y^C} \right) \varepsilon_t K_t + \mu(E) \tag{27}$$

The equilibrium values of (26) and (27) are

$$\bar{K} = \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4\mu(E)}{\lambda \sigma_2 Y^C}} \right) \frac{\sigma_1 Y^C}{\delta} \tag{28}$$

and

$$\bar{\varepsilon} = \frac{\delta}{\sigma_1} \tag{29}$$

As before, the equilibrium exists when Y^C is constant or asymptotically when Y^C grows at rate g (in which case δ is replaced in (28) and (29) by $g + \delta$ and $\mu(E)$ is set to 0).

A model similar to (26) and (27) was studied by Nijkamp et al. (1991), who considered productivity change in a multi-regional context with $\mu(E)$ modelled explicitly as the productivity change resulting from the diffusion of R&D output from other regions. In the multiregional context, it is difficult to derive theoretical results, but simulation showed that that model could exhibit widely varying behaviour, ranging from convergence to a steady-state equilibrium to persistent cycles and seemingly chaotic fluctuations.

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[11] See also Nijkamp and Reggiani (1992).

In the present context, some theoretical properties of (26) and (27) can be stated explicitly. While the equilibrium is the same as before, the dynamical properties are now more interesting than in the case of section III. Equation (27) can be rewritten as

$$\varepsilon_{t+1} = (1 + \lambda\, \sigma_2\, K_t)\, \varepsilon_t \left(1 - \frac{\lambda\, \sigma_2\, K_t^2}{(1 + \lambda\, \sigma_2\, K_t) Y^C}\, \varepsilon_t\right) + \mu(E) \tag{30}$$

If we introduce the variable $x_t = \dfrac{\lambda\, \sigma_2\, K_t^2}{(1 + \lambda\, \sigma_2\, K_t) Y^C}\, \varepsilon_t$, substitution into equation (30) gives:

$$x_{t+1} = \frac{(1 + \lambda\, \sigma_2\, K_t)^2 K_{t+1}^2}{(1 + \lambda\, \sigma_2\, K_{t+1}) K_t^2}\, x_t (1 - x_t) + \frac{\mu(E)\, \lambda\, \sigma_2\, K_{t+1}^2}{(1 + \lambda\, \sigma_2\, K_{t+1}) Y^C} \tag{31}$$

While the behaviour of the difference equation system (26) and (27) can obviously be studied by means of simulation, equation (31) provides some clues about the likely behaviour. First we note, from the definition of x_t and assuming that Y_t does not exceed Y^C too much, that $0 < x_t < 1$. Moreover, the process of capital accumulation described by (6) is such that for any realistic values of the parameters, the growth in the capital stock is only a small percentage each period. If the capital stock K_t is seen as a large but only gradually changing number Ψ, the time path of x_t can be approximately described by the nonlinear difference equation

$$x_{t+1} = \alpha\, x_t (1 - x_t) + \beta \tag{32}$$

For $\mu(E)=0$ we obtain the logistic equation (or referred to as Verhulst dynamics) with properties described in the influential article by May (1976). Nontrivial dynamics requires $1 < \alpha < 4$, otherwise x becomes at some stage zero or negative. When $1 < \alpha < 3$, x converges to the stable equilibrium $1 - 1/\alpha$, while for $3 < \alpha < 3.5700$ stable cycles emerge. Beyond this, chaotic fluctuations emerge which resemble random noise, although there are also stable cycles in this range (see e.g. Lorenz (1989) for a review).

In equation (32), $\alpha = 1 + \lambda\, \sigma_2\, \Psi$ and, hence, α increases over time due to the accumulation of capital described by (26). Eventually, for fixed λ and σ_2, α would exceed the value 4 and the model breaks down. However, we expect that the parameter λ would be sensitive in empirical situations to the scale of the economy. This is what we referred to in the simulation of section III as using a value for λ which generates empirically feasible fluctuations in ε. As long as $0 < \lambda\, \sigma_2\, \Psi < 2$, the model will converge. It is evident, however, that an economy which is on a balanced growth path in this model may after some time exhibit a phase of wild fluctuations when $\lambda\ \sigma_2\ \Psi$ exceeds 2.57.

