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Project valuation using state contingent claims

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ABSTRACT

This paper shows that the Banz and Miller framework for project valuation, using matrices of state contingent claim prices, is valid only if expectations of future payoffs are not revised. The appropriate general framework when such expectations are revised is then presented, followed by a computationally simpler version.

Banz and Miller (1978), hereafter BM, have derived a matrix of one-period state contingent claim prices and applied these to the valuation of project payoffs. However their formulation is valid only if expectations of future payoffs are never revised in accordance with the realisation of states prior to the payoff year. This paper verifies this and presents the appropriate general procedure if such revision occurs. However this general procedure is computationally too demanding to be useful. Accordingly a simplified procedure is offered which parallels the Myers and Turnbull (1977) simplification in multiperiod applications of the Capital Asset Pricing Model (CAPM).

I THE BM VALUATION PROCEDURE AND ITS GENERALISATION

Denoting the real project payoff at end year t by the random variable X_t, expectation now by E_0, and BM's nxn stationary valuation matrix by

$$V = [v_{ij}] = [\text{value now of \$1 in 1 year iff state j over year, given state i last year}]$$

then BM's value now for X_t is

$$V^t \begin{bmatrix} E_0(X_t \mid \text{state 1 over year t}) \\ \vdots \\ E_0(X_t \mid \text{state n over year t}) \end{bmatrix}$$

This is valid only if expectations of X_t will not be revised according to states realised in years 1 ... t -1. To demonstrate this, let $P_t(x)$ be the value of x at the end of year t.

Assuming, as BM do, that the market treats random variable X_1 as if it were the vector of conditional expectations

$$\begin{bmatrix} E_0(X_1 \mid \text{state 1 over year 1}) \\ \vdots \\ E_0(X_1 \mid \text{state n over year 1}) \end{bmatrix}$$

then

$$P_0(X_1) = V \begin{bmatrix} E_0(X_1 \mid \text{state 1 over year 1}) \\ \vdots \\ E_0(X_1 \mid \text{state n over year 1}) \end{bmatrix}$$

which is BM's formulation. By the same reasoning

$$P_1(X_2) = V \begin{bmatrix} E_1(X_2 \mid \text{state 1 over year 2}) \\ \vdots \\ E_1(X_2 \mid \text{state n over year 2}) \end{bmatrix} \tag{1}$$

and, since $P_1(X_2)$ is currently a random variable just as X_1 is, then

$$\begin{aligned} P_0(X_2) &= P_0(P_1(X_2)) \\ &= V \begin{bmatrix} E_0\{P_1(X_2) \mid \text{state 1 over year 1}\} \\ \vdots \\ E_0\{P_1(X_2) \mid \text{state n over year 1}\} \end{bmatrix} \end{aligned}$$

Using (1),

$$E_0\{P_1(X_2) \mid \text{state i over year 1}\} = (\text{V row i}) \begin{bmatrix} E_0(X_2 \mid \text{state i year 1, state 1 yr 2}) \\ \vdots \\ E_0(X_2 \mid \text{state i year 1, state n yr 2}) \end{bmatrix}$$

Thus

$$P_0(X_2) = V \begin{bmatrix} (\text{V row 1}) \begin{bmatrix} E_0(X_2 \mid \text{state 1 yr 1, state 1 yr 2}) \\ \vdots \\ E_0(X_2 \mid \text{state 1 yr 1, state n yr 2}) \end{bmatrix} \\ \vdots \\ (\text{V row n}) \begin{bmatrix} E_0(X_2 \mid \text{state n yr 1, state 1 yr 2}) \\ \vdots \\ E_0(X_2 \mid \text{state n yr 1, state n yr 2}) \end{bmatrix} \end{bmatrix}$$

