VICTORIA UNIVERSITY OF WELLINGTON

GRADUATE SCHOOL OF BUSINESS
AND GOVERNMENT MANAGEMENT

WORKING PAPER SERIES 7/90

Rational theories of the future in general equilibrium models

Stephen Burnell *

* Emmanuel College
Cambridge CB2 3AP
England

The Money and Finance Group
Faculty of Commerce and Administration
Victoria University of Wellington

September 1990

ISSN 0114-7420
ISBN 0-475-11426-4

RATIONAL THEORIES OF THE FUTURE IN GENERAL EQUILIBRIUM MODELS

Stephen Burnell

Emmanuel College
Cambridge CB2 3AP
England
and
Victoria University
Wellington
New Zealand

September 1989
Revised August, 1990

Abstract

This paper focusses upon three particular models - the basic Arrow-Debreu model, a simple model with two trading dates and asset markets, and the overlapping generations model - and looks at the theories of the future agents must hold in a rational expectations equilibrium. Not suprisingly, many of the difficulties of general equilibrium theory can be rephrased by asking: How could agents ever come to hold correct theories of the future?

Journal of Economic Literature Classification Numbers: 021, 022.

Key words: Rational expectations, general equilibrium.

RATIONAL THEORIES OF THE FUTURE IN GENERAL EQUILIBRIUM MODELS

I Introduction

In any dynamic microeconomic model, the actions of agents, today, will be influenced by their beliefs about the future. Consumers will attempt to forecast their preferences, tomorrow, as well as their endowments. Producers will attempt to forecast their technology, and everyone will be interested in future commodity and asset prices.

A theory of the future can be thought of as a list of propositions, describing how the world will be, tomorrow. Such propositions will, clearly, be influenced by the way the world is, today, and possibly by the history of the world, up till today. In a world with uncertainty, a theory describes how the world is influenced by the states of nature, tomorrow. A correct theory is one that correctly predicts the way the world will be, for every possible state of nature.

From the set of all possible theories, an agent chooses one. Her criterion of choice is not utility or profits, but truth. I will say that a theory is rationally chosen if it is true.

Now, it is possible to imagine an equilibrium situation in which every agent, today, correctly forecasts the future. In general equilibrium theory, most equilibrium concepts involve every agent choosing a rational theory. In an Arrow-Debreu equilibrium, consumers correctly predict their future preferences and endowments, producers correctly predict their production set and, further, every agent correctly predicts that commodity markets will not reopen. In a world with more than one trading date, a rational expectations equilibrium requires every agent to correctly predict future spot prices for commodities, and possibly assets, in addition to preferences, endowments and technologies. Notice, this requirement remains even if there are an infinite number of trading dates, incomplete asset markets and uncertainty. Hence, a rational, or correct, theory of the future is a fundamental concept in general equilibrium theory.

There are two main questions that I wish to consider in this paper. Firstly, what is a correct theory of the future, and what does it mean to say an agent chooses a correct theory? Secondly, how can an agent know, or believe she knows, that she has chosen a correct theory? Unless these questions can be satisfactorily answered, the concept of a rational expectations equilibrium, in general equilibrium theory, is particularly uninteresting.

