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Nominal Wage Stickiness and the Natural Rate Hypothesis: An Empirical Analysis

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Abstract

This paper studies a model in which nominal wages are predetermined at the expected market clearing level, firms bear costs of adjusting labor, and households have non–time–separable utility functions. It is shown that even if nominal wages are determined one period earlier and fixed only over a period, serially uncorrelated innovations in the price level affect aggregate employment arbitrarily far into the future. Nevertheless, the model satisfies the natural rate hypothesis. The empirical result shows that the slope of the statistical Phillips curve implied by the estimates of parameters is big. The result also shows enough serial persistence in employment.

Key Words: natural rate hypothesis, wage stickiness, Phillips curve

1. Introduction

The problem of identifying the characteristics of current market economies that account for the connection between nominal causes and real effects presents a difficult challenge to economic analysis. It seems to be true that we must assume some kind of rigidity in our economy in order to resolve this problem. Currently, there are basically two approaches in attacking this problem. One is based on the assumption of informational rigidity while the other is based on the assumption of price rigidity.

The first approach, as typified by Lucas (1972a, 1972b, 1973), relies on the assumption of incomplete information regarding the general price level. According to this approach, nominal shocks can affect real variables because people in local markets confuse changes in the general price level with shifts in relative price levels. But this approach has been criticized by some (e.g., Boschen and Grossman (1982), Grossman (1983), McCallum (1982)) in view of the fact that information on nominal aggregate variables is available relatively promptly.

The second approach incorporates wage rigidity by assuming the existence of multiperiod overlapping nominal wage contracts. This approach has been criticized for exogenously specifying the length and nature of wage contracts.

Perhaps the two most prominent models among the contract–based models are those of Fischer (1977) and Taylor (1979, 1980). As McCallum (1982) notes, even though both use basically the same aggregate demand function, there are several critical differences between them. Two of the major differences can be summarized as follows.

(1). In Fischer (1977), the nominal wage rate is predetermined at a level that is expected to clear the market in the relevant period. A contract may specify different wage rates in each subperiods of the contract. Whereas in Taylor (1979, 1980), a contract specifies the same wage rate in all subperiods of the contract. The level of wage rate in time t contract is set as weighted averages of the wage rates embedded in those past and future contracts which overlap with the time t contract, with an adjustment reflecting expected excess demand.

(2). In Fischer (1977), employment is determined at a level that equates the marginal product of labor to the real wage rate. In Taylor (1979, 1980), the price level is assumed to be a fixed mark–up of average wages.

Taylor's model has been particularly popular because it is testable and seems to be consistent with certain properties of the time series such as serial persistence in aggregate employment. Since each contract is written relative to other contracts in Taylor (1979, 1980), shocks are passed from one contract to another and there occurs serial persistence. However, as McCallum (1982) and Ashenfelter and Card (1982), among others, note, Taylor's model has some undesirable features. Among them are the following;

(1). The model does not possess the *natural rate property* as defined by Lucas (1972b) so that well articulated monetary policy is capable of yielding a permanent increase in actual employment level relative to its natural-rate value.[1] Essentially this feature arises because Taylor's specification implicitly assumes that labor supply and demand are functions of relative rather than real wage rates. Almost all of the results in Taylor (1979, 1980) depend heavily on this contract specification.

(2). Taylor's model is inconsistent with the data along some important dimensions. One implication of Taylor's model is that both nominal wages and unemployment follow an ARMA(n,n) with the same AR part, where n is equal to the length of wage contract minus one. However this is inconsistent with the results reported in Ashenfelter and Card (1982). In particular, (i) the quarterly wage data can be adequately described by AR1 process and there is no evidence of moving average errors in the AR representation of nominal wages, and (ii) the stochastic structure of nominal wages differs in important ways from that of both prices and employment.

In contrast to the model proposed by Taylor, the model developed by Fischer has not been extensively investigated empirically. Perhaps one reason for this is that the model as specified does not give rise to enough serial correlation in real variables. For example, Taylor (1980: p.2) claims that neither the information based models nor Fischer (1977) can explain the observed serial correlation in employment. Taylor (1985) motivates his model as one alternative to the expected market clearing approach (e.g., Fischer (1977)) that meets the empirical requirement of serial persistence.

In fact there are several ways in which one can generate serial persistence in real variables. For example, the existence of adjustment costs (Sargent 1978), capital formation lags (Kydland and Prescott 1982), non–time–separable utility functions (Eichenbaum, Hansen and Singleton 1988) or the inability of agents to distinguish between permanent and transitory shocks (Brunner, Cukierman and Meltzer 1980) all give rise to serially correlated economic time series. All of these features can be incorporated into a model in which nominal wages are set in the way described by Fischer (1977). Such models will satisfy the natural rate hypothesis but still allow for some rigidity of nominal wages and the possibility that purely nominal disturbances affect real variables in a persistent way.

The above considerations motivate the research agenda of this paper. In particular, we derive a model which is consistent with serial persistence in employment, allows for interactions between real and nominal variables and possesses the natural rate property.

In attacking this problem, we try to reconcile the equilibrium approach and the

[1] In Taylor's model, different acceleration magnitude of inflation is permanently associated with different levels of employment relative to its natural rate level. For a proof of this, see McCallum (1989).

contract–based approach. We use equilibrium real business cycle model in Sargent (1987: pp.472-478) and impose the assumption that nominal wages are predetermined at the expected market clearing level. The model is based on a specification of agents' intertemporal utility maximization problems. Serial persistence in variables occurs because of nontime-separability of utility from leisure and costs in adjusting employment levels.

We assume that agents set nominal wages at the beginning of each period (or, put it differently, one period in advance of the trading period), and wages remain fixed only over the period. Wage setting behavior depends on rational estimates of the next period's state variables. Because nominal wages are predetermined, the model gives rise to a positive correlation between aggregate employment and nominal disturbances.

The model is estimated and tested using post-World War II U. S. time series data. In order to disentangle the role of nontime-separability of preference, adjustment costs and nominal wage rigidities, we estimate the structural parameters of the model, rather than the reduced form.

The remainder of the paper is organized as follows. Section 2 describes our model. Section 3 explains the derivation of the closed form solution. Section 4 derives an estimable representation by assuming specific forms of the driving processes, and discusses the properties of it. It captures many of the key features of the general model. Section 5 describes estimation methodology. Section 6 presents the empirical results. Section 7 offers some concluding remarks.

2. The General Model

Our model is an extension of that in Sargent (1987, Chapter XVIII). We impose the assumption that nominal wages are predetermined.

There exist a large, fixed number of infinitely–lived households and firms in this economy. All firms are alike and households are all alike. There exists a homogeneous nonstorable consumption good. It is the only commodity in this economy. From now on, we follow the standard device of *representative agents* and assume, without loss of generality, that there exists a single representative household and a single representative firm and that each of them behaves as if prices were given.

Let

c_t consumption of the representative consumer at period t,

n_t^s the number of man hours supplied by the representative household during period t,

n_t^d the number of man hours demanded by the representative firm during period t,

n_t the number of man hours actually employed at period t,

W_t nominal wage rate at period t,

w_t log of the nominal wage rate at period t,

P_t price of a unit of consumption good at period t,

p_t log of the price of a unit of consumption good at period t,

Π_t dividends that the representative household receives from the representative firm at period t,

ε_t a random shock to preference,

a_t a random shock to technology,

η_t a nominal shock to price,

b discount factor, $0 < b < 1$.

The representative household gets utility from consuming the good and gets disutility from supplying labor. Its income consists of (1) wages earned from supplying labor to the firm and (2) the dividend received from the firm. It buys consumption goods from the firm. We can express the representative household's problem at period t ($t=0,1,\cdots$) as that of calculating a contingency plan for $\{n_{t+j}^s, j\geq 0\}$ and $\{c_{t+j}, j\geq 0\}$ so as to maximize the discounted present value of its expected utility at period t:

$$V_t^h = E_t \sum_{j=0}^{\infty} b^j [u_0 c_{t+j} - (\delta_0 + \varepsilon_{t+j}) n_{t+j}^s - \tfrac{1}{2}\delta_1 (n_{t+j}^s)^2 - \tfrac{1}{2}\delta_2 (n_{t+j}^s + \gamma n_{t+j-1}^s)^2] \qquad (1)$$

subject to $c_{t+j} = (W_{t+j}/P_{t+j}) n_{t+j}^s + \Pi_{t+j}$, n_{t-1} given. u_0, δ_0, δ_1, δ_2 are positive scalars and $0 < |\gamma| < 1$. For any random variable z, $E_t z = E[z|\Theta_t]$ where E is the mathematical expectations operator and Θ_t is the information set available at period t. We assume that Θ_t includes at least $\{n_{t-1}, n_{t-2}, \cdots, W_t, W_{t-1}, \cdots, P_t, P_{t-1}, \cdots, \varepsilon_t, \varepsilon_{t-1}, \cdots, a_t, a_{t-1}, \cdots, \eta_t, \eta_{t-1}, \cdots\}$. The random shock ε_{t+j} reflects changes in the marginal rate of substitution between consumption and leisure. We assume that $\{\varepsilon_t, t\geq 0\}$ is a sequence of exponential order less than $1/\sqrt{b}$. The parameter γ induces temporal nonseparabilities in the household's utility function. As in Kennan (1983) and Eichenbaum, Hansen and Singleton (1988), we do not impose any restriction on the sign of γ. A positive value of γ would imply that current leisure and leisure in adjacent periods are substitutes whereas, a negative value of γ would imply that those are complements. The importance of the intertemporal substitution factor in explaining fluctuations in economic variables has been stressed by many (Lucas and Rapping (1969), Kydland and Prescott (1982)). Intertemporal complementarity of leisure could be one of the sources of serial persistence in employment. In maximizing (1), the representative household considers the stochastic processes governing the behavior of $\{\varepsilon_{t+j}, j\geq 0\}$, $\{P_{t+j}, j\geq 0\}$, $\{W_{t+j}, j\geq 0\}$ and $\{\Pi_{t+j}, j\geq 0\}$ as beyond its control. Using the budget constraint to eliminate c_{t+j} from (1), the household's problem at period t becomes to choose a contingency plan for $\{n^s_{t+j}, j\geq 0\}$ to maximize

$$V^h_t = E_t \sum_{j=0}^{\infty} b^j[u_0(\frac{W_{t+j}}{P_{t+j}} n^s_{t+j} + \Pi_{t+j}) - (\delta_0 + \varepsilon_{t+j})n^s_{t+j} - \tfrac{1}{2}\delta_1(n^s_{t+j})^2 - \tfrac{1}{2}\delta_2(n^s_{t+j} + \gamma n^s_{t+j-1})^2] \quad (2)$$