To illustrate these points, we simulate the model (26) and (27). The parameter values are those used in section III. As before, $Y^C= 26000$. However, Y^C is now assumed constant to ensure that $\lambda\ \sigma_2\ K_t$ does not exceed 3. Hence g=0. All other parameters and starting values are as in the first simulation, i.e. $(\xi(0), \varepsilon(0)) = (3.5, 0.2)$. Figure 3 shows how the model converges also in difference equation form to constant capital productivity $\bar{\varepsilon} = \delta / \sigma_1 = 0.035 / 0.24 = 0.1458$. Since $K_0 = \xi_0\ Y^C = 3.5 \times 26000 = 91000$ and $\bar{K} = Y^C / \bar{\varepsilon} = 178286$, the May model "tuning parameter" increases from $1 + \lambda\ \sigma_2\ K_0 = 1 + 0.001 \times 0.01 \times 91000 = 1.91$ to $1 + \lambda\ \sigma_2\ \bar{K} = 2.78$. Exogenous shocks can therefore not generate cyclical or chaotic behaviour.

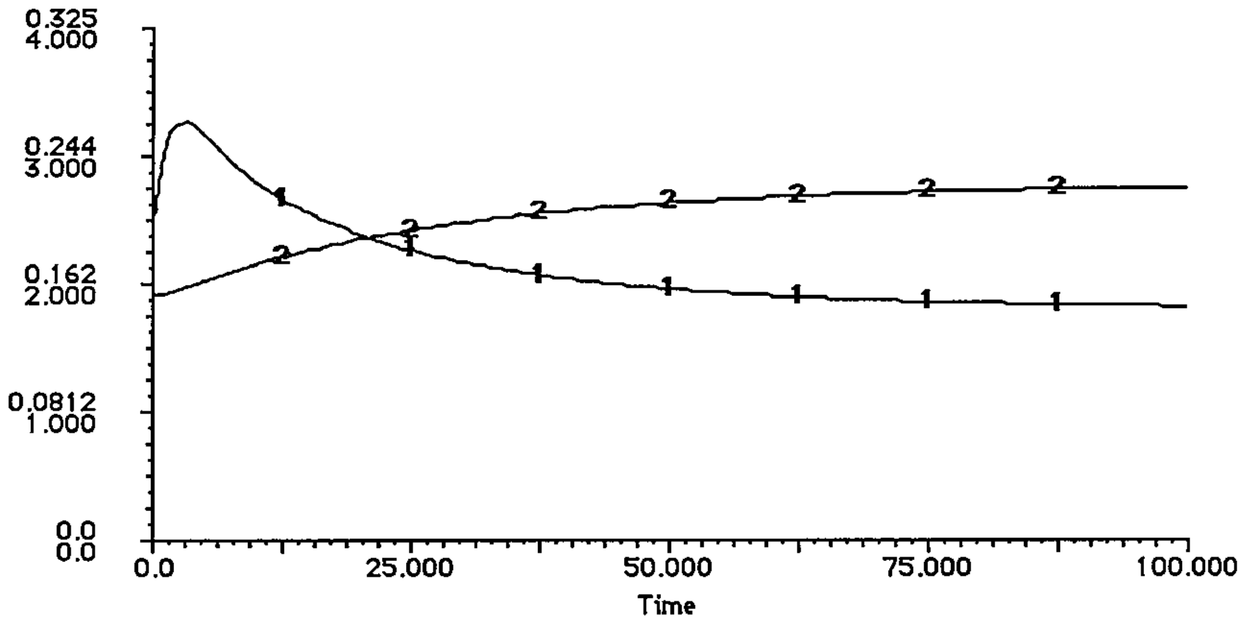

Figure 3
Capital productivity in a nonlinear growth model in difference equation form with convergence to a steady-state (1=capital productivity; 2= the approximate May model tuning parameter); logistic productivity response

If the parameter λ is increased slightly to 0.00163, $1 + \lambda\ \sigma_2\ \bar{K} = 3.906$ and chaotic fluctuations emerge when Y exceeds Y^C. This is illustrated in Figure 4. Since K changes over time, bifurcations occur each period.