which is equal to BM's formulation of

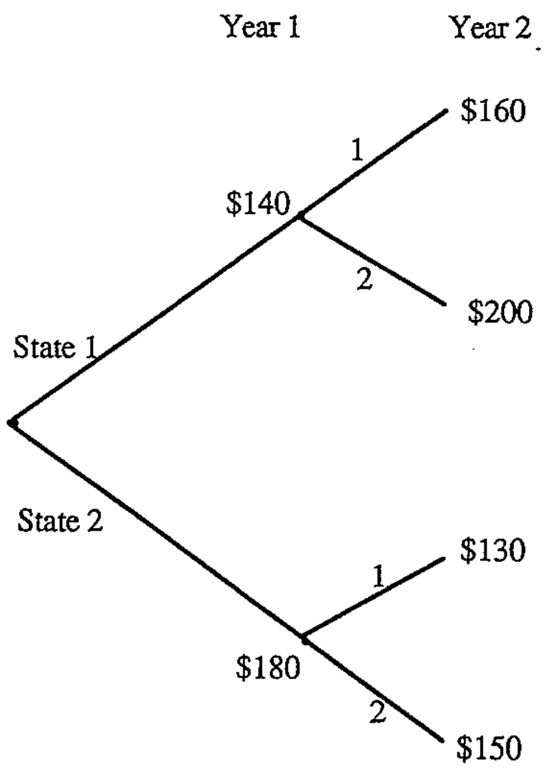
$$VV\begin{bmatrix} E_0(X_2 \mid \text{state 1 yr 2}) \\ \vdots \\ E_0(X_2 \mid \text{state n yr 2}) \end{bmatrix}$$

only if $E_0(X_2 \mid \text{state i yr 1, state j yr 2})$ is independent of the state in year 1, i.e. there is no revision in expectations according to the state in year 1. This is surely very restrictive. If states denote a diversified portfolio's rates of return, or GNP growth rates, and these are correlated with project payoffs, then a good year 1 state implies a high (or low) year 1 payoff, which should imply an upward (or downward) revision in expectations of payoffs in year 2 and beyond.

To illustrate the process let there be two equally probable states with

$$\dot{V} = \begin{bmatrix} .6 & .35 \\ .6 & .35 \end{bmatrix}$$

and the expected payoffs conditional on states in years 1, 2 be



Thus $E_o\{P_1(X_2) \mid \text{state 1 yr 1}\} = (.6 \quad .35)\begin{pmatrix}130\\150\end{pmatrix} = 130.5$

and $E_o\{P_1(X_2) \mid \text{state 2 yr 1}\} = (.6 \quad .35)\begin{pmatrix}160\\200\end{pmatrix} = 166$

Thus $$P_o(X_2) = V\begin{bmatrix}130.5\\166\end{bmatrix} = \begin{bmatrix}136.4\\136.4\end{bmatrix}$$

Since states 1, 2 are equally probable then

$$E_o(X_2 \mid \text{state 1 yr 2}) = .5(130) + .5(160) = 145$$

$$E_o(X_2 \mid \text{state 2 yr 2}) = .5(150) + .5(200) = 175$$

and hence BM's formula would yield a present value for X_2 of

$$VV\begin{bmatrix}145\\175\end{bmatrix} = V\begin{bmatrix}148.25\\148.25\end{bmatrix} = \begin{bmatrix}140.8\\140.8\end{bmatrix}$$

This value 140.8 overstates the correct value of 136.4 because it ignores the risk arising from uncertainty now about $P_1(X_2)$.

There is no disagreement about $P_o(X_1)$, which is

$$V\begin{bmatrix}140\\180\end{bmatrix} = \begin{bmatrix}147\\147\end{bmatrix}$$

Some additional points are as follows. The valuation process described above, when expectations are revised, involves separate valuations for $X_1, X_2, \ldots X_k$. This was employed to facilitate comparison with the BM method. A more efficient procedure is to firstly determine the conditional values $P_{k-1}(X_k)$, then $P_{k-2}\,[P_{k-1}(X_k) + X_{k-1}]$, then $P_{k-3}[P_{k-2}(\cdot) + X_{k-2}]$, etc. until $P_o[P_1(\cdot) + X_1]$ is determined. Using the previous example then

$$P_o(P_1(X_2) + X_1) = V\begin{bmatrix}130.5 + 140\\166 \quad + 180\end{bmatrix} = \begin{bmatrix}283.4\\283.4\end{bmatrix}$$

which agrees with the sum of $P_o(X_2)$ and $P_o(X_1)$ above i.e. $136.4 + 147 = 283.4$

Secondly, BM note that Bierman and Smidt [1975] have devised a similar valuation procedure to theirs, but based on the CAPM rather than the Option Pricing Model. However, Bierman and Smidt do acknowledge revision of expectations in their procedure.

