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Testing fractionally integrated time series

Ping X. Wu*

* Quantitative Studies Group
Faculty of Commerce and Administration
Victoria University of Wellington

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TESTING FRACTIONALLY INTEGRATED TIME SERIES*

Ping X. Wu

Quantitative Studies

Victoria University of Wellington

P.O.Box 600, Wellington

NEW ZEALAND

ABSTRACT

A generalization of the autoregressive integrated moving average (ARIMA(p,d,q)) model is the autoregressive fractionally integrated moving average (ARFIMA(p,d,q)) model in which d is allowed to take non-integer values. The continuum of values of d represents differences in the degree of long-run dependency of the time series. This paper is concerned with testing whether a time series is integer integrated or fractionally integrated. Without loss of generality, the typical case of testing d = 1 against d > 1 or d < 1 is considered. A locally best invariant test, based on King and Hillier (1985), is constructed for the simple ARFIMA(0,d,0) case. The test is then modified to test the general ARFIMA(p,d,q) series. The power of these tests is investigated using the Monte Carlo method. Also investigated is the power of a test based on Geweke and Porter-Hudak (1983)'s method, and an augmented Dickey-Fuller test. The invariant tests proposed here can also be used to test different integer values of d, such as d = 2 against d = 1.

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1. Introduction

Many time series are modelled as autoregressive integrated moving average (ARIMA(p,d,q)) processes:

$$\Phi(L)(1-L)^{d}(X_t - \mu_t) = \Theta(L)\varepsilon_t \tag{1}$$

where

$$\Phi(z) = 1 - \phi_1 z - \cdots - \phi_p z^p \quad ,$$

$$\Theta(z) = 1 + \theta_1 z + \cdots + \theta_q z^q \quad ,$$

and the polynomials have roots lying outside of the unit circle, L is the lag operator such that $LX_t = X_{t-1}$, μ_t is the mean of X_t and ε_t is white noise with $E(\varepsilon_t) = 0$ and $V(\varepsilon_t) = \sigma^2$.

Traditionally, d is restricted to be an non-negative integer, indicating that the data series is differenced before an ARMA(p,q) model is fitted. In recent years, there has been increased interest in generalizing this restriction. Possible non-integer values of d allow a much wider range of low frequency behaviour in the time series. Thus, long term dependency may also be modelled by (1) with a non-integer value of d. It is interesting to note that (1) includes the unit root process (integer value of d = 1) as a special case.

The generalization to non-integer values of d has important practical implications for the quantitative extent of long-run persistence of any shock to a time series. A shock will be less persistent for a smaller value of d. This in turn indicates that confidence intervals of forecasts can have smaller growth rates. Model (1) provides a possible base, not only for testing whether a time series has long-term dependency, but also for comparing the extent of this dependency. Further more, the model can also be used to test the unit root hypotheses against the trend stationary alternatives. Since non-integer values of d indicate fractional integration, (1) is called an ARFIMA(p,d,q) model.

The early work on ARFIMA(p,d,q) processes and their properties includes Granger and Joyeux (1980) and Hosking (1981). Much research has investigated the estimation and application of the ARFIMA(p,d,q) model. For example, Geweke and Porter-Hudak (1983), Boes, Davis and Gupta (1989) provide methods of estimation of d based on the frequency domain. Sowell (1992a) outlines maximum likelihood estimation procedures. In terms of applications, Nerlove (1964), Granger (1966), Granger and Joyeux (1980), Diebold and Rudebush (1989, 1991a), Sowell (1992b) and others indicate that a value of d between 0 and 1.5 is possible for some typical (often aggregate) economic time series. More detailed discussion can be found in a survey by Diebold and Nerlove (1990) and each section below of this paper.

Results of Sowell (1990) and Diebold and Rudebusch (1991b) show that the Dickey-Fuller test can have low power against the ARFIMA(0,d,0) alternatives. Diebold (1989) investigated the power properties of variance ratio tests in the ARFIMA(0,d,0) framework. However, it is not clear whether such tests can be applied or what power properties they have when the underlining time series is a general ARFIMA(p,d,q) process. Obviously, tests designed to specifically detect ARFIMA(p,d,q) alternatives are required.

This paper considers testing $d = 1$ in (1), i.e. a unit root hypothesis, against one-sided alternatives ($d < 1$ or $d > 1$) without knowledge of values of p and q. Two tests are considered with a focus on a new test proposed in this paper. This is a modified one-sided locally best invariant (MLBI) test for one-sided alternatives based on King and Hillier (1985)'s approach. An exact locally best invariant (LBI) test is constructed first for the ARFIMA(0,d,0) case. The test is then modified to test the general ARFIMA(p,d,q) alternatives. The second test is based on the estimate of d obtained using

Geweke and Porter-Hudak (1983)'s (GPH) method. The possibility of this approach was pointed out earlier by Diebold (1989). Also, in the Monte Carlo study later in this paper, we provide further evidence of the power properties of the augmented Dickey-Fuller (ADF) test in a wider setting. In the remainder of this paper, section 2 sketches properties of ARFIMA(p,d,q) process which will be useful for the development of tests in the sequent sections. The LBI and MLBI tests are constructed in section 3. Then follows a brief discussion of the test based on Porter-Hudak's method in section 4. Section 5 reports the results of a Monte Carlo study on power of the tests proposed in section 3 and 4, as well as those of the ADF test. Further discussion on the usefulness of the MLBI test and other comments are found in section 6.