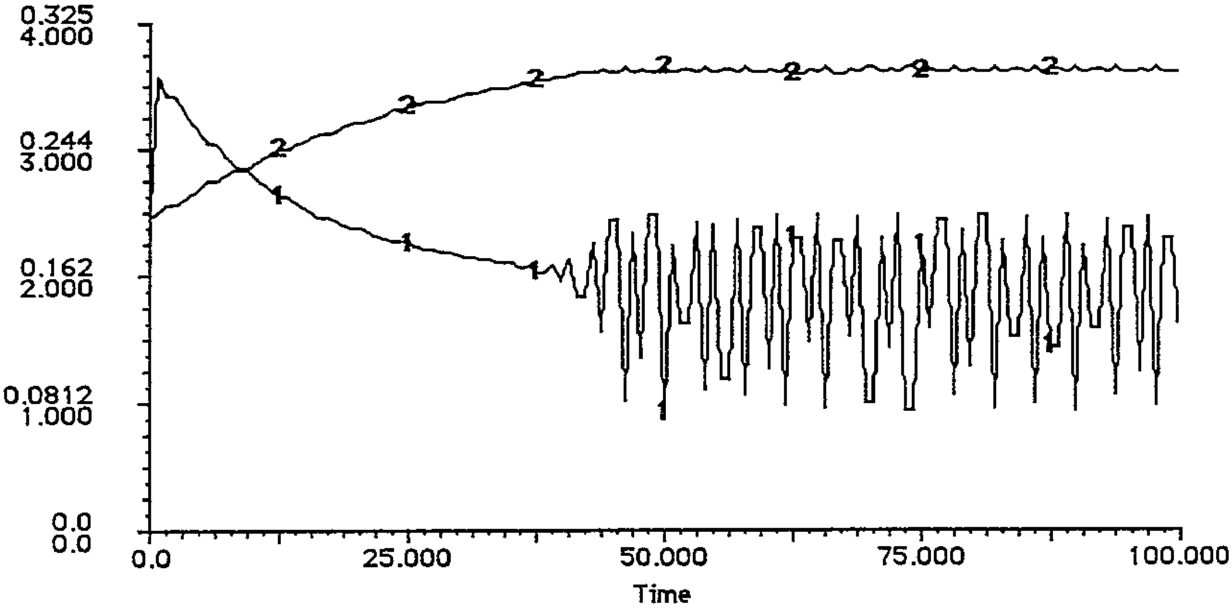

Figure 4
Capital productivity in a nonlinear growth model in difference equation form with chaotic fluctuations (1=capital productivity; 2= the approximate May model tuning parameter); logistic productivity response

However, Figure 4 shows that the process of capital accumulation leads to relatively minor variations in the tuning parameter. This suggests that returns to a stable or cyclical regime (which do exist within the chaotic range) would be shortlived. The bifurcations in the logistic model have been extensively studied. A plot of the attractors in this equation (the long-run set of values of x for a given parameter) when α increases from 2.8 to 3.9 is given in Figure 5.

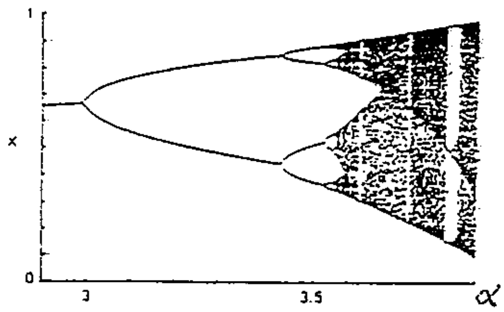

Figure 5
Numerical Plot of the Bifurcation in the Logistic Equation. Source: Holden (1986, p.46).

In summary, the model that was developed in section III can be interpreted in difference equation form as a logistic or May equation, but with a discrete jump of the tuning parameter (due to the change in the size of the capital stock) in each period.

A dynamic system which has the potential to generate chaotic fluctuations has the property of strong sensitivity to measurement of the initial conditions and the values of the parameters.[12] In the present context, this implies for example that the adoption of externally acquired innovations can have a destabilising effect on economic fluctuations. Up to now we have considered $\mu(E)=0$ in the simulations. If, however, in the case of the simulation displayed in Figure 3, $\mu(E)$ is increased to 0.08, chaotic fluctuations again emerge. See Figure 6. The behaviour of capital productivity in Figure 6 is qualitatively similar as in Figure 4, but chaos now results from the value of $\mu(E)$ rather than λ.