subject to n_{t-1} given. Household solves this maximization problem for each period $t=0,1,2,\cdots$. The sufficient condition for the household's problem consists of a set of Euler equations

$$\gamma b\delta_2 E_t n^s_{t+1} + (\delta_1 + \delta_2 + \delta_2\gamma^2 b)n^s_t + \delta_2\gamma n^s_{t-1} = u_0(W_t/P_t) - \delta_0 - \varepsilon_t, \quad t = 0,1,2,\cdots \quad (3)$$

and a side condition on the path of $\{n^s_t\}$

$$\sum_{t=0}^{\infty} b^t(\tfrac{1}{2})\delta_1(n^s_t)^2 < \infty.$$

Euler equation (3) is not linear with respect to W_t and P_t even though is linear with respect to the real wage W_t/P_t. This would make the derivation of the explicit equilibrium law of motion for W_t and n_t extremely difficult. Therefore, as in Sargent (1987, p.482) we use the following approximation. Suppose that units have been chosen to make W_t/P_t equal unity on average. Then by approximation,

$$\frac{W_t}{P_t} = 1 + \log\frac{W_t}{P_t} = 1 + w_t - p_t, \; t = 0,1,2,\cdots \quad (4)$$

which comes from the first two terms of a Taylor's series expansion of $\log(W_t/P_t)$ about $(W_t/P_t) = 1$. If we substitute (4) into (3), we get

$$\gamma b\delta_2 E_t n^s_{t+1} + (\delta_1 + \delta_2 + \delta_2\gamma^2 b)n^s_t + \delta_2\gamma n^s_{t-1} = u_0(1 + w_t - p_t) - \delta_0 - \varepsilon_t, \quad t = 0,1,2,\cdots \quad (5)$$

The representative firm produces consumption good using labor according to the quadratic production technology. And it bears real costs of having adjusted its labor demand. The problem of the representative firm at period t ($t=0,1,\cdots$) can be expressed as choosing contingency plans for $\{n^d_{t+j}, j\geq 0\}$ to maximize its expected present real value:

$$V^f_t = E_t \sum_{j=0}^{\infty} b^j \Pi_{t+j},$$

$$\Pi_{t+j} = (f_0 + a_{t+j})n^d_{t+j} - \tfrac{1}{2} f_1 (n^d_{t+j})^2 - \tfrac{1}{2} d(n^d_{t+j} - n^d_{t+j-1})^2 - \frac{W_{t+j}}{P_{t+j}} n^d_{t+j}, \qquad (6)$$

n_{t-1} given. f_1, f_2, d are positive scalars. $\{a_t\}$ is a sequence of random shocks to technology. It reflects stochastic fluctuations in the marginal product of labor. We assume that it is of exponential order less than $1/\sqrt{b}$. Positive value of d reflects the fact that there exist costs in adjusting labor force. This adjustment cost, together with the non-time–separable utility function of the household mentioned earlier, is the source of serial persistence in employment in our model. In maximizing (6), the firm considers the stochastic processes governing $\{W_{t+j}, j\geq 0\}$, $\{P_{t+j}, j\geq 0\}$ and $\{a_{t+j}, j\geq 0\}$ as beyond its control. The representative firm solves this maximization problem for each period $t=0,1,2,\cdots$. The Euler equations for the firm's problem are

$$bdE_t n^d_{t+1} - [f_1 + d(1+b)]n^d_t + dn^d_{t-1} = \frac{W_t}{P_t} - a_t - f_0, \quad t = 0,1,2,\cdots \qquad (7)$$

These Euler equations together with a side condition on the path of $\{n^d_t\}$

$$\sum_{t=0}^{\infty} b^t (\tfrac{1}{2}) f_1 (n^d_t)^2 < \infty.$$

are sufficient for an optimum.

If we use the same approximation as in the household's case, the Euler equation (7) becomes,

$$bdE_t n^d_{t+1} - [f_1 + d(1+b)]n^d_t + dn^d_{t-1} = (1 + w_t - p_t) - a_t - f_0, \quad t = 0,1,2,\cdots \qquad (8)$$

We suppose that there exists temporary wage inflexibility in this economy. That is, the nominal wages are determined one period in advance of the trading period. This assumption of predetermined nominal wage is based on the empirical observation that wages are usually set in advance of employment. We also assume that the nominal wages are set at the expected competitive market clearing level. Wage settlement is noncontingent in this economy. Perhaps transactions and information costs may explain this nonindexation but we do not pursue this problem in this paper.

As in Sargent (1987, Chapter XVIII), we exogenously specify a stochastic process for p_t. We assume that the price p_t is governed by the stationary stochastic process $p_t=V^1(L)\varepsilon_t+V^2(L)a_t+V^3(L)\eta_t$. $V^k(L)$'s k=1, 2, 3 are square–summable polynomials in the lag operator that is one–sided on the present and past. That is,

$$V^k(L) = \sum_{j=0}^{\infty} V_j^k L^j, \text{ where } \sum_{j=0}^{\infty} (V_j^k)^2 < \infty. \tag{9}$$

L is a lag operator which is defined by $L^jX_t=X_{t-j}$. $\{\eta_t\}$is a process of nominal shocks to price. One source of $\{\eta_t\}$ we can consider is the government money creation, which we do not explicitly model.[2] We specify p_t as dependent on ε_t and a_t as well as on η_t in order to capture the possibility of causality from $\{n_t\}$ or $\{w_t\}$ to $\{p_t\}$.

Because of our assumptions on nominal wage determination, we can let $w_t = E_{t-1}w_t^*$, where w_t^* is the market clearing nominal wage at period t. Since $\{w_t^*\}$ is the stochastic process for market clearing nominal wages at each period, it must satisfy Euler equations (5) and (8) when $n_t^d = n_t^s = n_t$ for all $t = 0, 1, 2,\cdots$. That is,

$$\gamma b\delta_2 E_t n_{t+1} + (\delta_1 + \delta_2 + \delta_2\gamma^2 b)n_t + \delta_2\gamma n_{t-1} = u_0(1 + w_t^* - p_t) - \delta_0 - \varepsilon_t,$$

$$bdE_t n_{t+1} - [f_1 + d(1+ b)]n_t + dn_{t-1} = 1 + w_t^* - p_t - a_t - f_0, \qquad t = 0,1,2,\cdots \tag{10}$$

If we choose expectation for each equation in (10) conditional on information set available one period earlier, we get

$$\gamma b\delta_2 E_{t-1}n_{t+1} + (\delta_1 + \delta_2 + \delta_2\gamma^2 b)E_{t-1}n_t + \delta_2\gamma n_{t-1}$$
$$= u_0(1 + E_{t-1}w_t^* - E_{t-1}p_t) - \delta_0 - E_{t-1}\varepsilon_t,$$

$$bdE_{t-1}n_{t+1} - [f_1 + d(1+ b)]E_{t-1}n_t + dn_{t-1}$$
$$= 1 + E_{t-1}w_t^* - E_{t-1}p_t - E_{t-1}a_t - f_0, \quad t = 0,1,2,\cdots \tag{11}$$

Since $w_t=E_{t-1}w_t^*$, equilibrium stochastic processes $\{w_t\}$ and $\{n_t\}$ must satisfy,

[2] It is desirable to have a model which can explain $\{p_t\}$ endogenously. But in order to do that we must explicitly introduce money into our model. This requires that we explain why agents want to keep money in our economy. Perhaps we may follow *cash–in–advance* or *money–in–the–utility–function* approach. But by following these approaches we would have to give up the linear–quadratic setup. This makes the derivation of an explicit closed form solution impossible and estimation of the model much more difficult.

$$\gamma b\delta_2 E_{t-1}n_{t+1} + (\delta_1 + \delta_2 + \delta_2\gamma^2 b)E_{t-1}n_t + \delta_2\gamma n_{t-1}$$

$$= u_0(1 + w_t - E_{t-1}p_t) - \delta_0 - E_{t-1}\varepsilon_t,$$

$$bdE_{t-1}n_{t+1} - [f_1 + d(1+b)]E_{t-1}n_t + dn_{t-1}$$

$$= 1 + w_t - E_{t-1}p_t - E_{t-1}a_t - f_0, \quad t = 0,1,2,\cdots \tag{12}$$

Since the negotiated wage is only the expected market clearing one, a labor rationing rule is necessary if agents misforcast (i.e., if the actually realized values of the forcasted variables are different from their conditional means). We shall follow convention in rational expectations macro literature on sticky prices (e.g., Fischer (1977), Gertler (1982)) by assuming that the employment is determined by demand side in this situation. Since actual employment level is determined by the demand side, equilibrium stochastic processes $\{w_t\}$ and $\{n_t\}$ must also satisfy Euler equation (8). And to facilitate our exposition, we set all constants to zero in the equations. This is related to our later assumption of zero means for driving processes $\{\varepsilon_t\}$, $\{a_t\}$ and $\{\eta_t\}$. If we estimate our model in terms of deviations from trend values, the removal of constant terms and the assumption of zero mean driving processes do not affect our result.

In sum, equilibrium in this economy is a pair of stochastic processes $\{w_t\}$ and $\{n_t\}$ which satisfy the following expectational difference equations simultaneously;

$$\gamma b\delta_2 E_{t-1}n_{t+1} + (\delta_1 + \delta_2 + \delta_2\gamma^2 b)E_{t-1}n_t + \delta_2\gamma n_{t-1} = u_0(w_t - E_{t-1}p_t) - E_{t-1}\varepsilon_t, \tag{13}$$

$$bdE_{t-1}n_{t+1} - [f_1 + d(1+b)]E_{t-1}n_t + dn_{t-1} = w_t - E_{t-1}p_t - E_{t-1}a_t, \tag{14}$$

$$bdE_t n_{t+1} - [f_1 + d(1+b)]n_t + dn_{t-1} = w_t - p_t - a_t, \tag{15}$$

where $p_t=V^1(L)e_t+V^2(L)a_t+V^3(L)\eta_t$, $t=0,1,2,\cdots$

In the next section, we derive the closed form solution for n_t and w_t.

3. Derivation of the Closed Form Solution

In solving equations (13), (14) and (15) simultaneously with respect to n_t and w_t, we follow the solution principle in Whiteman (1983).