In section II, I set up the basic Arrow-Debreu model and define a rational theory of the future. The conflict between a finite number of agents and the assumption of price taking behaviour is highlighted by this approach. Further, if the auctioneer uses "sunspots" to create contingent commodity markets, sufficient conditions can be found to ensure that these sunspots will matter, in equilibrium. In section III, a model with two trading dates is constructed. A rational theory of the future now requires agents, at date 1, to correctly predict spot prices, at date 2. I will call such a theory, about future prices, a rational theory of price formation. If the asset markets are incomplete, sufficient conditions can be found to ensure the existence of a continuum of rational expectations equilibria. In particular, it may be rational for agents to believe sunspots matter, even though the payoffs of assets are independent of the sunspots. Further, given commodity and asset prices, at date 1, sufficient conditions can be found to ensure every rational theory of price formation is locally unique. Crudely, even though there is a continuum of equilibrium starting points, once a particular starting point is specified, every equilibrium point, at date 2, is locally unique. Finally, in section IV, I consider a pure exchange, overlapping generations model. At date t, agents form a theory about commodity and asset prices, at t + 1. As the market clearing prices, at t + 1, will depend upon the theories of the young, at t + 1, a rational theory of price formation implicitly requires an infinite chain of beliefs. Suppose the asset markets are incomplete. Given the spot prices, consumption of the old, and the state of nature, at t, sufficient conditions can be found for the existence of a continuum of rational theories of price formation. In such a situation, a co-ordination story is required to explain why every young agent chooses the same rational theory. Agents do not choose a theory because it is correct; rather, it is correct because agents choose it. Clearly, this result does not support the claim that economics is a predictive science. Instead, it seems to indicate that the way forward will involve an analysis of co-ordination and the sociology of theory acceptance (see, for example, Arrow, 1987).

II One trading date

The starting point of my story is the beautiful Arrow-Debreu model (see Arrow and Debreu, 1954; Debreu, 1959). This model can be seen as a logically possible world, that firstly, clarifies many economic concepts and, secondly, serves as the foundation for more complex logically possible worlds.

In the Arrow-Debreu model, preferences, endowments and technologies are given. There is only one trading date. At this date, the auctioneer announces a price vector and agents announce their planned trades. In the simplest case, there is no time. This implies certainty and only one delivery date for commodities. Clearly, this is not compatible with any economically meaningful concept of production, for production is a causal process. Inputs precede outputs. Hence, for "real" production to take place, there must be at least two delivery dates. For convenience, suppose there are two dates, N perishable commodities at each date, I consumers, F producers, and no uncertainty. Inputs are delivered at date 1 and outputs at date 2. In the Arrow-Debreu world, there are 2N commodity markets. The endowments of consumer i is a vector of length 2N. Preferences over i's lifetime consumption plans can be represented by a utility function, from 2N-space to 1-

space. Further, i has shares in the F firms, which cannot be traded. Finally, the technology of firm f is a subset of 2N-space.

The trading date in this simple model is at date 1. I could have made the trading occur at date 0, but this would merely complicate the issue. At this date, agents must hold beliefs about what the world will look like at date 2.

Suppose agents trade only with the auctioneer. That is, an agent signs a contract, at date 1, guaranteeing to deliver commodities to the auctioneer, at dates 1 and 2, while the auctioneer signs a contract guaranteeing to deliver commodities to the agent. Further, suppose that every agent trusts the auctioneer to honour its contracts, at the announced prices (that the auctioneer guarantees will clear all commodity markets), but that the auctioneer trusts no-one. Firstly, an agent must believe that she knows whether her own planned trades are feasible. This requires consumer i to believe she knows what her own endowments will be, at date 2, and producer f to believe it knows its own production set. Further, if consumer i owns a share of firm f, she must believe she knows the profits of f (from the trades at date 1) before she can calculate her planned trades, at date 1. I will assume that every consumer observes the planned trades of all firms, before she announces her planned trades. Consumer i must therefore believe that firm f can honour all its obligations.

As things stand, some agents may have an incentive to break their contracts. For example, if the only punishment the auctioneer can administer is to refuse to honour its delivery contracts, at date 2, then the agent might prefer to receive delivery of commodities at date 1, and consume her endowment at date 2, rather than honour her contracts and deliver some commodities at date 2. A credible contract would then be one that is not only feasible, but also one which the agent in question has no incentive to break. Notice, the auctioneer must know the endowments, preferences and technology of every agent, in order to say whether a particular contract, with a particular agent, is credible. As this incentive constraint is not a feature of the Arrow-Debreu model, I will assume that the auctioneer has the ability to punish people who break their contracts. This punishment must be severe enough to ensure that no-one ever wants to cheat the auctioneer. The punishment for consumer i could be torture or forcing her to surrender all her endowments, at date 2. If, in addition, the auctioneer believes that every agent is rational and able to say whether a contract is feasible, then the auctioneer will believe that every agent can and will honour their contracts.