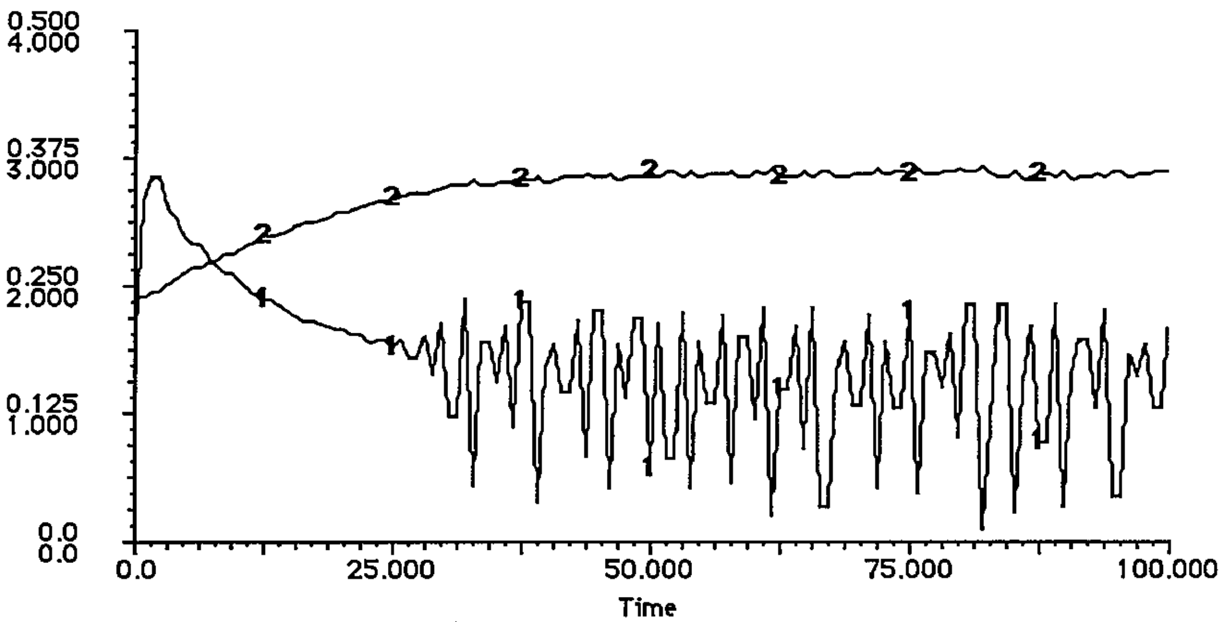

Figure 6
Capital productivity in a nonlinear growth model in difference equation form. Parameters are as for Figure 3, but now $\mu(E)=0.08$ instead of 0. (1=capital productivity; 2= the approximate May model tuning parameter); logistic productivity response

Even richer dynamical behaviour emerges when we allow for both floors and ceilings in the economy, i.e. return to the model of section V. In difference equation form (23) and (24) become

$$K_{t+1} - K_t = (\sigma_1 \varepsilon_t - \delta) K_t \tag{33}$$

and

$$\varepsilon_{t+1} - \varepsilon_t = \lambda \sigma_2 (\varepsilon_t K_t - Y^F) (Y^C - \varepsilon_t K_t) \varepsilon_t K_t + \mu(E) \tag{34}$$

We can rewrite (34) in the form

[12] See e.g. Nijkamp and Poot (1991b) for an overview of issues in empirical work with nonlinear dynamical models.

$$\varepsilon_{t+1} = \lambda\, \sigma_2\, K_t^3\, \varepsilon_t\, [\, (\, \varepsilon_t - Y^F/\ K_t\,)\, (\, Y^{C'}/\, K_t - \varepsilon_t\,)\,] + \mu(E) \tag{35}$$

where Y^F and $Y^{C'}$ are the two solutions to the quadratic equation $(x - Y^F) (Y^C - x) = - \frac{1}{\lambda\ \sigma_2\ K_t}$. Hence equation (35) is of the form

$$\varepsilon_{t+1} = a\, \varepsilon_t\, (\, \varepsilon_t - b\,)\, (\, c - \varepsilon_t\,) + \mu \tag{36}$$

with time-varying parameters a, b and c. The properties of cubic interative maps of this form, even with constant parameters, have not been studied as extensively as the May model. However, a very special case of (36) was analysed recently by Puu (1991). Introducing the parameter β, and letting $a = 1 + \beta$, $b = \sqrt{\frac{\beta}{1+\beta}}$, $c = -\sqrt{\frac{\beta}{1+\beta}}$ and $\mu = 0$, equation (36) can be reduced to

$$\varepsilon_{t+1} = \beta\, \varepsilon_t - (\, 1 + \beta\,)\, \varepsilon_t^3 \tag{37}$$