2. ARFIMA(p, $\tilde{d}$, q) Series

Let $\Delta = 1-L$ be the difference operator. Then (1) can be written as

$$\Phi(L)(1-L)^{\tilde{d}}(\Delta X_t - \Delta\mu_t) = \Theta(L)\varepsilon_t \tag{2}$$

where $-0.5 < \tilde{d} < 0.5$. Then $\tilde{d} = 0$ corresponds to a unit root process. Our interest is to test $\tilde{d} = 0$ against $\tilde{d} < 0$ or $\tilde{d} > 0$. Some properties of the ARFIMA(p, $\tilde{d}$, q) series $\{\Delta X_t - \Delta\mu_t\}$ defined by (2) are worth noting.

When $-0.5 < \tilde{d} < 0.5$, the ARFIMA(p, $\tilde{d}$, q) process is stationary, causal and invertible. Its spectral density is

$$f(\omega) = \frac{\sigma^2}{2\pi} \left|1-e^{-i\omega}\right|^{-2\tilde{d}} \frac{|\Theta(e^{-i\omega})|^2}{|\Phi(e^{-i\omega})|^2}$$

$$= \frac{\sigma^2}{2\pi} \left(2\sin\frac{\omega}{2} \right)^{-2\tilde{d}} \frac{|\Theta(e^{-i\omega})|^2}{|\Phi(e^{-i\omega})|^2} \tag{3}$$

where ω is the frequency, and

$$\frac{|\Theta(e^{-i\omega})|^2}{|\Phi(e^{-i\omega})|^2} = \frac{|1 + \theta_1 e^{-i\omega} + \cdots + \theta_q e^{-i\omega q}|^2}{|1 - \phi_1 e^{-i\omega} - \cdots - \phi_p e^{-i\omega p}|^2} .$$

It is assumed that $|\Theta(e^{i\omega})|^2/|\Phi(e^{i\omega})|^2$ is bounded from above and away from zero. When $\tilde{d} = 0$, (3) becomes the familiar spectral density of an ARMA(p,q) process. Since $\lim_{\omega\to 0} f(\omega) = \infty$ when $0 < \tilde{d} < 0.5$, and $\lim_{\omega\to 0} f(\omega) = 0$ when $-0.5 < \tilde{d} < 0$ (however $f(\omega)$ has $+\infty$ at $\omega = 0$), it is clear that the ARFIMA(p,$\tilde{d}$,q) model allows a wider range of low frequency behaviour. This, for example, includes situations where a once-differenced series shows no power at $\omega = 0$ in its estimated spectral density, or still exhibits a high peak at very low frequency.

In the time domain, the above low frequency behaviour is reflected in the slow hyperbolically decaying autocorrelation function

$$\rho_k \sim c_1 k^{2\tilde{d}-1}, \quad k \to \infty \tag{4}$$

in contrast to fast exponentially decaying $\rho(k) \sim c_2 h^k$, $0 < h < 1$, $k \to \infty$ of an ARMA process. Here c_1 and c_2 are constants whose values depend on $\tilde{d}$, ϕ and θ with $\phi = (\phi_1, \cdots, \phi_p)'$ and $\theta = (\theta_1, \cdots, \theta_q)'$.

It is obvious from (4) that an ARFIMA(p,$\tilde{d}$,q) process and an ARFIMA(0,$\tilde{d}$,0) have the same high lag correlation structures. Lower lag structures are dominated by values of ϕ and θ, in addition to values of p and q. This aspect is exploited in constructing the MLBI test in section 3.2.

Consider the moving average representation of $\Delta X_t - \Delta\mu_t$,

$$\Delta X_t - \Delta\mu_t = \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i} . \tag{5}$$

For $-0.5 < \tilde{d} \le 0$, $\sum_{i=0}^{\infty}|\psi_i| < \infty$. Then from time series theory, periodograms of $\{\Delta X_t - \Delta\mu_t\}$ are asymptotically independent and exponentially distributed. The GPH method is based on this asymptotic result. When $0 < \tilde{d} < 0.5$, $\sum_{i=0}^{\infty}|\psi_i| = \infty$.

In this case, although theorems assuming $\sum_{i=0}^{\infty}|\psi_i| < \infty$ are not directly applicable, Yajima (1989) show that the above asymptotic properties of periodograms still hold even when $\sum_{i=0}^{\infty}|\psi_i| = \infty$. Thus the GPH method is justified also for $0 < \tilde{d} < 0.5$.

3. The Modified LBI Tests

Although all the tests proposed in this section apply to general situations where $\Delta\mu_t$ can be a function of time t,[1] in this paper we restrict attention to those situations where $\Delta\mu_t = \mu$, an unknown constant. This corresponds to a level series $\{X_t\}$ with mean $E(X_t) = \upsilon + \mu t$, υ being a constant. In the remainder of the paper, we shall use the notation $Y_t = \Delta X_t$.