It is not clear how the auctioneer can punish a producer who fails to honour its contracts. Obviously, if firm f is able to honour its contracts, but chooses not to, then the auctioneer can simply confiscate all the outputs of the firm. If, however, f signs a contract that it cannot honour, who does the auctioneer punish? The difficulty here is that the firm acts like a person (it signs a contract), and yet it is not a person. There are a number of ways around this difficulty. Firstly, I could assume that every shareholder believes she knows the production set of firm f. The auctioneer tells all consumers the trading plans of the firm f, at date 1. Shareholders then have a responsibility to inform the auctioneer, if they believe the trading plan to be unfeasible. The auctioneer can then feel free to torture shareholders. Secondly, I could assume the auctioneer believes it knows the production set of every firm. It can then refuse to accept trading plans it considers unfeasible. Thirdly, I could assume firm f is run by a manager, who signs all the contracts. The manager then becomes the obvious target for any torture. However, if this manager is the only agent with information about the production set of f, conflicts between her and the shareholders could emerge. In particular, why should she choose a production plan that maximises profits? As these problems go well beyond the scope of this paper, I will simply assume that the auctioneer believes it is able to ensure that f only undertakes feasible production plans.

Further, agents must believe they know their optimal trading plan. Now, the utility function of a consumer represents preferences, held at date 1, over lifetime consumption plans. For example, if the agent consumes a lot of chocolate at date 1, she might believe, at date 1, that she will prefer fruit to chocolate, at date 2. In other words, a utility function can be used to construct a function from consumption, at date 1, to beliefs about preferences at date 2. Logically, of course, preferences over consumption at 1 could be expressed as a function of consumption at date 2. I will assume, however, the agent calculates the optimal trading plan, for delivery at date 2, as a function of the trading plan, for delivery at date 1. The agent then chooses the optimal plan for date 1. Notice, if the utility function is interpreted as representing preferences over lifetime consumption, held at date 1, without paying any attention to what preferences at date 2 will actually be, then there is a strong case for reopening commodity markets at date 2. In order to ensure that markets do not need to reopen, therefore, agents must correctly predict how consumption, at date 1, will affect preferences over consumption, at date 2. Further, to ensure markets do not reopen, it is necessary for agents to correctly predict their endowments, at date 2.

Assuming that firms seek to maximise profits, if a producer (or, rather, manager) believes she knows the set of technically feasible production plans and observes the prices announced by the auctioneer, then it can calculate the set of optimal production plans.

Finally, all agents must believe that markets will not reopen at date 2. At date 1, the auctioneer announces relative prices for the 2N commodities. Consumers then choose their optimal consumption plan, and producers choose their optimal production plan. As a result, all markets clear. Suppose that every agent is able to honour all their contracts, at both dates 1 and 2. Further suppose that some agents are unhappy, at date 2, with their chosen plan, because their preferences, endowments, or technology are not what they predicted. There is now room for mutual gains to be made through barter at date 2, for the trades made at date 1 yield an allocation that is no longer Pareto efficient. Notice, all these assumptions imply that any change in technology will be an improvement, because firms sell all their output, at date 1.

I will assume every agent believes commodity markets will reopen, at date 2, if, and only if, the old trades yield an allocation that is not Pareto efficient, when perceived by agents at date 2. If an agent believes that markets will not reopen, at date 2, then she must also believe that every consumer correctly predicts their preferences and endowments, and every producer correctly predicts their technology. Further, she must believe that every other agent is rational, and so will honour their debts.