Puu showed that this model has the interesting property of cyclical or chaotic movement between the two non-trivial equilibria for certain values of β. For $\beta > 1$ there are three equilibria ($-\sqrt{\frac{\beta-1}{\beta+1}}$, 0, $+\sqrt{\frac{\beta-1}{\beta+1}}$), of which the two nonzero ones are stable when $\beta < 2$. Chaos emerges when β is about 2.35, but is first confined to fluctuations around each of the two equilibria. Spillover (i.e. a shift from fluctuations around the low level equilibrium to fluctuations around the high level equilibrium or vice versa) emerges when β exceeds 2.6. Moreover, there are stable cycles in the chaotic region. For $\beta > 3$, the model breaks down. The behaviour of the equation over the whole range of parameter values is shown in Figure 7.

To study the properties of our own model, (33) and (34), we can again use simulation. As in Puu's specification, the dynamical properties will be particularly sensitive to the coefficient of the highest power of the variable. From (35) we see that the coefficient of ε_t^3 is $-\lambda\ \sigma_2\ K_t^3$ and to generate interesting behaviour, this needs to be sufficiently small. Hence we can use exactly the same parameter values as in the initial simulation in this paper in section III, but scale down λ sufficiently. A suitable value is $\lambda = 0.5 \times 10^{-11}$. The only new variable is Y^F, which we set equal to 10,000.

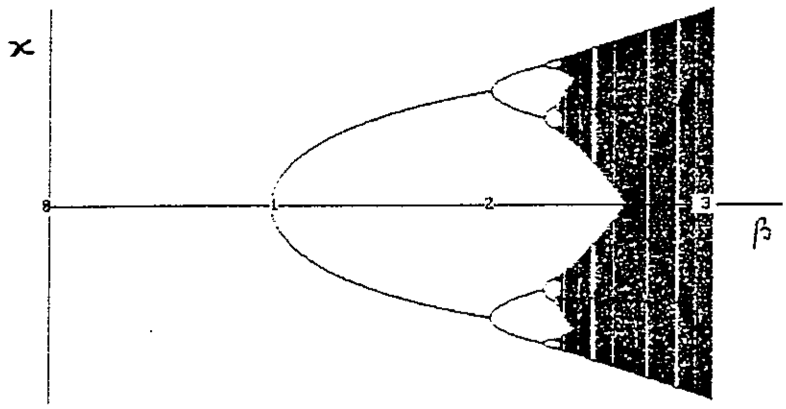

Figure 7
Numerical Plot of the Bifurcation in the Cubic Iterative Map. Source: Puu (1991, p.129).

Figure 8 shows how chaotic fluctuations emerge in the case of a cubic productivity response. This figure may be directly compared to the case of a quadratic productivity response in Figure 4. Although the fluctuations start earlier in the economy of Figure 8, productivity change is slower. Moreover, the difference between the two situations becomes clear when the initial conditions are varied. Figure 9 shows that if $K_0 = 48575$, the model still displays productivity growth up to the point where output exceeds Y^C with chaotic fluctuations subsequently. However, a small decrease in the initial capital stock to $K_0 = 48562$ moves the economy below its minimum sustainable level and creates negative net investment until eventually output becomes zero (see Figure 10).