3.1 The LBI Tests for ARFIMA(0,$\tilde{d}$,0) Model

Consider first the stationary ARFIMA (0,$\tilde{d}$,0) model

$$(1-L)^{\tilde{d}}(Y_t - \mu) = \varepsilon_t \quad , \qquad t = 1, \cdots, n \tag{6}$$

where $\varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$, and one-sided alternative hypotheses. Specifically, the testing problems are

$$H_0: \quad \tilde{d} = 0 \qquad \text{against} \qquad H_a: \quad \tilde{d} > 0 \quad , \tag{7}$$

$$\text{or} \qquad H_a^-: \quad \tilde{d} < 0 \quad . \tag{8}$$

Equivalently, the model can be written in vector form as

$$Y = \mu l + u \quad , \tag{9}$$

where $Y = (Y_1, \cdots, Y_n)'$, $u = (u_1, \cdots, u_n)'$ with $u_t = (1-L)^{-\tilde{d}}\varepsilon_t$, and l is an

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[1]This is then a regression setting where other variables can enter the equation as regressors. Estimation of $\tilde{d}$ in linear regression models has been considered by Yajima (1988).

$n \times 1$ vector of ones. Then the covariance matrix of u is

$$V(u) = \sigma^2 \gamma_0(\tilde{d}) \Omega(\tilde{d}) \tag{10}$$

where

$$\gamma_0(\tilde{d}) = \frac{\Gamma(1-2\tilde{d})}{(\Gamma(1-\tilde{d}))^2} \quad , \tag{11}$$

and the ij-th element of $\Omega(\tilde{d})$,

$$\rho_k(\tilde{d}) = \frac{\tilde{d}(1+\tilde{d}) \cdots (k-1+\tilde{d})}{(1-\tilde{d})(2-\tilde{d}) \cdots (k-\tilde{d})} \quad , \qquad k = |i-j| = 1, 2, \cdots, n-1 \tag{12}$$

is the k-th order autocorrelation coefficient of $\{u_t\}$. Since μ and σ^2 are nuisance parameters, we seek to construct a test based on a density of a certain random variable function of Y_t which is invariant with respect to μ and σ^2.

Let $M = I_n - \mathit{1}(\mathit{1}'\mathit{1})^{-1}\mathit{1}'$, P be an $(n-1) \times n$ matrix such that $PP' = I_{n-1}$ and $P'P = M$. The testing problems (7) and (8) are invariant under the group of transformations

$$Y \to \tau_0 Y + \tau \mathit{1}$$

where τ_0 and τ are scalars with $\tau_0 > 0$. Then

$$v = \frac{P\hat{u}}{(\hat{u}'M\hat{u})^{1/2}}$$

is a maximal invariant which is free from μ and σ^2, where $\hat{u} = MY$ is the OLS residual from (9). Based on the density of v (see King (1980)), and following King and Hillier (1985), a LBI test against H_a is to reject the null for small values of

$$s = \frac{\hat{u}'A\hat{u}}{\hat{u}'\hat{u}} = -2 \sum_{k=1}^{n-1} \frac{1}{k} \hat{\rho}_k \quad , \tag{13}$$

where, noting that $d\gamma_0(\tilde{d})/d\tilde{d}|_{\tilde{d}=0} = 0$,

$$A = -\frac{d}{d\tilde{d}}\left(\gamma_0(\tilde{d})\Omega(\tilde{d})\right)\Big|_{\tilde{d}=0}$$

$$= -\frac{d\,\Omega(\tilde{d})}{d\tilde{d}}\Big|_{\tilde{d}=0}$$

$$= -\begin{bmatrix} 0 & 1 & \frac{1}{2} & \cdot & \cdot & \cdot & \frac{1}{n-1} \\ & 0 & 1 & \frac{1}{2} & & & \cdot \\ & & \cdot & \cdot & \cdot & & \cdot \\ & & & \cdot & \cdot & \cdot & \frac{1}{2} \\ & & & & \cdot & \cdot & 1 \\ & " & & & & \cdot & 0 \end{bmatrix}, \qquad (14)$$

and

$$\hat{\rho}_k = \frac{\sum_{j=k+1}^{n} \hat{u}_j \hat{u}_{j-k}}{\sum_{j=1}^{n} \hat{u}_j^2} \qquad (15)$$

is the sample autocorrelation coefficient. Note that the test statistic is a weighted sum of sample autocorrelation coefficients with more weight being given to the lower lags. For H_a^-, the LBI test is to reject the null for small values of $s' = -s$.

Given the ARFIMA$(0,\tilde{d},0)$ model, a locally best test is particularly appropriate to distinguish a unit root process ($\tilde{d} = 0$) from its close alternatives. The LBI test (13) is quite powerful for the given sample sizes as shown in Table 1 on the next page.[2]

3.2 The Modified LBI Tests

The ARFIMA$(0,\tilde{d},0)$ model has limited applicability in practice as with only one parameter $\tilde{d}$, it is impossible to capture a variety of short-run

[2]The procedure of Monte Carlo simulation will be discussed in section 5.

Table 1. Power of LBI tests against the ARFIMA(0,$\tilde{d}$,0) process (second row: power against H_a^- using s′)

n	$\tilde{d}$ = .0	.05	.2	.4	critical values
	.0	-.05	-.2	-.4	
50	.050	.109	.449	.868	-0.36
	.050	.088	.348	.781	-0.56
100	.050	.145	.716	.990	-0.31
	.050	.134	.650	.990	-0.41

behaviour. However for an ARFIMA(p,$\tilde{d}$,q) series the s-test in (13) cannot be used directly. The weights are obtained from knowledge of the particular covariance structure (12) of Y_t. For an general ARFIMA(p,$\tilde{d}$,q) process, its covariances are dependent on nuisance parameters ϕ and θ in the AR and MA polynomials of (1), and knowledge of p and q. For example, since the ρ_k's in (12) are all positive for $\tilde{d}$ > 0 and $\hat{\rho}_k$'s in (13) have larger weights for lower lags, the s-test will have larger size for an ARFIMA(1,d,0) series with ϕ_1 > 0, and smaller size for ϕ_1 < 0.