Suppose $E\varepsilon_t = Ea_t = E\eta_t = 0$ [3] and the driving process $(\varepsilon_t\ a_t\ \eta_t)'$ has the following Wold moving average representation:

$$\begin{bmatrix} \varepsilon_t \\ a_t \\ \eta_t \end{bmatrix} = \begin{bmatrix} A^{11}(L) & A^{12}(L) & A^{13}(L) \\ A^{21}(L) & A^{22}(L) & A^{23}(L) \\ A^{31}(L) & A^{32}(L) & A^{33}(L) \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix} = A(L)e_t, \tag{16}$$

where $A(L) = \sum_{j=0}^{\infty} A_j^{gh} L^j$ and $\sum_{j=0}^{\infty} (A_j^{gh})^2 < \infty$ for g, h = 1, 2, 3;

e_{1t}, e_{2t} and e_{3t} are serially uncorrelated processes with $Ee_{1t}e_{2s}=0$, $Ee_{1t}e_{3s}=0$, $Ee_{2t}e_{3s}=0$ for all $t \neq s$;

$$Ee_te_t' = V = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix};$$

and where the one–step–ahead prediction errors are given by

$$\begin{aligned} \varepsilon_t - E[e_t \mid \varepsilon_{t-1},\ldots,a_{t-1},\ldots,\eta_{t-1},\ldots] &= A_{11}e_{1t}+A_{12}e_{2t}+A_{13}e_{3t}, \\ a_t - E[a_t \mid \varepsilon_{t-1},\ldots,a_{t-1},\ldots,\eta_{t-1},\ldots] &= A_{21}e_{1t}+A_{22}e_{2t}+A_{23}e_{3t}, \\ \eta_t - E[h_t \mid \varepsilon_{t-1},\ldots,a_{t-1},\ldots,\eta_{t-1},\ldots] &= A_{31}e_{1t}+A_{32}e_{2t}+A_{33}e_{3t}, \end{aligned}$$

i.e., e_{1t}, e_{2t}, and e_{3t} are jointly fundamental for ε_t, a_t and η_t.
We assume that individual agents (i.e., representative firm and household) know the above law of motion for $(\varepsilon_t\ a_t\ \eta_t)'$.

If we impose the restriction that the solution for n_t and w_{t+1} lie in the linear space spanned by current and lagged $(\varepsilon_t\ a_t\ \eta_t)'$s,[4] it can be shown that there is unique solution in our model.[5] The moving average representation for n_t is the following.

[3] This assumption, together with the fact that we dropped the constants from (12) and (8), implies that our model is estimated in terms of deviations from trend values.

[4] Therefore we exclude 'bubble' or 'bootstrap' effects.

[5] The proof of the existence and uniqueness of solution and the derivation of that solution is available on request.

$$n_t = \sum_{k=1}^{3} \Big\{ \frac{1}{(1-\rho_1 L)(1-\rho_2 L)} \frac{1}{b(u_0 d - \gamma\delta_2)} \{A^{1k}(L) - A^{1k}(\rho_2^{-1})$$

$$- u_0[A^{2k}(L) - A^{2k}(\rho_2^{-1})]\}L$$

$$+ \frac{1}{1-\rho_1 L} \frac{1}{\rho_1 - [f_1 + d(1+b)]/bd} \{ \frac{1}{b(u_0 d - \gamma\delta_2)} [A^{1k}(\rho_2^{-1}) - A_0^{1k}$$

$$-u_0(A^{2k}(\rho_2^{-1}) - A_0^{2k})] - \frac{V_0^1 A_0^{1k} + (V_0^2+1)A_0^{2k} + V_0^3 A_0^{3k}}{bd} \} \Big\} e_{kt}. \quad (17)$$

where $(1-\rho_1 L)(1-\rho_2 L) = 1 + \frac{\Omega}{b}L + \frac{1}{b}L^2$ and $|\rho_1| < 1 < \frac{1}{b} < |\rho_2|$ if we let

$$\Omega = - \frac{u_0[f_1 + d(1+b)] + \delta_1 + \delta_2 + \delta_2\gamma^2 b}{u_0 d - \gamma\delta_2}.$$

The solution for w_t can be derived by substituting (17) to any one of (13), (14) or (15).

4. An Estimable Model

We have derived a unique solution for our model. But the closed form solution given at the end of the previous section looks very complex and uninformative about its properties. Moreover we cannot estimate our model at this very general level. In this section, we consider the equilibrium of this economy under some special assumptions on shock processes.

First, we assume that the matrix polynomial A(L) in equation (16) is diagonal (i. e., $A^{jk}(L)=0$ for $j \neq k$). This assumption still permits ε_t, a_t and η_t to be correlated contemporaneously, since e_{1t}, e_{2t} and e_{3t} are allowed to be contemporaneously correlated. But this rules out correlation among them at any non-zero lags. Also, this does not exclude the possibility of $\{p_t\}$ being Granger caused by $\{n_t\}$ or $\{w_t\}$, since $\{p_t\}$ not only depends on the nominal shock process $\{\eta_t\}$ but also on $\{\varepsilon_t\}$ and $\{a_t\}$. Under this assumption, we can let A_0^{11}, A_0^{22}, A_0^{33} and V_0^1 all equal to 1 without loss of generality.[6]

Now we assume further that e_t, a_t and h_t follow AR(1) processes. Let

$$A^{11}(L) = \frac{1}{1-a_{11}L},\ A^{22}(L) = \frac{1}{1-a_{22}L} \text{ and } A^{33}(L) = \frac{1}{1-a_{33}L}$$

[6] We can make this by normalization.

so that $\varepsilon_t - a_{11}\varepsilon_{t-1} = e_{1t}$, $a_t - a_{22}a_{t-1} = e_{2t}$ and $\eta_t - a_{33}\eta_{t-1} = e_{3t}$, where $|a_{11}| < 1$, $|a_{22}| < 1$ and $|a_{33}| < 1$. We also assume that p_t is only a function of current levels of shocks so that $V^1(L)=V_0^1$, $V^2(L)=V_0^2$ and $V^3(L)=V_0^3=1$ in (9)[7]. That is,

$$p_t = V_0^1\varepsilon_t + V_0^2 a_t + \eta_t. \tag{18}$$

Then the closed form solution for n_t becomes

$$n_t = \rho_1 n_{t-1} + \frac{a_{11}}{b\phi_1\phi_2(\rho_2 - a_{11})}\varepsilon_t - \frac{a_{11}(\phi_2 + a_{11})}{b\phi_1\phi_2(\rho_2 - a_{11})}\varepsilon_{t-1} - \frac{\phi_1\rho_2 + \gamma\delta_2 a_{22}}{bd\phi_1\phi_2(\rho_2 - a_{22})}a_t$$
$$+ \frac{\phi_1\rho_2 a_{22} + \gamma\delta_2(a_{22})^2 + u_0 d a_{22}\phi_2}{bd\phi_1\phi_2(\rho_2 - a_{22})}a_{t-1} - \frac{V_0^1}{bd\phi_2}e_{1t} - \frac{V_0^2}{bd\phi_2}e_{2t} - \frac{1}{bd\phi_2}e_{3t}, \tag{19}$$

where $\phi_1 = u_0 d - \gamma\delta_2$ and $\phi_2 = \rho_1 - [f_1 + d(1+b)]/(bd)$.[8]

We can derive the closed form solution for w_t by substituting (19) into (13), (14) or (15). The closed form solution for w_t is represented as

$$w_t = (bd\phi_2\rho_1 + d)n_{t-1} - \frac{da_{11}(\phi_2 + a_{11})}{\phi_1(\rho_2 - a_{11})}\varepsilon_{t-1} + \frac{\phi_1\rho_2 a_{22} + \gamma\delta_2(a_{22})^2 + u_0 d a_{22}\phi_2}{\phi_1(\rho_2 - a_{22})}a_{t-1}$$
$$+ V_0^1 a_{11}\varepsilon_{t-1} + V_0^2 a_{22}a_{t-1} + a_{33}\eta_{t-1}. \tag{20}$$

Note that the innovation in the nominal shock, $\eta_t - E_{t-1}\eta_t = e_{3t}$, appears in the closed form solution for n_t. That is, an unexpected nominal shock affects the real variable n_t. There exists a so called *Monetary Business Cycle* phenomenon in this economy. The covariance between n_t and $\eta_t - E_{t-1}\eta_t = e_{3t}$ given $\varepsilon_t - E_{t-1}\varepsilon_t = e_{1t} = 0$ and $a_t - E_{t-1}a_t = e_{2t} = 0$ is

$$E[n_t(\eta_t - E_{t-1}\eta_t)] = E[n_t e_{3t}] = -\frac{E[e_{3t}^2]}{bd\phi_2} = -\frac{\sigma_3^2}{bd\phi_2}.$$

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[7] This assumption doesn't seem to be an ideal one in the light of the fact that employment depends on all the current and past levels of shocks in our model. Since we are assuming AR(1) process for the shocks, the product price also follows AR(1) process in our model. According to Ashenfelter and Card (1982), the price level follows a higher order AR. But the assumption of higher order AR will make our model much more complex and difficult to be estimated. Therefore, at this stage, we assume that the price level is only a function of current levels of shocks.

[8] We use this notation from now on.

But $\phi_2 = \rho_1$ - $[f_1+d(1+b)]/(bd) = \rho_1 - 1 - (f_1+d)/(bd)$, and we know that $|\rho_1| < 1$, so $\rho_1 - 1 < 0$. And the parameters f_1, b, d are all positive. So we get $\phi_2 < 0$. Therefore there is positive correlation between the employment level n_t and the surprise in the nominal shock e_{3t} ($= \eta_t - E_{t-1}\eta_t$). In our model, the nominal wage at period t is determined one period in advance at the expected market clearing level. After a shock occurs in period t, quantity (i.e., employment level) adjustment is made along the labor demand schedule with the predetermined nominal wage. If there is an unexpected price increase due to a nominal shock, the realized real wage becomes less than the one expected at period t-1. And employment is adjusted to a higher level by moving down the labor demand schedule.

In (32) and (33), there exist <u>exact</u> relationships among the variables n_t, w_t, p_t, ε_t, a_t, η_t, ε_{t-1}, a_{t-1}, η_{t-1} and n_{t-1}.[9] This is because dynamic economic theory implies that agents' decision rules are exact functions of the information they possess about the relevant state variables governing the dynamic process they wish to control. In order to do econometric analysis, we must resort to some device to convert the exact equations delivered by economic theory into inexact (i.e., stochastic) equations. We achieve this by following Hansen and Sargent (1980) and assuming that the shocks ε_t, a_t and η_t are not observable to the econometrician even though those are observable to the private agents in our model. The econometrician just observes n_t, w_t and p_t.

5. Estimation Methodology

5.1 Trivariate ARMA Representation

To simplify the exposition of the derivation of the ARMA representation for $(n_t\ w_{t+1}\ p_t)'$ process[10], we rewrite (18), (19), and (20) as

$$A_0 y_t + A_1 y_{t-1} = B_0 \Lambda_t + B_1 \Lambda_{t-1} + C_0 e_t, \tag{21}$$

where $y_t=(n_t\ w_{t+1}\ p_t)'$, $\Lambda_t=(\varepsilon_t\ a_t\ \eta_t)'$, and $e_t=(e_{1t}\ e_{2t}\ e_{3t})'$. Elements of the 3×3 matrices A_0, A_1, B_0, B_1 and C_0 are functions of the structural parameters of our model.