Now, the above story calls the assumption of price taking, on the part of every agent, into question. For an agent believes that if she incorrectly predicts the future, then the commodity markets will reopen, at date 2, and the relative spot prices, then, will differ from the relative futures prices, at date 1. This problem, however, is nothing more than the recognition that price taking behaviour is incompatible with a finite number of agents. For example, suppose agents believe that the auctioneer will not rest until market clearing prices are found. Market clearing is not an approximate concept; if one billionth of a unit of two commodities are left over, then the markets have not cleared. Suppose, therefore, that the auctioneer calls prices that ensure every agent can realize their plans. Now, any agent knows that if she changes her planned trades, at these prices, then the markets will not clear. Hence, the auctioneer will be forced to announce a new list of relative prices. In this sense, therefore, the agent can influence prices, and she knows this. This problem is well-known and has been analysed in detail, elsewhere (see, for example, Makowski, 1980; Ostroy, 1980).

In summary, in the world of Arrow-Debreu, with N perishable commodities, two delivery dates and no uncertainty, a theory of the future, for consumer i, is a vector of endowments, at date 2, and a function from consumption, at date 1, to preferences over consumption vectors, at date 2. Similarly, a theory of the future, for firm f, is nothing more than a production set. Further, to ensure that agents believe markets will not reopen, at date 2, every agent must believe that all agents are rational and hold a correct theory of the future. Finally, for the auctioneer to believe it can find market clearing prices, it must be able to punish agents who do not honour their contracts, and, further, it must believe that every agent is rational and able to say whether a contract is feasible.

An equilibrium, in the Arrow-Debreu world, is given by a price vector, announced by the auctioneer, and a theory of the future for each agent, such that:

(i) every agent is willing and able to honour their obligations,

(i) all markets clear, and

(ii) every agent's theory is correct.

Now, introduce uncertainty to this basic model. For simplicity, suppose there is no uncertainty at date 1, and two states of nature (s is either 1 or 2), at date 2. Nature chooses the state of nature, possibly using an objective probability distribution, that may or may not be known to agents, or possibly using a deterministic formula, that is not known to agents. The important point is, at date 1, agents do not know which state of nature will occur and, further, they cannot influence Nature's choice, either individually or collectively. Whether an agent is able to speak of the probability of state 1 occurring, and whether this probability is to be thought of as subjective or objective, is irrelevant. Once the state of nature is given, "atmospheric conditions, natural disasters, technical possibilities,... are determined" (Debreu, 1959, p.98). In the Arrow-Debreu world there are now 3N markets, at date 1; N spot markets and N contingent markets, guaranteeing delivery of commodities if state s occurs, for s being 1 or 2.

In this situation, a theory of the future, held by consumer i, is a function from the two states, at date 2, to her endowments, and a function from consumption, at date 1, to preferences over consumption, at date two. Notice, these preferences are over vectors of length 2N. Put another way, i's planned consumption in state 1 may influence her preferences over consumption in state 2. In such a situation, the optimal consumption plans, in states 1 and 2, can be thought of as being determined simultaneously, as a function of the consumption at date 1. Similarly, a theory of the future, held by firm f, is a subset of 3N-space.

As before, the auctioneer must believe that the planned trades of every agent are feasible. To ensure this, I will continue to assume that the auctioneer has the power to severely punish any agent who breaks their contract and, further, that the auctioneer believes every agent is rational and able to say whether a particular trading plan is feasible.

Finally, to ensure that every agent believes that markets will not reopen, at date 2, every agent believes that all agents hold a correct theory of the future.

The assumptions of this model are not only unrealistic, but also logically troublesome. In particular, suppose the only uncertainty in the model is the psychological well-being of consumer i, at date 2. For example, at date 1, i reasons that at date 2, she will be either happy (state 1) or sad (state 2). Further, suppose that the state of nature affects i's preferences. That is, i's preferences over consumption, at state 1, can be thought of as being generated by a function from consumption at date 1 and consumption at state 2. Similarly, preferences over consumption, at state 2, can be thought of as a function from consumption at date 1 and state 1. If these two functions are not identical, then the state of nature affects i's preferences. Formally, this is merely a special case of the Arrow-Debreu model. There are two main problems with this interpretation, however.