To tackle this problem we note that, although the short-run behaviour of an ARFIMA(p,d,q) process is characterized by ϕ and θ, it is obvious from (3) that the spectral density at low frequency (therefore long-run behaviour of dependency) is dominated by values of $\tilde{d}$. Thus one way to deal with the nuisance parameters is to discard the high frequency density and therefore make ϕ and θ less relevant. This would mean applying a filter which only allows low frequency components of the series to pass. One simple procedure for this is to take a moving average of the series. After applying an equal weight moving average of order r, the transformed series will have the spectral density

$$f(\omega) = \frac{\sigma^2}{2\pi} \left|1-e^{-i\omega}\right|^{-2d} \frac{|\Theta(e^{-i\omega})|^2}{|\Phi(e^{-i\omega})|^2} \frac{1}{r^2}\left(\frac{\sin(r\omega/2)}{\sin(\omega/2)}\right)^2 . \tag{16}$$

From the shape of the curve $(\sin(r\omega/2)/\sin(\omega/2))/r^2$, it is clear that $f(\omega)$ is dampened quickly for large ω when r becomes large. For any given $|\omega_0| > 0$, $f(\omega_0)$ can be made arbitrary close to zero as $r \to \infty$. Thus only smaller neighbourhood of $\omega = 0$ will be effectively relevant for inference when r increases. The neighbourhood may be conveniently defined as the interval $(-2\pi/r,\ 2\pi/r)$, as the filter $(\sin(r\omega/2)/\sin(\omega/2))/r^2$ decreases monotonically and reaches zero for the first time at $\omega = -2\pi/r,\ 2\pi/r$ when $|\omega|$ increases from zero. Since $|\Theta(e^{-i\omega})|^2/|\Phi(e^{-i\omega})|^2$ has zero derivative at $\omega = 0$, an ARFIMA$(p,\tilde{d},q)$ process and an ARFIMA$(0,\tilde{d},0)$ process have the same local spectral power at $\omega = 0$ up to a constant scalar. However a LBI test constructed in the way of section 3.1 is invariant to this constant as it is invariant to values of σ^2. Based on this argument, a MLBI test, embedding the filtering structure, will be asymptotically robust to values of the nuisance parameters ϕ and θ.

Let F be the matrix of a data series transformation which dampens the high frequency density. In the case of the equal weight moving average transformation with order r, F is the $(n-r+1) \times n$ matrix

$$F = \frac{1}{r}\begin{bmatrix} 1 & \cdots & 1 & & & 0 \\ & 1 & & 1 & & \\ & & \ddots & & \ddots & \\ & 0 & & 1 & \cdots & 1 \end{bmatrix} .$$

Note that if $r = 1$, F becomes the identity matrix. Premultiplying F to both sides of (9), we have

$$Y^* = \mu l^* + u^* \quad ,$$

where $Y^* = FY$, $u^* = Fu$ and $l^* = Fl$. From (9) and (10) the transformed error vector u^* has the covariance matrix $\sigma^2\gamma_0(d)F\Omega(d)F'$. Thus, following the same procedure as in section 3.1, we get the MLBI test against H_a which will reject the null for small values of

$$s_m = \frac{Y'F'M^*FAF'M^*FY}{Y'F'M^*FY} , \tag{17}$$

where M^* is as M, but has dimension (n-r+1) × (n-r+1). When r = 1, $s_m = s$. For H_a^-, the MLBI test is to reject the null for small values of $s_m' = -s_m$.

The question remains as to how to determine the value of r. For any given sample size n, a large value of r is not necessarily good because not many effective observations will be available to ensure reasonable power. Yet a value of r which is not large enough indicates that the tests can not be sufficiently robust to the influence of nuisance parameters ϕ and θ. Thus the conditions $r \to \infty$ and $r/n \to 0$ as $n \to \infty$ are necessary for the robustness and power considerations. In our Monte Carlo study, $r = n^\alpha$, $0 < \alpha < 1$ is used.

Finally we note that there is trade-off in gains of robustness of the MLBI test to the nuisance parameters ϕ and θ. The test would be less powerful than the LBI test when applied to an ARFIMA(0,$\tilde{d}$,0) process.