Because of our assumption on the shock process $(\varepsilon_t\ a_t\ \eta_t)'$, we can also let

[9] Note that $e_{1t}=\varepsilon_t-a_{11}\varepsilon_{t-1}$, $e_{2t}=a_t-a_{22}a_{t-1}$ and $e_{2t}=\eta_t-a_{33}\eta_{t-1}$.

[10] Remember the fact that w_t is determined at period t-1, not at period t. Therefore if we use an ARMA representation for $(n_t\ w_t\ p_t)'$, the time domain approximation to maximum–likelihood won't work because we will always get a zero root in the moving average part. This is the reason why we derive an ARMA representation for $(n_t\ w_{t+1}\ p_t)'$ instead of $(n_t\ w_t\ p_t)'$.

$$\Lambda_t = \Upsilon\Lambda_{t-1} + e_t, \qquad (22)$$

$$\text{where } \Upsilon = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

By substituting (22) into (21), we get

$$A_0y_t + A_1y_{t-1} = (B_0\Upsilon + B_1)\Lambda_{t-1} + (B_0 + C_0)e_t. \qquad (23)$$

Now by multiplying both sides of (23) by $(I-\Upsilon L)(B_0\Upsilon+B_1)^{-1}$, substituting (22), multiplying the result by $(B_0\Upsilon + B_1)$ and rearranging, we get the following ARMA(2,1) representation for the y_t process:

$$A_0y_t + [A_1 - (B_0\Upsilon + B_1)\Upsilon(B_0\Upsilon + B_1)^{-1}A_0]y_{t-1} - (B_0\Upsilon + B_1)\Upsilon(B_0\Upsilon + B_1)^{-1}A_1y_{t-2}$$
$$= (B_0 + C_0)e_t + (B_0\Upsilon + B_1)[I - \Upsilon(B_0\Upsilon + B_1)^{-1}(B_0 + C_0)]e_{t-1}. \qquad (24)$$

To facilitate exposition, we rewrite (24) as

$$S(L)y_t = R(L)e_t, \qquad (25)$$

where $S(L) = \sum_{j=0}^{2} S_jL^j$, $R(L) = \sum_{j=0}^{1} R_jL^j$,

$$\text{and } Ee_te_t' = V = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix}.$$

5.2 Bivariate ARMA Representation

In section 6, we will report the estimation result of both our predetermined wage model and Sargent's ((1979, Chapter XVI)) competitive equilibrium labor market model and compare their implications. Since the variables in Sargent's model are employment and real wages, we derive the closed form ARMA representation of those two variables implied by our model.

Note the fact that if W_t/P_t=1 on average, $W_t/P_t=1+\log(W_t/P_t)=1+w_t-p_t$. We used this fact in deriving the closed form solution for n_t and w_t. We linearly transform price data so that $W_t/P_t = 1$ holds on average. Also, since we estimate our model in terms of deviations from trend values, we can ignore the constant 1. Therefore the closed form for real wages implied by our model can be derived by subtracting equation (18) from equation (20). That is, the closed form real wage W_t/P_t in our model is the following.

$$W_t/P_t = (bd\phi_2\rho_1+d)n_{t-1} - \frac{da_{11}(\phi_2+a_{11})}{\phi_1(\rho_2-a_{11})}\varepsilon_{t-1}$$

$$+\frac{\phi_1\rho_2a_{22}+\gamma\delta_2(a_{22})^2+u_0da_{22}\phi_2}{\phi_1(\rho_2-a_{22})}a_{t-1} - V_0^1e_{1t} - V_0^2e_{2t} - e_{3t}. \quad (26)$$

We rewrite (19) and (26) as

$$\bar{y}_t + \bar{A}_1\bar{y}_{t-1} = \bar{B}_0\bar{\Lambda}_t + \bar{B}_1\bar{\Lambda}_{t-1} + \bar{C}_0e_t, \quad (27)$$

where $\bar{y}_t = (n_t\ W_t/P_t)'$, $\bar{\Lambda}_t = (\varepsilon_t\ a_t)'$, and $e_t = (e_{1t}\ e_{2t}\ e_{3t})'$. Elements of the matrices $\bar{A}_1$, $\bar{B}_0$, $\bar{B}_1$ and $\bar{C}_0$ are functions of the structural parameters of our model. Because of our assumption on shock process $(\varepsilon_t\ a_t)'$, we can also let

$$\bar{\Lambda}_t = \bar{\Upsilon}\bar{\Lambda}_{t-1} + I^*e_t, \quad (28)$$

where $\bar{\Upsilon} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$ and $I^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

By substituting (28) into (27), we get

$$\bar{y}_t + \bar{A}_1\bar{y}_{t-1} = (\bar{B}_0\bar{\Upsilon} + \bar{B}_1)\bar{\Lambda}_{t-1} + (\bar{B}_0^* + \bar{C}_0)e_t \quad (29)$$

where

$$\bar{B}_0^* = \bar{B}_1I^* = \begin{bmatrix} \frac{a_{11}}{b\phi_1\phi_2(\rho_2-a_{11})} & -\frac{\phi_1\rho_2+\gamma\delta_2a_{22}}{bd\phi_1\phi_2(\rho_2-a_{22})} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Note that $\bar{C}_1$ and $\bar{B}_1^*$ are of dimension 2×3 and therefore are non–square. Now by multiplying both sides of (29) by $(I - \bar{\Upsilon}L)(\bar{B}_0\bar{\Upsilon} + \bar{B}_1)^{-1}$, substituting (28), multiplying the result by $(\bar{B}_0\bar{\Upsilon} + \bar{B}_1)$ and rearranging, we get the following representation for the $\bar{y}_t$ process:

$$\bar{y}_t + [\bar{A}_1 - (\bar{B}_0\bar{\Upsilon} + \bar{B}_1)\bar{\Upsilon}(\bar{B}_0\bar{\Upsilon} + \bar{B}_1)^{-1}]\bar{y}_{t-1} - (\bar{B}_0\bar{\Upsilon} + \bar{B}_1)\bar{\Upsilon}(\bar{B}_0\bar{\Upsilon} + \bar{B}_1)^{-1}\bar{A}_1\bar{y}_{t-2}$$
$$= (\bar{B}_0^* + \bar{C}_0)e_t + (\bar{B}_0\bar{\Upsilon} + \bar{B}_1)[I^* - \bar{\Upsilon}(\bar{B}_0\bar{\Upsilon} + \bar{B}_1)^{-1}(\bar{B}_0^* + \bar{C}_0)]e_{t-1}. \tag{30}$$

To facilitate exposition, we rewrite (30) as

$$\bar{S}(L)\bar{y}_t = Q(L)e_t, \tag{31}$$

where $\bar{S}(L) = \sum_{j=0}^{2} \bar{S}_j L^j$, $Q(L) = \sum_{j=0}^{1} Q_j L^j$. Multiplying both sides of (31) by $\bar{S}(L)^{-1}$, we get

$$\bar{y}_t = \bar{S}(L)^{-1}Q(L)e_t.$$

Note that matrix polynomial $\bar{S}(L)^{-1}Q(L)$ is nonsquare (of dimension 2×3) so that the process $\{e_t\}$ is not fundamental for the $\{\bar{y}_t\}$ process. Our next step is to replace $\bar{S}(L)^{-1}Q(L)e_t$ by its Wold representation $\bar{S}(L)^{-1}\bar{R}(L)\bar{e}_t$ where

$$\underset{2\times2}{\bar{S}(L)^{-1}}\underset{2\times3}{Q(L)}\underset{3\times1}{e_t} = \underset{2\times2}{\bar{S}(L)^{-1}}\underset{2\times2}{\bar{R}(L)}\underset{2\times1}{\bar{e}_t}$$

where $\bar{R}(L) = I + \bar{R}_1 L$, $\det \bar{R}(z) = 0 \Rightarrow |z| > 1$, $\bar{e}_t = \bar{y}_t - E[\bar{y}_t | \bar{y}_{t-1}, \cdots, \bar{y}_0]$, and $E\bar{e}_t\bar{e}_t' = \Xi$. To compute $\bar{R}(L)$ we solve the spectral factorization equation

$$\bar{S}(L)^{-1}Q(L)VQ(L^{-1})'\bar{S}(L^{-1})^{-1\prime} = \bar{S}(L)^{-1}\bar{R}(L)\Xi\bar{R}(L^{-1})'\bar{S}(L^{-1})^{-1\prime}$$

subject to the zeros of $\det \bar{R}(z)$ not being inside the unit circle. The white noise vector $\bar{e}_t$ is fundamental for $\bar{y}_t$ and, therefore, is in the space spanned by current and lagged e_t's. We can then rewrite (39)' as

$$\bar{S}(L)\bar{y}_t = \bar{R}(L)\bar{e}_t.$$

This is the ARMA(2,1) representation for the $\bar{y}_t$ process.

5.3 Likelihood Function

We have 18 structural parameters: b, u_0, δ_1, δ_2, γ, f_1, d, a_{11}, a_{22}, a_{33}, V_0^1, V_0^2, σ_1^2, σ_2^2, σ_3^2, σ_{12}, σ_{13} and σ_{21}. But we can normalize u_0 to be 1. Also, we set the discount rate b to 0.991 *a priori*.[11] Therefore we have 16 parameters to estimate. Let

$$\Phi = (\delta_1\ \delta_2\ \gamma\ f_1\ d\ a_{11}\ a_{22}\ a_{33}\ V_0^1\ V_0^2\ \sigma_1^{\ 2}\ \sigma_2^{\ 2}\ \sigma_3^{\ 2}\ \sigma_{12}\ \sigma_{13}\ \sigma_{21})'$$

to be the vector of parameters to be estimated.

By multiplying both sides of (25) by $S(L)^{-1}$ we can derive the following Wold Moving Average representation for $y_t=(n_t\ w_{t+1}\ p_t)'$.

$$y_t = S(L)^{-1}R(L)e_t.$$

Also we know that[12]

$$Ee_te_t' = V = \begin{bmatrix} \sigma_1^{\ 2} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^{\ 2} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^{\ 2} \end{bmatrix}.$$

Then the covariance generating function for $y_t=(n_t\ w_{t+1}\ p_t)'$ process is

$$g_y(z) = S(z)^{-1}R(z)VR(z^{-1})'S(z^{-1})^{-1\prime} \tag{32}$$

Now suppose that we have a sample on $y_t=(n_t\ w_{t+1}\ p_t)'$ for $t=1,\cdots,T$. Let $\mathbf{y}_T = (y_1\ y_2\ \cdots\ y_T)'$. Then $L_T = E\mathbf{y}_T\mathbf{y}_T'$ is the covariance matrix of $\mathbf{y}_T$. The log–liklihood function for $\mathbf{y}_T$ is given by

$$£_T(\Phi) = -\frac{3T}{2}\log 2\pi - \frac{1}{2}\log(\det L_T) - \frac{1}{2}\mathbf{y}_T' L_T^{-1}\mathbf{y}_T$$

The elements of L_T can be computed as functions of the vector Φ of underlying parameters using the covariance generating function $g_y(z)$ defined in (32). However, directly maximizing the log–likelihood function $£_T(\Phi)$ with respect to Φ is computationally difficult, because L_T is a complicated function of Φ, and since inversion of the large matrix L_T is required for each evaluation of $£_T(\Phi)$. In estimating our model, we use time domain approximation to $£_T(\Phi)$.