A third point concerns leverage. Assuming that the payoffs $X_1......X_k$ are unlevered then the valuation procedure described above yields unlevered present value, since the valuation matrix V does not allow for any advantage to debt financing. Accordingly, if an advantage to debt financing is considered to exist, in the form of an interest tax shield, then the present value of the interest tax shield should be added to the value derived above.

II A SIMPLIFIED PROCEDURE

Implementation of the BM generalised procedure described above requires a tree of conditional expected payoffs. Even with only 3 branches per year the number of conditional expectations required rapidly becomes unmanageably large. For example, for a 4 year project the number required for the fourth year cash flow alone would be 81. The same problem arises in seeking to apply the CAPM to multiperiod projects, as discussed by Myers and Turnbull. Their solution is to analytically specify the process by which expectations are revised as cash flows are realised. Consequently the only expectations required are the expectations now of each future cash flow. A parallel procedure is offered below for the BM framework. In this procedure dependence of state contingent claim prices on prior year states will be disregarded, as BM show that the degree of dependence is trivial.

Consider random real cash flow X_2 arising in 2 years time. Its value in 1 year, $P_1(X_2)$, is

$$P_1(X_2) = \Sigma V_j E_1(X_{2j})$$

where X_{2j} is X_2 if state j occurs in year 2, and V_j is the value of a claim paying a real \$1 in 1 year if state j prevails over that year and nothing otherwise.

Assume the distribution of X_2 is normal. Then

$$E_1(X_{2j}) = E_1(X_2) + \sigma_2 Z_{2j}$$

where σ_2 is the standard deviation of the distribution, $E_1(X_2)$ the mean of the distribution and Z_{2j} the standard normal value corresponding to $E_1(X_{2j})$.

Thus
$$\begin{aligned} P_1(X_2) &= \Sigma V_j\{E_1(X_2) + \sigma_2 Z_{2j}\} \\ &= E_1(X_2)[\Sigma V_j + \theta_2 \Sigma V_j Z_{2j}] \ , \ \theta_2 \equiv \sigma_2/E_1(X_2) \end{aligned}$$

Now assume, as Myers and Turnbull do, that expectations are revised as follows

$$E_1(X_2) = E_o(X_2)[1 + a\Delta_1] \ , \ \Delta_1 \equiv \frac{X_1 - E_o(X_1)}{E_o(X_1)}$$

with coefficient $0<a<1$ expressing the degree to which deviations in X_1 from $E_o(X_1)$ prompt revision in the expectation of X_2. Thus

$$P_1(X_2) = E_o(X_2)[1 + a\Delta_1][\Sigma V_j + \theta_2 \Sigma V_j Z_{2j}]$$

Now $E_o[P_1(X_2)]$ depends upon the state realised in period 1, via Δ_1. Thus

$$E_o[P_1(X_2) \mid \text{state i yr 1}] = E_o(X_2)[1 + a\Delta_{1i}][\Sigma V_j + \theta_2 \Sigma V_j Z_{2j}]$$

where
$$\begin{aligned} \Delta_{1i} &= \frac{E_o(X_{1i}) - E_o(X_1)}{E_o(X_1)} \\ &= \frac{\sigma_1 Z_{1i}}{E_o(X_1)} \\ &= \theta_1 Z_{1i} \end{aligned}$$

and σ_1, Z_{1i}, θ_1 have analagous definitions to σ_2, Z_{2j}, θ_2.