4. The Regression Method

Geweke and Porter-Hudak (1983) proposed a method of estimation of $\tilde{d}$ based on frequency domain approach. Let

$$I(\omega_j) = \frac{1}{n}\left|\sum_{t=1}^{n} Y_t e^{-i\omega_j t}\right|$$

be the periodogram where

$$\omega_j = 2\pi j/n, \qquad j = 0,\ 1,\ \cdots,\ [n/2]$$

are Fourier frequencies and [x] is the integer part of x. Also let

$$g(\omega) = \frac{\sigma^2}{2\pi} \frac{|\Theta(e^{-i\omega})|^2}{|\Phi(e^{-i\omega})|^2}$$

so by (3) $f(\omega) = |1-e^{-i\omega}|^{-2\tilde{d}} g(\omega)$, and

$$\log f(\omega) = \log g(0) + \tilde{d}.\log |1-e^{-i\omega}|^{-2} + \log (g(\omega)/g(0)) \quad .$$

To use the periodogram, this expression can be written as

$$\log I(\omega_j) = \log g(0) + \tilde{d}(-\log |1-e^{-i\omega_j}|^2) + \log (I(\omega_j)/f(\omega_j)) +$$

$$\log (g(\omega_j)/g(0)) \quad . \qquad (18)$$

When ω_j is close to zero the last term in (18) is negligible compared to other terms. As discussed in section 2, $I(\omega_j)/f(\omega_j)$ are asymptotically independent over j when $-0.5 < \tilde{d} < 0.5$. Then (18) can be viewed as a simple regression model

$$y_j = \beta + \tilde{d}\, x_j + \varepsilon_j \qquad (19)$$

where the meaning of each term is obvious, and the OLS estimate $\tilde{d}$ is $\tilde{d}_e$. The calculation is based on ω_j with j up to m(n): m(n) being a function of n such that $m(n) \to \infty$ and $m(n)/n \to 0$ as $n \to \infty$. Geweke and Porter-Hudak show that there exists such m(n) which ensures plim $\tilde{d}_e = \tilde{d}$. If further m(n) also satisfies $(\log n)^2/m(n) \to 0$, then

$$\tilde{d}_e \sim AN\left(d, \frac{\pi^2}{6} \left(\sum_{j=1}^{m(n)} (x_j-\bar{x})^2\right)^{-1}\right)$$

where $\pi^2/6$ is the variance of the asymptotic distribution of the error term $\varepsilon_j = \log(I(\omega_j)/f(\omega_j))$.

A one-sided t-test, termed GPH test, then can be used to test $\tilde{d} = 0$. However there is no obvious way to determine the function m(n). It is not guaranteed that any function m(n) satisfying $m(n) \to \infty$, $m(n)/n \to 0$, $(\log n)^2/m(n) \to 0$ will ensure consistency and asymptotical normality. Simulation results of Geweke and Porter-Hudak (1983) indicate that $m(n) = n^{1/2}$ may be a good choice for their particular case of simulation. Our Monte Carlo study in the next section provide further evidence on this. Finally we note that the t-test test is also invariant to the nuisance parameters μ and σ^2.

5. Empirical Power of the Tests

This section reports some simulation results on the power of the MLBI test, the GPH test and the ADF test.

5.1 Experiment Set-up

For the MLBI test, we tentatively choose $r = n^{\alpha}$ and report the power result for $\alpha = 0.4, 0.5, 0.6$. For the GPH test, we use $m = r$. The sample sizes are set at $n = 100, 200, 400$.

Experiments were carried out with respect to the following four ARFIMA$(p,\tilde{d},q)$ processes:

$$\text{p1:}\qquad (1 - 0.3L)(1 - L)^{\tilde{d}}\,(Y_t - \mu) = \varepsilon_t \qquad ;$$

$$\text{p2:}\qquad (1 + 0.3L)(1 - L)^{\tilde{d}}\,(Y_t - \mu) = \varepsilon_t \qquad ;$$

$$\text{p3:}\qquad (1 - 0.2L)(1 - L)^{\tilde{d}}\,(Y_t - \mu) = (1 + 0.6L)\varepsilon_t \qquad ;$$

$$\text{p4:}\qquad (1- 0.7L + 0.5L^2)(1-L)^{\tilde{d}}\,(Y_t - \mu) = \varepsilon_t \qquad .$$

These represent a variety of situations. When $\tilde{d} = 0$, p1 has a spectral density which attains maximum at $\omega = 0$. Thus, it is expected that the size of the LBI test will be biased upward in testing against $\tilde{d} > 0$, and downward against $\tilde{d} < 0$ alternatives. The opposite situations are expected for p2. Bias of the MLBI test is expected to be much smaller, as α increases. The ARFIMA$(1,\tilde{d},1)$ process p3 is a slightly more general case. For p4, the theoretical spectral density has a peak at the frequency $\omega = 1.018$ (see Harvey 1981, p.73). This corresponds to a cycle of around 6.2 years for yearly data. The purpose is to examine the influence of short-run cycles on the power of the tests considered.

Since the LBI and MLBI test statistics are in ratios of quadratic form in normal variates, the power of these tests may be evaluated by the subroutines used in calculating p-values of the Durbin-Watson statistic. However, to conform to the power evaluation needs of the GPH test, we use the Monte Carlo method throughout the simulations. To generate the required ARFIMA(p, $\tilde{d}$, q) series $\Phi(L)(1-L)^{\tilde{d}}(Y_t - \mu) = \Theta(L)\varepsilon_t$, we set $\mu = 0$ and $\sigma^2 = 1$ since the MLBI tests and GPH test are invariant to values of these nuisance parameters. We first calculate the covariances $\gamma_u(j) = \gamma_0(\tilde{d})\rho_j(\tilde{d})$ of $u_t = (1-L)^{-\tilde{d}}\varepsilon_t$, using (11) and (12). Writing the model as $Y_t = \Phi(L)^{-1}\Theta(L)u_t = \Psi(L)u_t = \sum_{j=0}^{\infty}\psi_j u_{t-j}$, the covariances of Y_t are calculated as