From (25),

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[11] In our actual estimation, several different values of b were tried. Basic qualitative results were not affected by the changes in b.

[12] See p.9.

$$S_0y_t+S_1y_{t-1}+S_2y_{t-2} = R_0e_t+R_1e_{t-1}.$$

Multiplying both sides by S_0^{-1},

$$y_t+S_0^{-1}S_1y_{t-1}+S_0^{-1}S_2y_{t-2} = S_0^{-1}R_0e_t+S_0^{-1}R_1e_{t-1}.$$

Let us rewrite it as

$$y_t+\tilde{S}_1y_{t-1}+\tilde{S}_2y_{t-2} = \tilde{R}_0e_t+\tilde{R}_1e_{t-1}.$$

Let $\tilde{R}_0e_t=u_t$. Then $e_t=\tilde{R}_0^{-1}u_t$. u_t is also a vector white noise and the covariance matrix of u_t is $Eu_tu_t' = V_u = \tilde{R}_0Ee_te_t'\tilde{R}_0' = \tilde{R}_0V\tilde{R}_0'$. The above equation becomes

$$y_t+\tilde{S}_1y_{t-1}+\tilde{S}_2y_{t-2} = u_t+\tilde{R}_1\tilde{R}_0^{-1}u_{t-1}.$$

Let $\tilde{R}_1\tilde{R}_0^{-1}=\tilde{R}_1^{*}$. Then we have

$$y_t+\tilde{S}_1y_{t-1}+\tilde{S}_2y_{t-2} = u_t+\tilde{R}_1^{*}u_{t-1}. \qquad (33)$$

We use this equation to calculate the u_t vector implied by a given set of parameter values F (Let us denote it $\hat{u}_t$). Now we assume that $\hat{u}_2=\hat{u}_1=0$. The impact of these initial û's becomes negligible as $T \to \infty$ if none of the roots of the determinant of $\tilde{R}^*(z)=I+\tilde{R}_1^{*}z$ lie on the open unit disk $|z| < 1$. Beginning with this initial û's, we can solve for $\hat{u}_t$ by using recursions on (33):

$$\hat{u}_t = y_t+\tilde{S}_1y_{t-1}+\tilde{S}_2y_{t-2}-\tilde{R}_1^{*}\hat{u}_{t-1}.$$

Assuming that the e_t process is multivariate normal, the log likelihood function can be approximated by

$$£_T^{**}(\Phi) = -\frac{3T}{2}\log 2\pi - \frac{T}{2}\log(\det V_u) - \frac{1}{2}\sum_{t=1}^{T} \hat{u}_t'V_u^{-1}\hat{u}_t.$$

As shown by Wilson (1973) and Bard (1974), maximum–likelihood estimates of

$$\tilde{\Phi} = (\delta_1\ \ \delta_2\ \ \gamma\ \ f_1\ \ d\ \ a_{11}\ \ a_{22}\ \ a_{33}\ \ V_0^1\ \ V_0^2)'$$

can be obtained by minimizing $\det\hat{V}_u$ with respect to $\tilde{\Phi}$, where $\hat{V}_u$ is the sample covariance matrix of u_t,

$$\hat{V}_u = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_t \hat{u}_t'.$$

Note that the parameters σ_1^2, σ_2^2, σ_3^2, σ_{12}, σ_{13}, and σ_{23}, which are elements of the covariance matrix V of e_t, are not included in $\underset{\sim}{\Phi}$. Maximum-likelihood estimates of σ_1^2, σ_2^2, σ_3^2, σ_{12}, σ_{13}, and σ_{23} can be obtained from $\tilde{R}_0^{-1}\hat{V}_u\tilde{R}_0^{-1\prime}$.[13]

6. Empirical Results

In this section, we report the estimation result of both our predetermined wage model and that of Sargent's competitive equilibrium model. We use our estimation result of Sargent's model as one of the benchmarks for evaluating the performance of our contract model.

Two models are basically the same except in the following respect. In our model, nominal wages are predetermined so that a nominal shock to price can affect real variables. Whereas in Sargent's model, all prices are perfectly flexible and therefore nominal variables cannot affect real variables. Sargent's is a real business cycle model which explains the dynamics between the real variables: employment and real wages.

The set of structural parameters in Sargent's model is a subset of ours. Under our assumption that preference shocks and technology shocks follow AR(1) processes with autoregressive coefficients a_{11} and a_{22} respectively, the structural parameters in Sargent's model are δ_1, δ_2, γ, f_1, d, a_{11}, a_{22}, σ_1^2, σ_2^2 and σ_{12}. (See Sargent 1987: pp.472-478.) The closed form solution for employment and real wage processes in Sargent's model can be represented as VAR(2).[14]

Estimations were obtained using quarterly data for the period 1954:1 through 1980:4. Before estimating structural models, the data on all variables were detrended by regressing them on a constant, three seasonal dummies, linear trend and trend squared, and then using the residuals from those regressions as the data for estimating models.[15]

Unfortunately, the structural parameters in Sargent's model are not identified. Several sets of structural parameter values were found that maximize the likelihood function value.[16] But all these different sets of parameter values implied the same vector autoregressive representation. Therefore, we can check the performance of Sargent's

[13] Given the extremely complicated form of the cross-equation restrictions in (25), analytically verifying the sufficient conditions for identification is not possible. However, it can be shown that the necessary condition for identification is satisfied.

[14] It can be derived by following the similar procedure used in section 5.1.

[15] Appendix 1 shows details of data used.

[16] In Kennan (1988), Sargent's model is converted into a partial adjustment model and estimated. There are three possible estimates for each parameter.

model by looking at the implied VAR(2) representation.

The vector autoregression implied by the structural parameter values that maximize the likelihood function is reported in Table 1. Notice that there are two zero restrictions.[17] Coefficients on twice lagged real wages for both employment and real wages are zero.

In order to test the restrictions imposed by Sargent's competitive equilibrium model, we estimated the unrestricted VAR(2) for employment and real wages. The result appears in Table 2. When we compare the results in Table 1 and Table 2, we see that the R^2's (particularly, the one for real wages) are close. But, there are some important differences in autoregressive coefficients. First, in unrestricted case, the coefficient for W_{t-2}/P_{t-2} is significantly different from zero in the n_t equation. Whereas in the restricted case, it is equal to zero *a priori*. To remedy the constrained model, one may want to specify higher order lags for the shocks or higher order adjustment costs. Second, in Table 2, the coefficient for W_{t-1}/P_{t-1} in the n_t equation is big (0.84902) and significant, whereas in Table 1, it is very small (0.93E–6). Above facts indicate that the response of employment to a shock in real wages implied by the estimates of Sargent's model would be very small compared to the one shown by an unrestricted VAR. This will become clearer when we look at the vector moving average representation.

Since there are 7 free parameters in the restricted model whereas there are 8 free parameters in the unrestricted one, $-2(\pounds_r-\pounds_u)$ is asymtotically distributed as $\chi^2(1)$ under the null hypothesis that the restricted model is consistent with the data. Since the value is –2(692.6933-699.2825)=13.1784, the restricted model is rejected at the .01 significance level.

In order to check whether the Sargent model is rejected basically because of the zero restrictions, we test the model of VAR(2) with only zero restrictions. Our estimation result of the model with zero restrictions is in Table 3. Since there are 6 free parameters in the zero-restricted model whereas there are 8 free parameters in the unrestricted one, $-2(\pounds_{zr}-\pounds_u)$ is asymtotically distributed as $\chi^2(2)$ under the null hypothesis that the zero restriction is consistent with the data. Since the value is –2(694.9060-699.2825)=8.7530, the zero-restriction is rejected at .025 significance level but cannot be rejected at .01 level. This seems to imply that the cross equation restrictions as well as zero restrictions contribute to the rejection of the Sargent model.

We can also see whether Sargent's model can generate reasonable dynamics between real wages and employment by looking at the moving average representation implied by the estimates of the model. It appears in Figure 1. Bivariate moving average representations implied by our estimates of unrestricted VAR(2) model and VAR(2)

[17] The closed form solution for the Sargent model can be derived by following a similar procedure to that used in deriving that of our model. According to the closed form solution derived, there are two zero restrictions.

model with zero restrictions appear in Figure 2 and Figure 3 respectively. As expected, the response of employment to a one standard deviation innovation in real wages implied by the estimates of Sargent's model is very small. The one implied by the VAR(2) with zero restrictions is not that small but the characteristic of it is quite different from the one implied by the unrestricted VAR(2).

Table 4 contains the maximum likelihood estimates of the structural parameters of our contract model[18]. All of the parameters are significant[19] except V_0^1 and V_0^2, which represent the dependence of prices on preference shocks and technology shocks. Parameters for the labor supply side (δ_1 and δ_2) are meager compared to the parameters for the labor demand side (f_1 and d). $\gamma = -0.7154$ indicates that leisure time in adjacent periods are complements according to our estimation. That means that the non-time-separable utility function for household can contribute in generating the serial persistence of employment. The pattern of estimates for f_1 and d is quite different from those reported in Sargent (1978). The coefficient for the quadratic term is bigger than the coefficient for the adjustment cost. All three shocks are highly positively serially correlated. r_1 represents endogenous dynamics generated by the adjustment cost and non-time-separable utility function. Relatively small value of ρ_1 (=0.1687) implies that those factors don't contribute much in generating the serial persistence of employment. That is, the persistence in our model comes from the positive serial correlation of exogenous shocks rather than from the endogenous dynamics. Small ρ_1 comes from the fact that the coefficients for quadratic terms (δ_1 and f_1) are bigger than the coefficients for inter-temporal adjustments (δ_2 and d). Examination of the variances shows that preference shocks don't fluctuate much compared to technology shocks and nominal shocks.

Table 5 reports the implied vector ARMA representation of our system. Note that there are six zero restrictions on the AR2 part. MA1 parameters are all very small. These facts make the employment process essentially an AR(2). Therefore the impulse response of employment to its own innovation would be hump-shaped. This agrees with the empirical regularities found by others (including Ashenfelter and Card (1982)). According to their result, employment process can be adequately described as AR(2) or ARMA(2,1). Coefficient for w_t in the w_{t+1} equation is slightly above one. It might be

[18] In our estimation, we set the discount factor b = .991 a priori. Several different values for b were tried. But, the qualitative results were not affected.