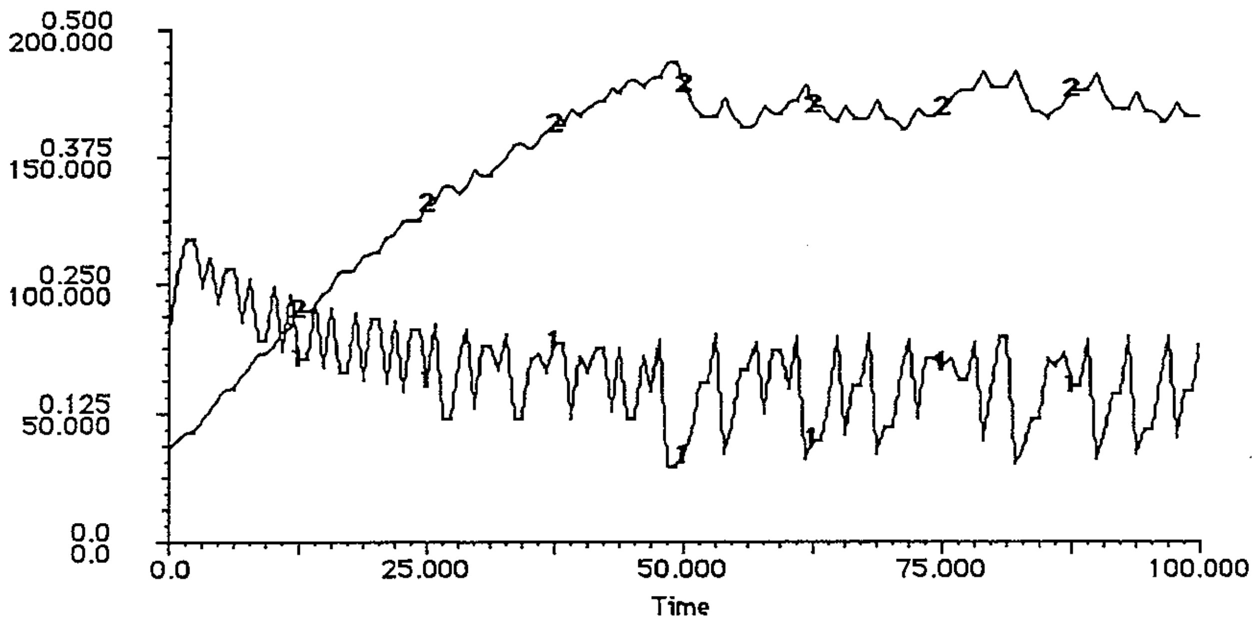

Figure 8
Capital productivity in a nonlinear growth model in difference equation form with chaotic fluctuations (1=capital productivity; 2= $\lambda\ \sigma_2\ K_t^3$); cubic productivity response; initial conditions as in Figure 4

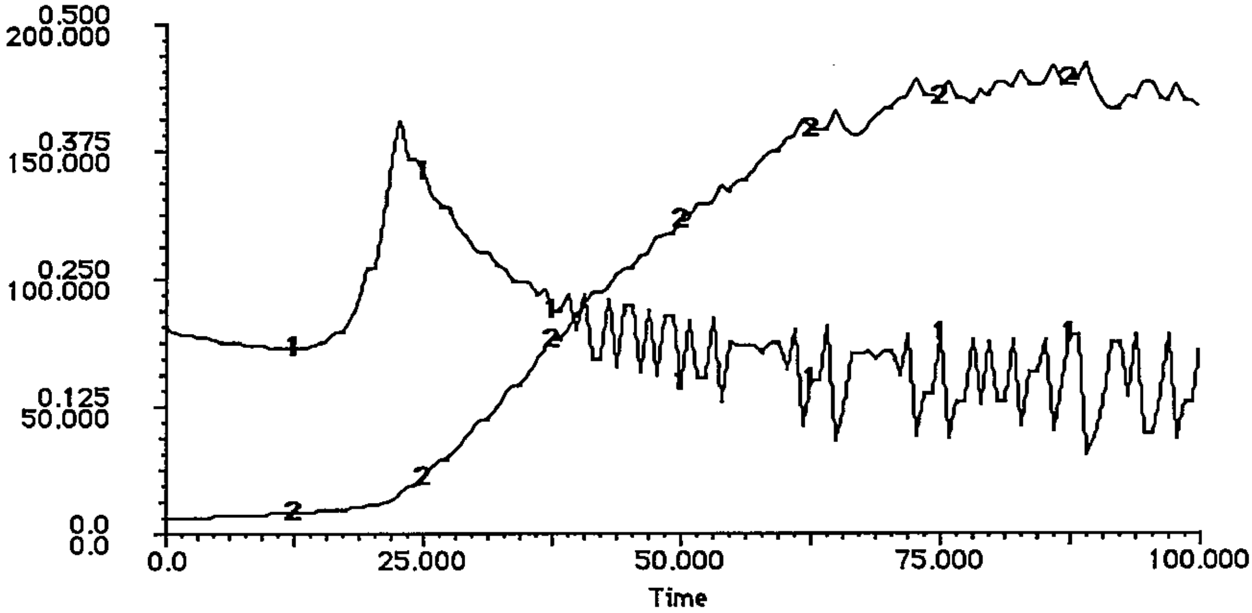

Figure 9
Capital productivity in a nonlinear growth model in difference equation form with chaotic fluctuations
(1=capital productivity; 2= $\lambda\ \sigma_2\ K_t^3$); cubic productivity response; K_0 = 48575.