$$\gamma_Y(h) = \sum_{k=-\infty}^{\infty} \tilde{\gamma}(k)\gamma_u(h-k)$$

where

$$\tilde{\gamma}(k) = \sum_{j=-\infty}^{\infty} \psi_j\psi_{j+k},$$

see Hosking (1984) or Brockwell and Davis (1986, p.470 and p.91). Once the covariance matrix of Y, say V(Y), is calculated, its Cholesky decomposition V(Y) = R′R is then found by the IMSL subroutine LFTDS. The subroutine RNMVN is then used to generate the Gaussian ARFIMA series. The number of replications is N = 5000. Power is calculated for points of $\tilde{d}$ = -1.0, -0.4, -0.2, -0.05, 0.0, 0.05, 0.2, 0.4. The ARFIMA(p,-1,q) series are obtained by taking the first order difference of the ARFIMA(p,0,q) series generated.

The ADF test ($\hat{\tau}_\tau$) is the t-test of $\rho = 1$ in the regression

Table 2. Power of the tests against $(1-0.3L)(1-L)^{\tilde{d}}(Y_t-\mu) = \varepsilon_t$ *

(one-sided test against $\tilde{d} > 0$ and $\tilde{d} < 0$)

n	r	$\tilde{d}$ = .0	.05	.2	.4	$\tilde{d}$ = .0	-.05	-.2	-.4	-1.0
100	6	.112	.160	.408	.697	.018	.027	.107	.464	1.000
		.048	.053	.124	.253	.053	.055	.093	.181	.554
		.057	.065	.114	.163	.049	.054	.089	.210	
	10	.082	.109	.266	.474	.024	.035	.094	.358	1.000
		.059	.084	.218	.492	.041	.057	.123	.279	.859
	16	.073	.082	.182	.321	.033	.040	.088	.279	1.000
		.093	.143	.398	.763	.030	.041	.126	.381	.980
200	8	.107	.189	.531	.870	.018	.034	.216	.821	1.000
		.040	.058	.138	.340	.051	.066	.120	.255	.687
		.050	.083	.146	.211	.052	.057	.153	.499	
	14	.076	.144	.366	.668	.027	.040	.189	.682	1.000
		.052	.079	.249	.612	.048	.068	.177	.440	.952
	24	.065	.097	.232	.408	.037	.049	.137	.486	1.000
		.085	.138	.476	.884	.035	.052	.227	.649	.999
400	11	.097	.221	.681	.959	.018	.047	.410	.992	1.000
		.037	.066	.177	.474	.052	.072	.171	.354	.762
		.050	.081	.167	.240	.047	.067	.240	.773	
	20	.070	.144	.478	.799	.030	.060	.318	.942	1.000
		.044	.087	.327	.758	.050	.082	.357	.642	.984
	36	.063	.116	.293	.554	.040	.066	.240	.769	1.000
		.066	.147	.584	.955	.038	.077	.378	.894	1.000

* MLBI test first row, GPH test second row, ADF test third row. Maximum width of 95% confidence interval of estimates of power is smaller than ∓0.014.

$$X_t = \beta_0 + \vartheta t + \rho X_{t-1} + \sum_{i=1}^{g-1} \beta_i \Delta X_{t-i} + v_t \quad .$$

The ARFIMA(p,d,q) series X_t with $d = 1 + \tilde{d}$ are generated by $X_t = X_{t-1} + 2 + Y_t$, $X_0 = 2$. The values of g are chosen as $g = [4(n/100)^{1/4}]$, [x] being the integer part of x. The test is one-sided. The critical values for n = 100 are from Fuller (1976, p.372). Critical values for n = 200, 400 are obtained using interpolation.

Table 3. Power of the tests against $(1+0.3L)(1-L)^{\tilde{d}}(Y_t-\mu) = \varepsilon_t$ *

(one-sided test against $\tilde{d} > 0$ and $\tilde{d} < 0$)

n	r	$\tilde{d}$ = .0	.05	.2	.4	$\tilde{d}$ = .0	-.05	-.2	-.4	-1.0
100	6	.021	.050	.234	.593	.121	.191	.571	.979	1.000
		.042	.046	.107	.231	.055	.062	.104	.184	.405
		.058	.066	.124	.187	.048	.053	.109	.324	
	10	.027	.052	.185	.423	.084	.121	.369	.873	1.000
		.040	.057	.155	.407	.057	.080	.156	.330	.752
	16	.034	.050	.144	.296	.076	.101	.233	.648	1.000
		.029	.047	.202	.579	.075	.098	.242	.576	.963
200	8	.021	.067	.380	.817	.116	.212	.783	1.000	1.000
		.039	.055	.131	.331	.054	.067	.122	.254	.464
		.048	.083	.164	.245	.049	.064	.206	.680	
	14	.032	.079	.298	.630	.080	.124	.536	.984	1.000
		.039	.062	.212	.567	.057	.081	.197	.457	.829
	24	.039	.068	.202	.395	.069	.093	.309	.857	1.000
		.039	.066	.320	.795	.064	.105	.342	.774	.989
400	11	.013	.026	.111	.294	.111	.258	.929	1.000	1.000
		.036	.065	.173	.467	.053	.073	.172	.347	.533
		.052	.083	.182	.273	.045	.072	.298	.881	
	20	.035	.089	.394	.777	.073	.152	.686	1.000	1.000
		.039	.077	.301	.735	.057	.089	.278	.636	.879
	36	.041	.087	.272	.544	.064	.109	.420	.975	1.000
		.039	.090	.468	.924	.063	.118	.469	.921	1.000

* MLBI test first row, GPH test second row, ADF test third row. Maximum width of 95% confidence interval of estimates of power is smaller than ∓0.014.