[19] The likelihood function was maximized by using a derivative-free Davidon-Fletcher-Powell algorithm. Numerical difficulty in inverting high dimensional information matrix in our case generated a few negative diagonal elements. So I could not calculate asymtotic standard errors from the inverse of the information matrix. Therefore I calculated the asymtotic standard errors by assuming that the likelihood-ratio test statistic is equal to the Wald test statistic. In general, those two statistics are asymtotically similar. (See p.784 of Engel (1984).) Appendix 2 provides details of the method used.

worthwhile to modify our model and estimate using first differenced data.[20]

In order to test our model, we estimated the vector ARMA(2,1) process for employment, nominal wages and price with only zero restrictions. The result is in Table 6. Since the likelihood ratio is –2(1142.6266-1190.8255)=96.3978, our model is rejected decisively.

We also check the performance of our model by comparing the vector moving average representation for employment and real wages implied by our model and Sargent's competitive equilibrium model. In order to compare ours with that of Sargent's competitive equilibrium model, bivariate ARMA(2,1) representation for employment and real wages implied by our model was derived using the method descrided in Section 5.2. It is reported in Table 7. Basically, our model has the same problem that appears in the Sargent model in explaining the effect of real wages on employment. That is, the coefficient for W_{t-1}/P_{t-1} in the n_t equation is very small (–0.23E–5). This, together with the zero restriction, implies that the impulse response of employment to real wages is very small according to the estimates of our model. Another problem with our model is that the coefficient for n_{t-1} in the W_t/P_t equation is large compared to the one in the unrestricted model. MA1 coefficients for real wages are large. This is in contradiction to empirical regularities found by others (including Ashenfelter and Card (1982)). That is, the real wage process can be adequately described as AR(1).

The moving average representation derived from that bivariate ARMA process appears in Figure 4. Comparison of Figure 4, Figure 1 and Figure 2 indicates that both our contract model and Sargent's competitive equilibrium model capture reasonably well the response of employment and real wages to their own innovations. One thing that should be pointed out is that both of the constrained models substantially underestimate the response of employment to innovations in real wages. This is more severe in the competitive equilibrium model.[21] This fact was also pointed out in Sargent (1978).

In sum, our contract model couldn't make a noticeable improvement in explaining the dynamics between employment and real wages.

Until now, we have checked whether our model can generate reasonable dynamics for employment and real wage processes. Our next step is to see whether estimates of our model show enough of a statistical Phillips curve phenomenon (i.e., positive correlation between the inflation rate and employment). Table 8 shows that the slope of

---

[20] Modification of our model is straightforward. We can achieve this by assuming that the first difference of the shocks (driving processes) are zero mean, linearly regular, covariance stationary stochastic processes with known Wold representations.

[21] While estimating Sargent's competitive equilibrium model, I found a couple of other convergence points where the response of employment to real wages was big enough. But, at all those points, $|\gamma| > 1$ which contradicts our assumption.

the Phillips curve implied by our estimates of parameters[22] is actually bigger than the one shown by data[23]. Remember the fact that we only assumed that the nominal wages are determined one period ahead. We didn't assume any long–term contract. It is encouraging that we could have enough positive correlation between inflation rate and employment with this weak assumption.

7. Concluding Remarks

In this paper, we have examined whether an expected–market clearing, sticky wage contract model which satisfies the natural rate hypothesis can explain serial persistence in employment as well as the Phillips curve phenomenon. We assumed one period contracts with nominal wages determined one period in advance. As a mechanism for generating serial persistence of employment endogenously, we incorporated the features of adjustment cost and non–time–separable utility function into our model. We also assumed that the preference, technology, and nominal shocks all follow AR(1) processes. We solved and estimated the model and checked whether the estimates of structural parameters imply enough serial persistence in employment and the positive correlation between inflation rate and emploment level.

Our model seemed successful in generating serial persistence in employment. The impulse response of employment to its own innovation increased for the first few quarters before it started to decrease. One thing to be noted is the fact that the endogenous dynamics in our model didn't contribute much in generating this serial persistence. Most of this persistence seemed to come from the exogenously specified AR(1) shocks in our model. But our result is encouraging in the sense that the slope of the statistical Phillips curve implied by our estimates was big even though we made a very weak assumption of nominal wages determined only one period in advance.

There are several things that could be done for our model. First, we exogenously specified a form of stochastic process for price level because of the technical difficulty in estimation. We have to check the robustness of our result with respect to the specification of the price process. Further, it would be desirable if we could have a model which explains p_t endogenously. Second, we saw that the zero restrictions in our model and the Sargent's model are damaging. It is likely that by specifying higher orders of lags for stochastic disturbances and adjustment costs the performance of our model can be improved. Third, it might be worthwhile to estimate with first differenced data.

[22] $E[(p_t-p_{t-1})n_t]/E[(p_t-p_{t-1})^2]$

[23] $\Sigma[(p_t-p_{t-1})n_t]/\Sigma[(p_t-p_{t-1})^2]$

Tables

Table 1: Vector Autoregression Implied by the Values of the Structural Parameters that Maximize the Likelihood Function for the Competitive Equilibrium Model

$$\begin{bmatrix} n_t \\ W_t/P_t \end{bmatrix} + \begin{bmatrix} -1.21227 & -0.93E\text{-}6 \\ 0.01066 & -0.98262 \end{bmatrix} \begin{bmatrix} n_{t-1} \\ W_{t-1}/P_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0.35292 & 0.0 \\ 0.03102 & 0.0 \end{bmatrix} \begin{bmatrix} n_{t-2} \\ W_{t-2}/P_{t-2} \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$£_r = 692.6933$

R^2 for Employment : 0.8352942

R^2 for Real Wage : 0.9028930

Note -- $£_r$ is the value of likelihood function.

Table 2: Solution of Likelihood Equations for Unrestricted VAR(2)

$$\begin{bmatrix} n_t \\ W_t/P_t \end{bmatrix} + \begin{bmatrix} \underset{(0.08930)}{-1.17937} & \underset{(0.25924)}{-0.84902} \\ \underset{(0.03384)}{0.00821} & \underset{(0.09823)}{-1.07379} \end{bmatrix} \begin{bmatrix} n_{t-1} \\ W_{t-1}/P_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} \underset{(0.08863)}{0.29574} & \underset{(0.27011)}{0.73390} \\ \underset{(0.03358)}{0.02902} & \underset{(0.10235)}{0.10184} \end{bmatrix} \begin{bmatrix} n_{t-2} \\ W_{t-2}/P_{t-2} \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$\pounds_u = 699.2825$

R^2 for Employment : 0.8525764

R^2 for Real Wage : 0.9038272

Note -- $\pounds_u$ is the value of likelihood function.
Figures in parentheses are standard errors of the corresponding estimates.

Table 3: Solution of Likelihood Equations for VAR(2) with Zero Restrictions

$$
\begin{bmatrix} n_t \\ W_t/P_t \end{bmatrix} + \begin{bmatrix} \underset{(0.09052)}{-1.16343} & \underset{(0.08789)}{-0.18505} \\ \underset{(0.03328)}{0.01043} & \underset{(0.03232)}{-0.98165} \end{bmatrix} \begin{bmatrix} n_{t-1} \\ W_{t-1}/P_{t-1} \end{bmatrix}
$$

$$
+ \begin{bmatrix} \underset{(0.08983)}{0.31163} & 0.0 \\ \underset{(0.03303)}{0.03122} & 0.0 \end{bmatrix} \begin{bmatrix} n_{t-2} \\ W_{t-2}/P_{t-2} \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}
$$

$£_{zr} = 694.9060$

R^2 for Employment : 0.8419065

R^2 for Real Wage : 0.9028938

Note -- $£_{zr}$ is the value of likelihood function.

Figures in parentheses are standard errors of the corresponding estimates.

Table 4: Solution of Likelihood Function for Predetermined Wage Model

$\delta_1 = 4.8885$ (0.9409)	$a_{11} = 0.9882$ (0.0028)	$\rho_1 = 0.1687$
$\delta_2 = 2.1626$ (0.5908)	$a_{22} = 0.9842$ (0.0059)	$\rho_2 = 5.9808$
$\gamma = -0.7154$ (0.0855)	$a_{33} = 0.9839$ (0.0091)	$\phi_1 = 354.0015$
$f_1 = 1447.4501$ (317.4063)	$\sigma_1^2 = 0.0113$	$\phi_2 = -5.9844$
$d = 352.4543$ (63.1187)	$\sigma_2^2 = 904.4849$	$b = 0.991$
$V_0^1 = 4.6237$ (3.5215)	$\sigma_3^2 = 89.2164$	
$V_0^2 = 0.3303$ (0.3051)	$\sigma_{12} = -3.1858$	
	$\sigma_{13} = 1.0004$	
	$\sigma_{23} = -284.0671$	

$£_T^{**}(\tilde{\Phi}) = 1142.6266$

Note -- Numbers inside the parentheses are the standard errors of the corresponding estimates.

Table 5: Vector ARMA Representation
Implied by Estimates of Table 7

$$\begin{bmatrix} n_t \\ w_{t+1} \\ p_t \end{bmatrix} + \begin{bmatrix} -1.15294 & 0.23E\text{-}5 & -0.22E\text{-}5 \\ -0.00204 & -1.00779 & 0.02353 \\ -0.06445 & -0.01993 & -0.96426 \end{bmatrix} \begin{bmatrix} n_{t-1} \\ w_t \\ p_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0.16606 & 0.0 & 0.0 \\ -0.03026 & 0.0 & 0.0 \\ 0.01015 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} n_{t-2} \\ w_{t-1} \\ p_{t-2} \end{bmatrix}$$

$$= \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} + \begin{bmatrix} -0.84E\text{-}4 & -0.47E\text{-}3 & -0.11E\text{-}4 \\ 0.15E\text{-}4 & 0.85E\text{-}4 & 0.21E\text{-}5 \\ -0.52E\text{-}5 & -0.29E\text{-}4 & -0.70E\text{-}6 \end{bmatrix} \begin{bmatrix} u_{1t-1} \\ u_{2t-1} \\ u_{3t-1} \end{bmatrix}$$

Table 6: Solution of Likelihood Equations for Vector ARMA(2,1) with Zero Restrictions

$$\begin{bmatrix} n_t \\ w_{t+1} \\ p_t \end{bmatrix} + \begin{bmatrix} \underset{(0.075)}{-1.66685} & \underset{(0.054)}{0.17384} & \underset{(0.031)}{-0.02084} \\ \underset{(0.048)}{0.10630} & \underset{(0.041)}{-0.99884} & \underset{(0.025)}{0.05439} \\ \underset{(0.058)}{0.18184} & \underset{(0.058)}{-0.13014} & \underset{(0.036)}{-0.91593} \end{bmatrix} \begin{bmatrix} n_{t-1} \\ w_t \\ p_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} \underset{(0.066)}{0.76469} & 0.0 & 0.0 \\ \underset{(0.043)}{-0.12525} & 0.0 & 0.0 \\ \underset{(0.053)}{-0.24083} & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} n_{t-2} \\ w_{t-1} \\ p_{t-2} \end{bmatrix}$$

$$= \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} + \begin{bmatrix} \underset{(0.084)}{-0.73748} & \underset{(0.247)}{0.59532} & \underset{(0.234)}{-0.76492} \\ \underset{(0.053)}{0.10557} & \underset{(0.107)}{-0.13008} & \underset{(0.111)}{0.31746} \\ \underset{(0.061)}{0.24423} & \underset{(0.098)}{-0.03279} & \underset{(0.100)}{0.25412} \end{bmatrix} \begin{bmatrix} u_{1t-1} \\ u_{2t-1} \\ u_{3t-1} \end{bmatrix}$$

$\pounds_{zr} = 1190.8255$

Note -- $\pounds_{zr}$ is the value of likelihood function.