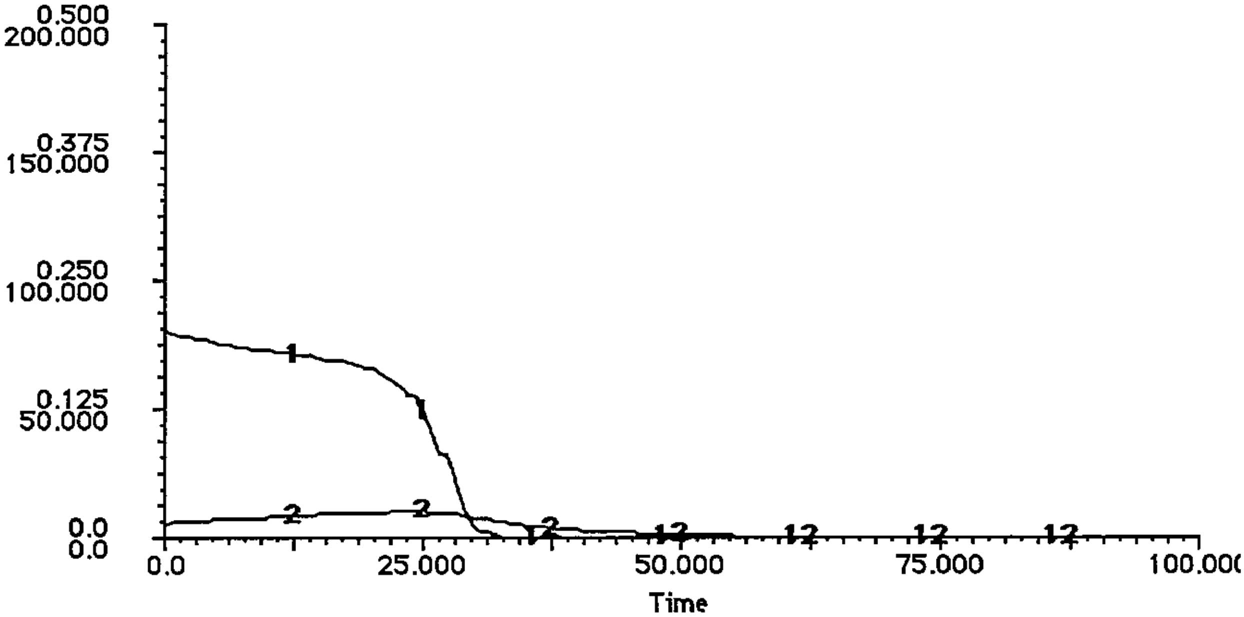

Figure 10
Capital productivity in a nonlinear growth model in difference equation form with chaotic fluctuations
(1=capital productivity; 2= $\lambda\ \sigma_2\ K_t^3$); cubic productivity response; K_0 = 48562.

As in the case of the quadratic productivity response it is again fruitful to consider the influence of exogenous productivity shocks on the dynamic properties of the model. What happens, for example, if more resources are allocated to the acquisition of new technological blueprints from abroad? We have then the same situation as in Figure 2, but with the possibility of bounded irregular movement around either of the two equilibria. Simulations carried out by Puu (1991) show that it is in this case possible that a small increase in $\mu(E)$ may generate a jump from a stable high level equilibrium to chaotic fluctuations in the region of the low level equilibrium. Figure 11 illustrates this for the equation

$$x_{t+1} = \beta x_t - (1 + \beta) x_t^3 + \mu \tag{38}$$

where β=2.00 and μ moves across the interval [-0.5,0.5]. The system (33) and (34) would display qualitatively similar behaviour.

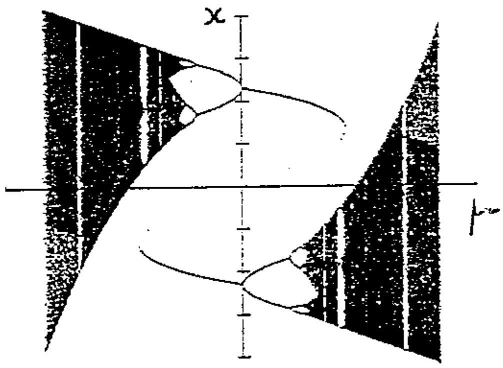

Figure 11
The effect of exogenous shocks in the Cubic Iterative Map. Source: Puu (1991, p.143).