Thus

$$E_o[P_1(X_2) \mid \text{state i yr 1}] = E_o(X_2)[1 + a\theta_1 Z_{1i}][\Sigma V_j + \theta_2 \Sigma V_j Z_{2j}]$$

Then $P_o(X_2) = \Sigma V_i E_o[P_1(X_2) \mid \text{state i yr 1}]$

$$= E_o(X_2)[\Sigma V_i + a\theta_1 \Sigma V_i Z_{1i}][\Sigma V_j + \theta_2 \Sigma V_j Z_{2j}]$$

By extension the value now of a cash flow X_k arising in k years is

$$P_o(X_k) = E_o(X_k) \prod_{t=1}^{k-1} [\Sigma V_i + a\theta_t \Sigma V_i Z_{ti}][\Sigma V_j + \theta_k \Sigma V_j Z_{kj}]$$

If it is now assumed that θ_t and $\Sigma V_i Z_{ti}$ are the same for all t (namely θ and $\Sigma V_i Z_i$), which is not implausible, then

$$P_o(X_k) = E_o(X_k)[\Sigma V_i + a\theta \Sigma V_i Z_i]^{k-1}[\Sigma V_i + \theta \Sigma V_i Z_i]$$

Thus the only project specific parameters to be estimated are $E_o(X_k)$, the expectations revision coefficient "a", the relative standard deviation θ , and the sign of $\Sigma V_i Z_i$ (which will be negative if X_k is positively correlated with the return on the portfolio whose outcomes constitute the "states"). This result closely resembles the Myers and Turnbull result of

$$P_o(X_k) = E_o(X_k) \left[\frac{1-a\lambda\sigma}{1+R_f}\right]^{k-1} \left[\frac{1-\lambda\sigma}{1+R_f}\right]$$

where R_f = riskless rate

$$\lambda = \frac{E_m - R_f}{\sigma_m}, \quad \text{m being the market portfolio}$$

$$\sigma = \frac{Cov(X_t, R_{mt})}{E_{t-1}(X_t)}, \quad \text{assumed same for all t}$$

To clarify the calculation of $\Sigma V_i Z_i$ consider a three state world, with BM state contingent claim values of $V_1 = 53¢$, $V_2 = 29¢$, $V_3 = 17¢$ (states 1, 2, 3 are the equally probable low, medium and high returns on some portfolio). If the project payoffs are positively correlated with this portfolio's return then Z_1, Z_2, Z_3 will be in ascending order. Since the 3 states are equally probable then the Z range must be partitioned accordingly into $< -.43$, $-.43 \rightarrow .43$, $> .43$ and hence

$$Z_1 = E(Z| Z< .43) = -1.1$$
$$Z_2 = E(Z| -.43 \leq Z \leq .43) = 0$$
$$Z_3 = E(Z| Z>.43) = 1.1$$

Thus $$\Sigma V_j Z_j = -1.1(.53) + 0(.29) + 1.1(.17) = -.41$$

An example of cash flow valuation now follows. Consider a project with real cash flows X_1, X_2 in 1,2 years time. The conditional expectations for X_1 are 80,100,120 according to which of the 3 states prevails, the unconditional expectation now for X_2 is 110, and

a = .7. Thus
$$\theta = \frac{\text{Standard deviation of (80,100,120)}}{\text{Expectation of (80,100,120)}}$$

$$= .16$$

Thus $$P_o(X_1) = E_o(X_1) [\Sigma V_j + \theta \Sigma V_j Z_j]$$

$$= 100 [.99 + .16(-.41)]$$

$$= 92$$

and $$P_o(X_2) = E_o(X_2) [\Sigma V_j + a\theta \Sigma V_j Z_j][\Sigma V_j + \theta \Sigma V_j Z_j]$$

$$= 110 [.99 + (.7)(.16)(-.41)] [.99 + .16(-.41)]$$

$$= 96$$

The unlevered project value is then $92 + 96 = 188$.

IV CONCLUSION

This paper has generalised the BM valuation procedure to the case where expectations are revised prior to their realisation, and then offered a simplified procedure. This simplified procedure parallels the Myers and Turnbull simplification in multiperiod applications of the CAPM.

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