Table 7: Bivariate Vector ARMA Representation Implied by the Values of the Structural Parameters that Maximize the Likelihood Function for the Predetermined Wage Model

$$\begin{bmatrix} n_t \\ W_t/P_t \end{bmatrix} + \begin{bmatrix} -1.15294 & 0.23E\text{-}5 \\ 0.24156 & -0.98818 \end{bmatrix} \begin{bmatrix} n_{t-1} \\ W_{t-1}/P_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0.16606 & 0.0 \\ -0.21788 & 0.0 \end{bmatrix} \begin{bmatrix} n_{t-2} \\ W_{t-2}/P_{t-2} \end{bmatrix}$$

$$= \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} + \begin{bmatrix} -0.87E\text{-}4 & 0.65E\text{-}4 \\ 0.18170 & -0.13684 \end{bmatrix} \begin{bmatrix} u_{1t-1} \\ u_{2t-1} \end{bmatrix}$$

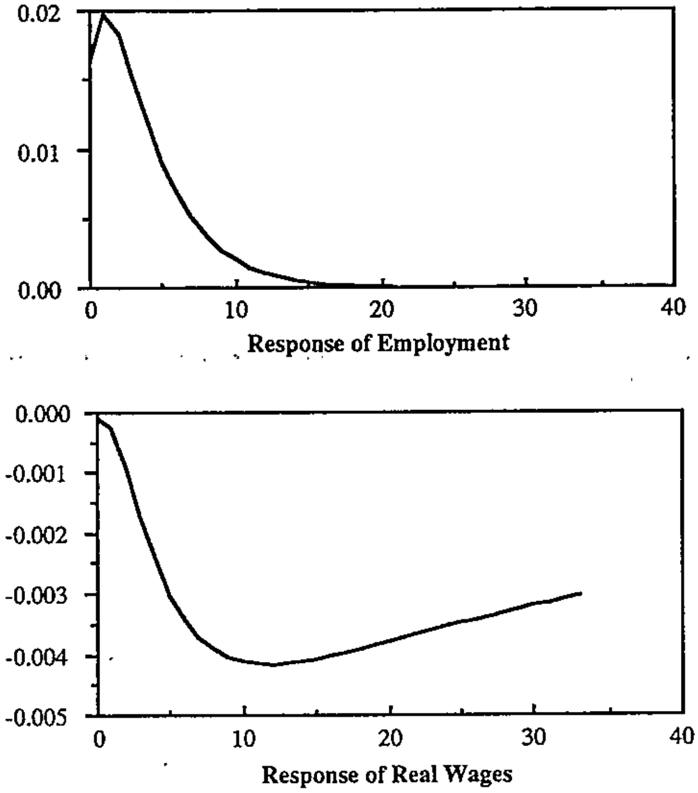
Table 8: Sample and Estimated Slope of Statistical Phillips Curve

Estimated	4.8034
Sample	2.8255 (0.6501)

Note -- Number inside the parentheses is a standard error.

Figure 1: Dynamic Responses of Employment and Real Wages Implied by Maximum-Likelihood Estimates of the Sargent's Model

Responses to Employment Innovation



Responses to Real Wage Innovation

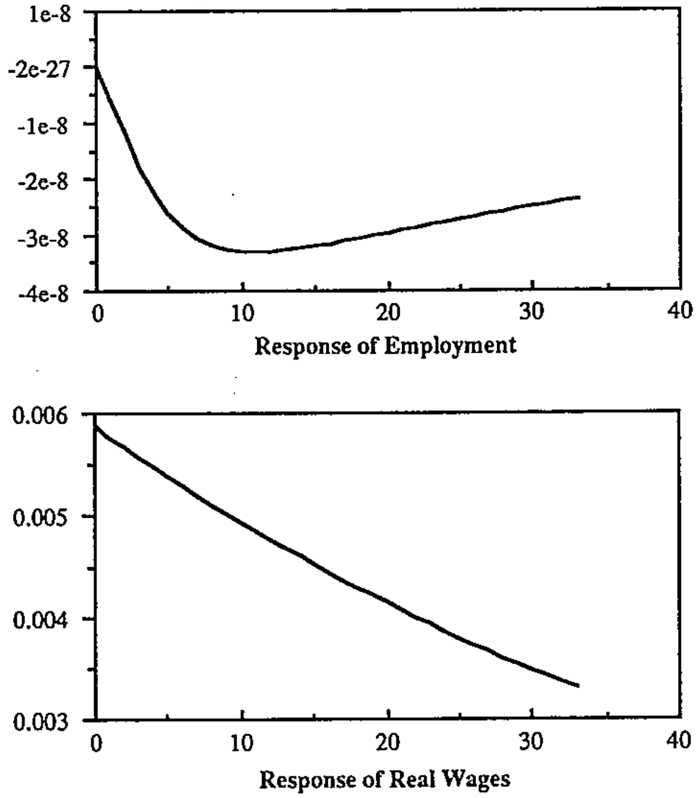


Figure 2: Dynamic Responses of Output and Real Wages Implied by Maximum-Likelihood Estimates of VAR (2)

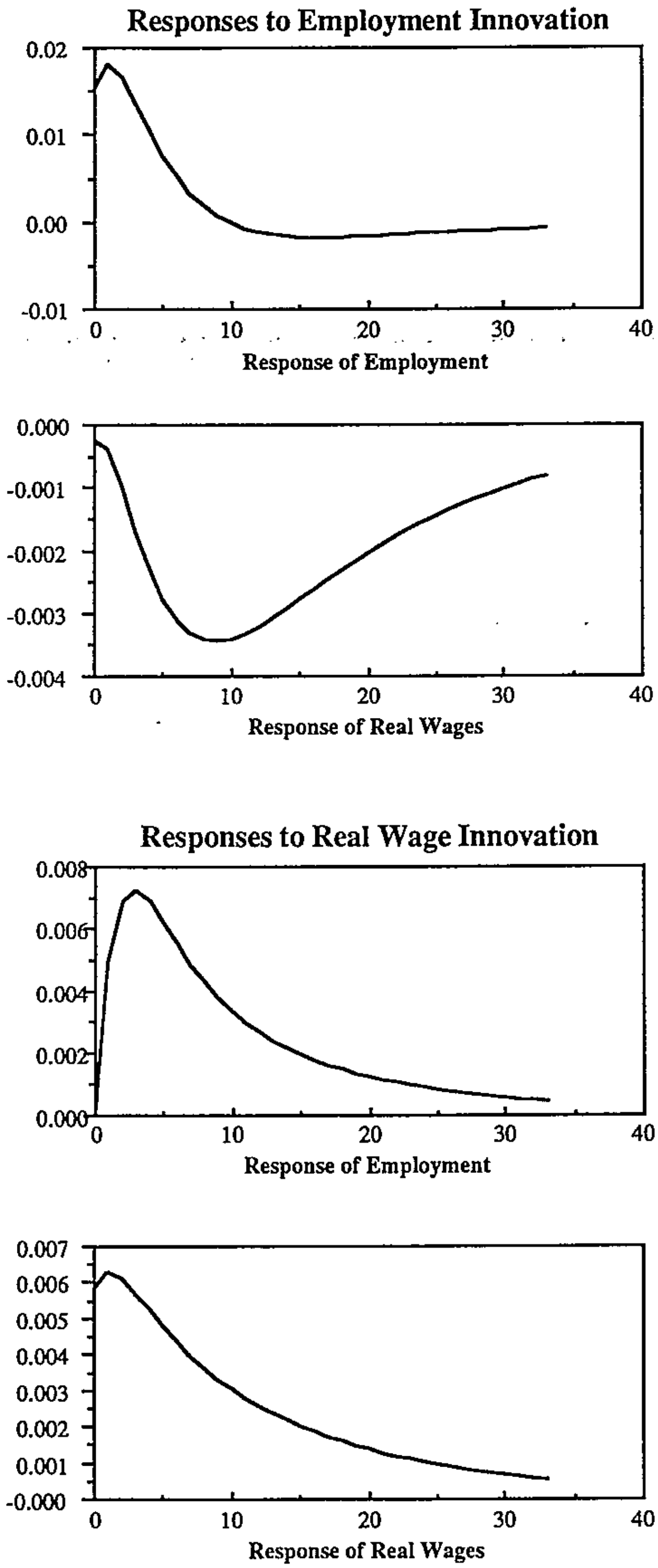


Figure 3: Dynamic Responses of Employment and Real Wages Implied by Maximum-Likelihood Estimates of the VAR (2) with Zero Restrictions