5.2 Size and Power of the Tests

The results are given in Tables 2 to 5. Critical values are given in Table 6. Consider the size first. The sizes of the MLBI test are typically either above or below the nominal size, which is 0.05. The differences are not large. However it is conceivable that if $\alpha = 0$ (or $r = 1$, so the MLBI test becomes the LBI test) were chosen, the size would be seriously distorted. With a fixed sample size n, the size of the test becomes closer to 0.05, when α (or r) increases from $\alpha = 0.4$ to $\alpha = 0.6$. Fixing the value of α at $\alpha = 0.5$ or 0.6,

Table 4. Power of the tests against $(1-0.2L)(1-L)^{\tilde{d}}(Y_t-\mu)=(1+0.6L)\varepsilon_t$*

(one-sided test against $\tilde{d} > 0$ and $\tilde{d} < 0$)

n	r	$\tilde{d}$ = .0	.05	.2	.4	$\tilde{d}$ = .0	-.05	-.2	-.4	-1.0
100	6	.130	.178	.424	.701	.014	.019	.077	.346	1.000
		.047	.053	.124	.251	.053	.057	.095	.184	.576
		.047	.050	.091	.134	.071	.085	.125	.266	
	10	.088	.116	.273	.477	.022	.032	.076	.292	.999
		.058	.082	.215	.488	.042	.057	.125	.283	.878
	16	.076	.085	.184	.321	.029	.036	.080	.242	.993
		.099	.149	.409	.770	.027	.039	.122	.374	.985
200	8	.118	.203	.540	.872	.016	.026	.173	.721	1.000
		.039	.057	.138	.339	.051	.065	.120	.258	.722
		.037	.064	.112	.174	.073	.088	.210	.561	
	14	.084	.150	.370	.671	.025	.037	.164	.605	1.000
		.050	.078	.248	.609	.048	.068	.180	.444	.967
	24	.067	.099	.232	.408	.035	.046	.124	.439	1.000
		.086	.135	.476	.884	.035	.052	.230	.656	1.000
400	11	.106	.233	.686	.960	.015	.039	.357	.977	1.000
		.037	.066	.176	.471	.052	.073	.171	.359	.812
		.059	.091	.177	.253	.036	.053	.186	.689	
	20	.073	.147	.450	.798	.028	.056	.288	.913	1.000
		.043	.086	.323	.755	.052	.082	.260	.646	.991
	36	.065	.118	.294	.552	.038	.064	.224	.735	1.000
		.064	.143	.578	.955	.039	.079	.381	.895	1.000

* MLBI test first row, GPH test second row, ADF test third row. Maximum width of 95% confidence interval of estimates of power is smaller than ∓ 0.014.

the size also generally approaches 0.05 as n increases. The exception happens at $\alpha = 0.4$ for cases p2 and p4 as can be seen from Tables 3 and 5. For the GPH test, its sizes are generally closer to 0.05, compared to the MLBI test. For fixed n, the size either increases or decreases as α increases, and $\alpha = 0.4$ often gives the closest size to 0.05. In general, as α increases, the size of the MLBI test improves while the size of the GPH test worsens.

One feature of the results is that the ADF has uniformly lower power than the MLBI test and the GPH test in testing against $\tilde{d} > 0$ alternatives. However

Table 5. Power of the tests against $(1- 0.7L + 0.5L^2)(1-L)^{\tilde{d}}(Y_t-\mu) = \varepsilon_t$ *

(one-sided test against $\tilde{d} > 0$ and $\tilde{d} < 0$)

n	r	$\tilde{d}$ = .0	.05	.2	.4	$\tilde{d}$ = .0	−.05	−.2	−.4	−1.0
100	6	.019	.040	.185	.512	.121	.173	.440	.883	1.000
		.031	.035	.086	.200	.067	.072	.121	.212	.508
		.058	.064	.137	.203	.046	.058	.126	.376	
	10	.020	.037	.149	.370	.116	.157	.383	.806	1.000
		.018	.025	.075	.266	.102	.131	.253	.471	.896
	16	.025	.041	.124	.267	.093	.118	.252	.593	1.000
		.002	.004	.027	.213	.274	.361	.624	.883	.998
200	8	.014	.043	.296	.754	.166	.272	.782	.998	1.000
		.033	.050	.117	.310	.060	.075	.134	.272	.591
		.046	.084	.176	.268	.050	.063	.233	.745	
	14	.025	.060	.252	.592	.113	.165	.568	.974	1.000
		.022	.040	.154	.486	.077	.108	.264	.540	.934
	24	.030	.056	.183	.380	.082	.112	.334	.825	1.000
		.005	.010	.105	.543	.193	.276	.640	.937	1.000
400	11	.010	.056	.457	.920	.176	.348	.942	1.000	1.000
		.033	.062	.164	.457	.054	.077	.177	.360	.647
		.049	.084	.188	.286	.044	.070	.314	.906	
	20	.025	.068	.353	.758	.106	.202	.724	.999	1.000
		.030	.059	.259	.694	.071	.108	.314	.685	.956
	36	.035	.076	.253	.535	.074	.128	.452	.967	1.000
		.010	.030	.292	.845	.142	.237	.662	.996	1.000