The models discussed in this section suggested that in many situations the long-run behaviour of capital productivity exhibits irregular, but trendless, oscillations. Recalling that the parameter values in the simulations were based on New Zealand macroeconomic data, the question arises whether such simulations are at least consistent with observed fluctuations in capital productivity. While formal econometric work is beyond the scope of the present paper, capital productivity indeed appeared to exhibit trendless fluctuations during the last forty years. This can be seen from Figure 12. The vertical axis has a similar scale as in the figures displaying the simulations. While there is obviously strong autocorrelation, an oscillatory pattern emerges at roughly five-year intervals. It is readily admitted that a large part of the variation in capital productivity in Figure 5 may be due to business cycle fluctuations and that the causes of these may be much broader than the nonlinear feedback of the endogenous R&D sector on aggregate productivity. In a recent econometric study of key features of the New Zealand business cycle, Kim et al. (1992) noted that it is difficult to establish separate roles for demand and supply factors and to capture the influence of technological change based supply shocks. Further work on the role and potential endogeneity of technological change is obviously warranted.

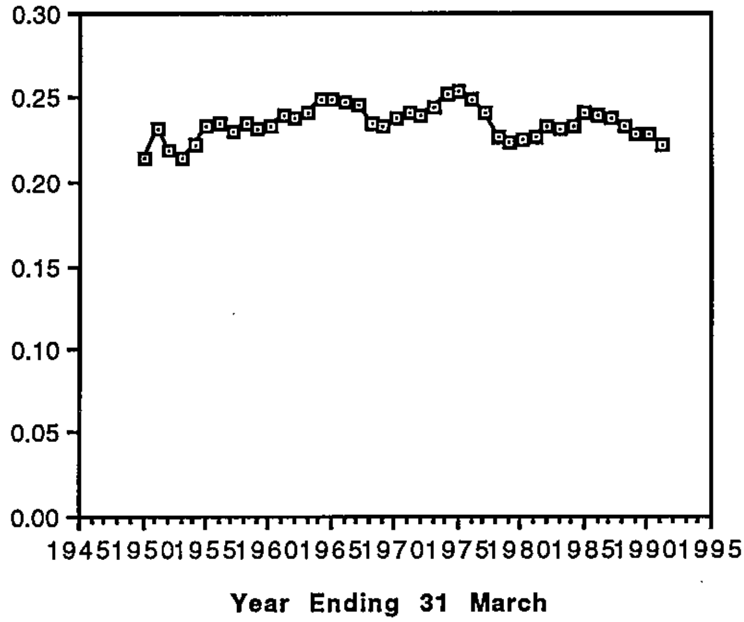

Figure 12
Aggregate capital productivity in New Zealand. Data source: Research Project on Economic Planning, Victoria University of Wellington.

VII. Reflections

Regime switches from stable to unstable behaviour in an economy such as those discussed in this paper are increasingly receiving attention. Although earlier attempts date back to the fifties (notably Goodwin's non-linear accelerator-multiplier model; see Goodwin, 1982), economists have had great difficulties in incorporating discontinuities and instabilities in their modelling efforts. Modern approaches to self-organising systems have convincingly demonstrated that - after initial shocks - a new equilibrium will not automatically arise but, instead, can only emerge after a qualitative shift in the system (so-called structure dynamics). The transition path in such situations is of course of eminent importance, for both analytical and predictive reasons (see also Baumol and Benhabib, 1989; Benhabib and Day, 1981; Rosser, 1991; Zhang, 1991).

The present analytical apparatus still has important shortcomings. Model specifications are often semantically insufficient, econometric estimation procedures for non-linear dynamic models are often inadequate, and the statistical tests on validity of model results need much improvement (see also Brock et al. 1987; Ornstein and Weiss, 1991; and Sayers, 1991). Other important items on the broader research agenda of non-linear dynamic equilibrium analysis are: the impact of time lags on stability, the effect of an unstable niche in a globally stable dynamic system, the links to self-organising models and the use of experimental designs for analysing unstable behaviour. In our model of productivity change, a complete description of accumulation and technological change in an open economy setting would be a first requirement.

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