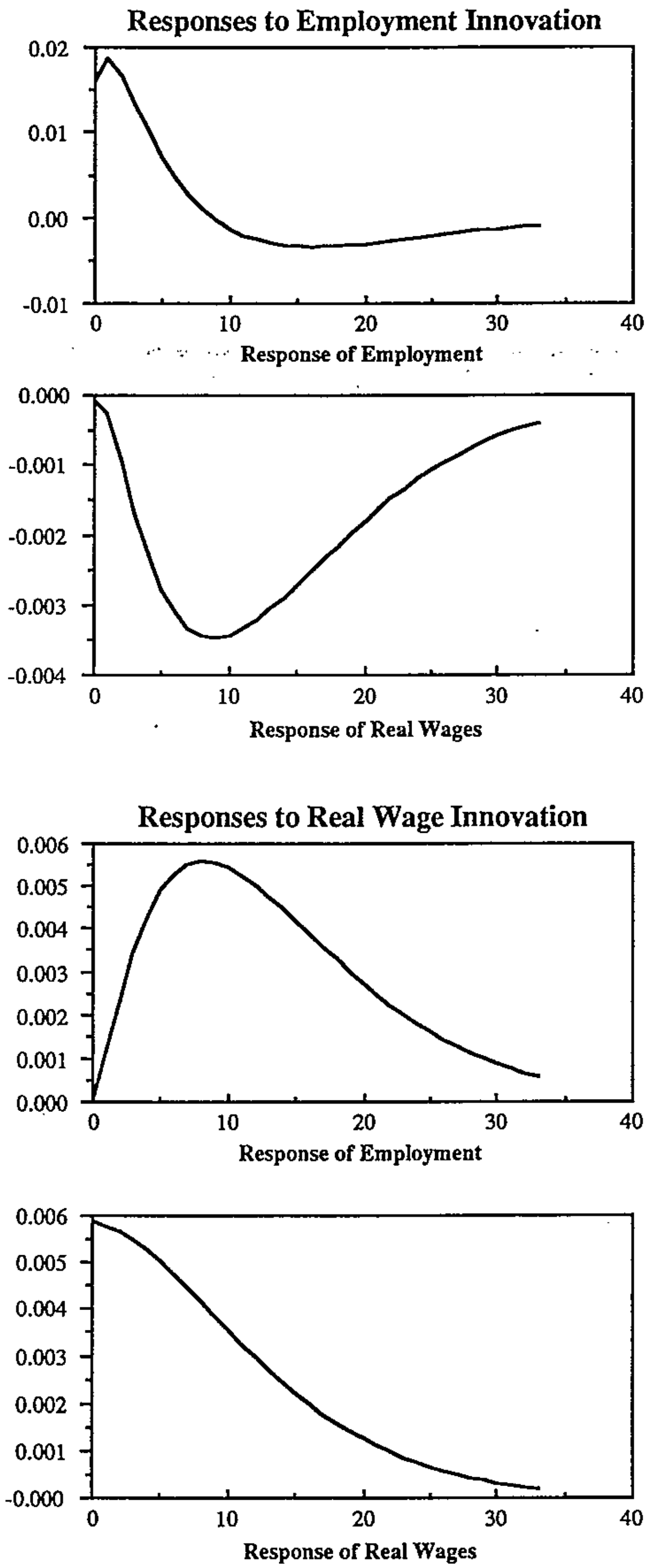
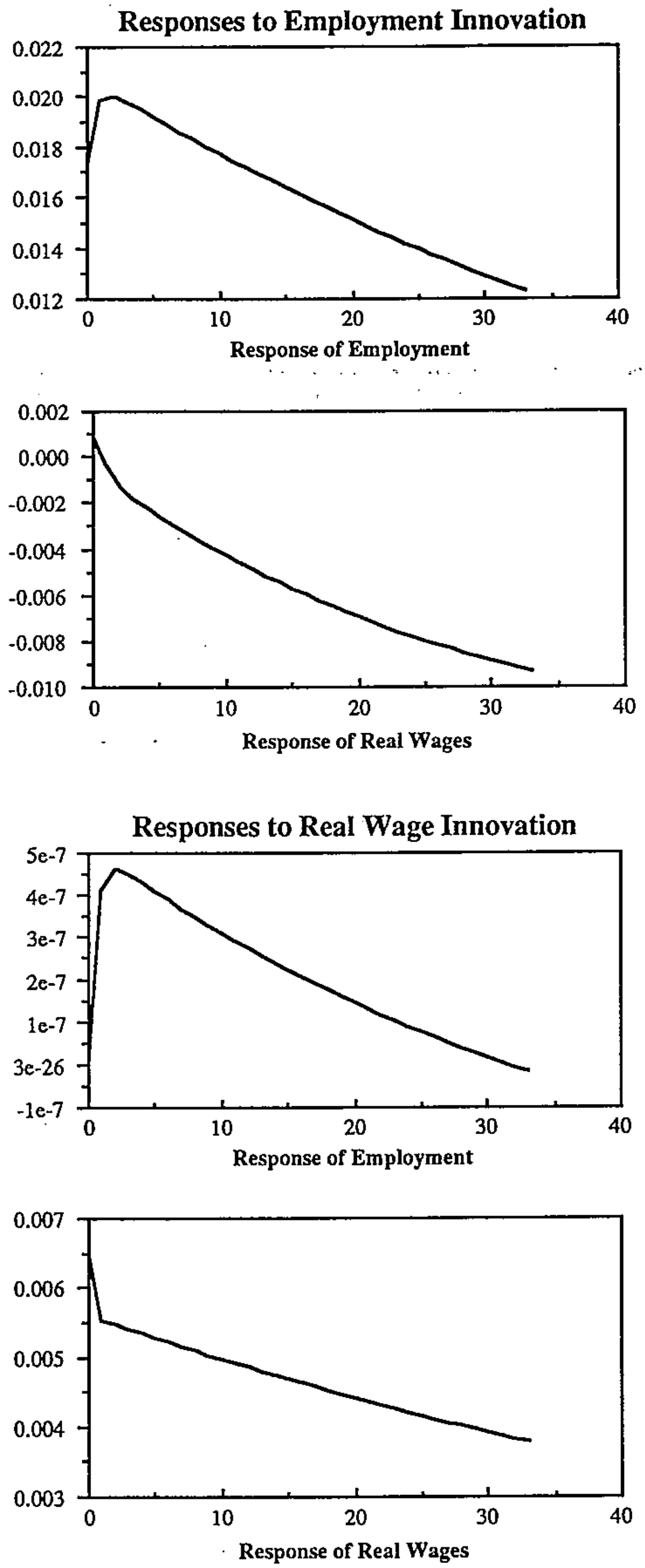


Figure 4: Dynamic Responses of Employment and Real Wages Implied by Maximum-Likelihood Estimates of Predetermined Wage Model



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Appendix 1: Description of Data

The variables used in estimation are the hours worked n_t, log of the nominal wage level w_t, and log of the price level p_t. Data on these variables were obtained by following several steps.

(1). The following series were obtained from the Citibank Economic Database.

(i) Man–hours of Employed Labor Force per Week (Household Data) (**LHOURS**).

(ii) Total Non–institutional Population Including Armed Forces (**POPT16**).

(iii) Average Hourly Compensation for All Employees on Non–Agricultural Payrolls (**LPCNAG**).

(iv)Consumer Price Index: All Urban Consumers; All Items (1967=100) (**PZU**).

All of the series were monthly. The sample period was 1954,1 - 1980,12. All those series were transformed into quarterly by choosing the arithmetic average of respective 3 month values.

(2). Remember the fact that we assumed[24] that "$W_t/P_t=1$ on average" in order to do some approximation. So each element in series (iv) were transformed by

$$P_t \frac{\sum_{t=1}^{T} W_t}{\sum_{t=1}^{T} P_t}$$

where P_t is a representative element of series (iv). Let the resulting series be (iv)'.

(3). Series (iii) and (iv)' were transformed by using the natural log for each element. Let the resulting series be (iii)' and (iv)" respectively.

(4). n_t were measured by dividing series (i) by series (ii), while w_t were measured by series (iii)'. And p_t were measured by series (iv)".

(5). All the above data were detrended by using a trend, a quadratic trend, and 3 seasonal dummies.

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24 See p.9.

Appendix 2: Note on the Calculation of Approximate Standard Errors Using Likelihood–Ratio and Wald Test Statistics

Suppose the likelihood function satisfies standard regularity conditions and the information matrix is nonsingular so that the parameters are locally identified. Then the variance covariance matrix, $V(\hat{\Phi})$, of the maximum likelihood estimator $\hat{\Phi}$ is easily calculated as the inverse of the information matrix. That is,

$$V(\hat{\Phi}) = \frac{R^{-1}(\Phi)}{T}$$

where, $R(\Phi) = -\frac{1}{T}\frac{\partial^2 £_T(\Phi)}{\partial\Phi\partial\Phi'}$.

Since the consistent estimate of $R(\Phi)$ is

$$R(\hat{\Phi}) = -\frac{1}{T}\frac{\partial^2 £_T(\hat{\Phi})}{\partial\Phi\partial\Phi'},$$

standard errors of the elements of $\hat{\Phi}$ are given by the square root of the diagonal elements of $[R^{-1}(\hat{\Phi}) / T]$. However, when we have quite a few number of parameters to estimate, the dimension of $R(\hat{\Phi})$ is very big. The numerical difficulty of inverting large matrix $R(\hat{\Phi})$ sometimes results in negative diagonal elements which cause problem in calculating standard errors. In this note it is shown that approximate standard error of $\hat{\Phi}$ can be obtained indirectly by using the likelihood–ratio test statistic and the Wald's test statistic.

Suppose we want to calculate the standard error of Φ_j. Then let our null hypothesis be

$$H_0 = h(\Phi) = \Phi_j - \Phi_j^0 = 0.$$

Among the asymtotic tests of the above hypothesis are the likelihood–ratio test and the Wald's test. The relevant test statistics are[25]

$$LR = -2\,[\,\underset{h(\Phi)=0}{MAX}\, £_T(\Phi) - \underset{\Phi}{MAX}\, £_T(\Phi)\,]$$

25 See p.142 of Amemiya [1985].

$$\text{Wald} = -h(\hat{\Phi})' \left\{ \frac{\partial h(\hat{\Phi})}{\partial \Phi'} \left[\frac{\partial^2 £_T(\hat{\Phi})}{\partial \Phi \partial \Phi'} \right]^{-1} \frac{\partial h(\hat{\Phi})'}{\partial \Phi} \right\}^{-1} h(\hat{\Phi}).$$

Since $h(\Phi) = \Phi_j - \Phi_j^0$,

$$\frac{\partial h'}{\partial \Phi} = e_j = (0, \cdots, 0, 1, 0, \cdots, 0)'$$

where e_j is a unit vector with 1 for the j'th element and 0's for all the other elements. Therefore, in our case, Wald test statistic becomes

$$\text{Wald} = (\Phi_j - \Phi_j^0)^2 \,/\, \text{j'th diagonal element of } [T\,R(\hat{\Phi})]^{-1}.$$

If LR = Wald[26], the standard error of Φ_j is

square root of the j'th diagonal element of $[R^{-1}(\hat{\Phi}) / T]$
= square root of $\{(\Phi_j - \Phi_j^0)^2 / LR\}$.

[26] If the log likelihood function is in the form of $c - .5(\phi-\hat{\phi})A(\phi-\hat{\phi})$ where A is a symmetric positive definite matrix which may depend upon the data and upon unknown parameters and c is a scalar, then LR = Wald. Our log likelihood function is not of this form. But, in general LR and Wald are aymtotically similar. (See p.784 of Engel [1984].) Therefore, by assuming that LR = Wald, we can get approximate standard errors.