* MLBI test first row, GPH test second row, ADF test third row. Maximum width of 95% confidence interval of estimates of power is smaller than ∓ 0.014.

for testing against $\tilde{d} < 0$, the result is not unambiguous. Depending on the values of α, relative power of the ADF test and the MLBI test or the GPH test differs. While size of the MLBI test improves as α increases from $\alpha = 0.4$ to $\alpha = 0.6$ for fixed n, the test becomes generally less powerful for $|\tilde{d}| \neq 0$. For example, from Table 2, for $\tilde{d} = -0.2$ and n = 100, the power of the MLBI test drops from 0.107 to 0.088, despite improvement in size from 0.018 to 0.033. It is not so for $\tilde{d} = -0.05$, as at this value, the power is sensitive to the size. Exactly the opposite situations happen to the GPH test. From these

Table 6. Critical values of the tests at 5% significance level*

test	n = 100 (r = 6, 10, 16)			200 (r = 8, 14, 24)			400(r = 11, 20, 36)		
ADF	-0.90			-0.9133			-0.926		
	-3.45			-3.4376			-3.424		
GPH	1.645			1.645			1.645		
	-1.645			-1.645			-1.645		
MLBI	-2.98	-3.54	-3.83	-3.42	-4.10	-4.68	-3.87	-4.72	-5.40
	1.33	1.42	1.26	1.85	2.03	1.90	2.39	2.66	2.64

* Testing against H_a: $\tilde{d} > 0$ on first row, against H_a^-: $\tilde{d} < 0$ on second row.

observations, one possible approach, in practice, is to use the MLBI test with $\alpha = 0.6$ or the GPH test with $\alpha = 0.4$ in testing against either $\tilde{d} > 0$ or $\tilde{d} < 0$ alternatives. The ADF test may be used in testing against $\tilde{d} < 0$ only.

Finally, we note that at $\tilde{d} = -1$, the MLBI test is very powerful. It has power exceeding 0.99 in all the cases (and is uniformly not less powerful than the GPH test). This indicates that the MLBI test can also be used to test, for example, d = 2 against d = 1. Further usefulness of this test is discussed in the next section.

6. Remarks

Lo (1991) suggests a modified R/S test for long-run dependency, that is robust to short-run behaviour, in a rather general framework. As he pointed out, specialization to parametric models may reveal more evidence of long-run memory in time series. Our paper develops a class of locally best invariant tests for the particular ARFIMA(p,d,q) models. They can be used to test whether a time series is integer integrated (having unit root) or fractionally integrated. A test based on the GPH method is also included in the Monte Carlo power study.

Unless it is specifically known that a time series can be modelled by an

ARFIMA(0,d,0) process, the exact LBI test is subject to a serious size problem. With the MLBI test, the wrong real size is largely corrected. Although the real size is still not very reliable in small samples, simulation results for our particular cases show that this problem is diminished as the order of smoothing r becomes large. When the sample size n becomes larger at the faster rate $n/r = n^{1-\alpha}$ with $0 < \alpha < 1$, reasonable power is also achieved. For $-0.5 < \tilde{d} < 0.5$, there is no dominance of the MLBI test over the GPH test or *vice versa* in any case. Both can be used in the way described in the last section. The merit of the two tests lie in their robustness to the nuisance parameters ϕ and θ, and polynomial orders p and q,[3] that characterize short-run behaviour, while maintaining reasonable power. Our Monte Carlo results also reveal clear power advantage of the MLBI test and the GPH test over the ADF test for the range $d > 1$. When $0.5 < d < 1$, the ADF test has competitive power.

Additional advantages of the ARFIMA(p,d,q) framework and the usefulness of the tests proposed in this paper should be mentioned. Since d nests integer values, and because the MLBI test and the GPH test are invariant to the mean of time series, hypotheses such as $d = 2$ (or $\tilde{d} = 0$) against $d = 1$ (or $\tilde{d} = -1$) may be tested formally, using the two tests. In this case, our Monte Carlo study shows that the MLBI test is very powerful. This is a significant result. As is well known, see for example Sowell (1992b, pp.281-282), if $\{X_t\}$ is a unit root process with draft, then $\Phi(L)(\Delta X_t - \mu) = \Theta(L)\varepsilon_t$. If $\{X_t\}$ is a trend (time trend with coefficient μ^*) stationary process, then $\Phi^*(L)(\Delta X_t - \mu^*) = (1-L)\Theta^*(L)\varepsilon_t$, so a moving average unit root is present. Note that * indicates that the mean of ΔX_t can be different under the different hypotheses, and different short-run characteristics may be mingled with different trends,

[3]When p and q are known, a more powerful test (see Wu (1992a)) is available.

namely, stochastic and deterministic trends. Testing the unit root hypothesis against the trend stationary hypothesis, is equivalent to testing an ARFIMA(p,0,q) process against an ARFIMA(p^*,-1,q^*) process. Since the the MLBI test is invariant to mean and is robust to short-run behaviour, it is very suitable for such problems. The high power of the test revealed in our experiment guarantees its usefulness. Some applications of the test in this context and tabulated small sample critical values of the test are found in Wu (1992b).

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