First, it is not clear how agents, other than consumer i, observe the state of nature. Here it is necessary to draw a distinction between what I will call external and internal states of nature. An external state can be observed by everyone; for example, whether it is raining, or not. An internal state of nature, however, affects only the preferences of one consumer, call her i, and is directly observable only by i. Notice, there may be states that are neither external nor internal. If the state of nature is an internal one, consumer i alone is able to verify which state has occurred. Hence no-one else will be willing to sign a contract, conditional on this state. Notice, this is not because i can influence which state occurs, but because i's announcement, of which state has occurred, cannot be verified. This problem was first pointed out by Radner (1968).

Second, suppose the internal state of i is determined by an external state. For example, if it does not rain, i will be happy (not-rain is the external state while i's happiness is the internal state). If i is the only agent to be affected by this external state, the existence of commodity markets, at date 1, contingent on this state occurring, would seem to defy the assumption that every agent is a price taker. For all other agents, the state is a "sunspot", as it does not affect their preferences, endowments or technology. Remember there is no other uncertainty. I will say that a utility function strongly depends upon a state of nature, 1, if, for all consumption plans with the property that the consumption vector, at date 2, contingent on state 1, is equal to the vector, contingent on not-1, the two corresponding vectors of relative marginal utilities are not equal. Suppose consumer i's utility function strongly depends upon state 1. Either contingent markets exist, or they do not. If they do not, then at any price vector, i will have an incentive to barter with other agents. If contingent markets exist, then every equilibrium, in the Arrow-Debreu world, has the consumption plan of at least one consumer, other than i, changing as the state of nature changes. For, if i consumes the same in both states, relative prices must differ across the states. Hence, every other consumer will have different plans, depending on the state. Further, suppose every agent, other than i, consumes the same in both states. Market clearing then requires that i does as well, which is a contradiction.

Hence, even though the state is like a sunspot for every agent other than i, in any Arrow-Debreu equilibrium, the consumption allocation will depend upon the state of nature. This result certainly goes against the spirit of the price taking assumption, because it requires every consumer to take notice of a state that directly affects only consumer i.

An more interesting example is one in which every state of nature is a sunspot for every agent. There is no other uncertainty. Suppose every consumer acts as though she is trying to maximise an expected utility function. That is, each consumer i has a subjective probability distribution over the two states, and a strictly concave utility function over lifetime plans held with certainty, such that i's preference ordering can be represented by the expected utility function. Further, suppose at least two consumers have different subjective probability distributions over the states. If the auctioneer chooses to open the contingent commodity markets, then every Arrow-Debreu equilibrium will be associated with consumption plans that differ with the state (for at least some agents). On the other hand, if the auctioneer chooses not to open the contingent markets, then every equilibrium will be Pareto inefficient.

Finally, it is difficult to see how anyone can verify that a particular agent's theory of the future is correct, in the world with uncertainty. At date 2, only one state of nature actually occurs.

Clearly, agents can verify whether they correctly predicted (at date 1) what would happen if this state occurred. They cannot, however, say whether they correctly predicted what would have happened, if the other state had occurred. A theory can be consistent with the facts of the world, but it is not possible to say that it is correct. Of course, if there is an indefinite number of worlds, each with the same parameter values, prices and theories, then a theorist observing all the worlds will be able to verify whether an agent's theory is correct. This is because the theorist observes what actually happens at every state of nature. Any agent, whose vision is limited to her world, will be unable to attain such knowledge.

III Two trading dates

Consider now a world with N perishable commodities, I consumers, F firms, two delivery dates, no uncertainty, no futures markets and one asset, called an Arrow security. At date 1, agents buy and sell commodities on the spot markets, and either buy or sell units of the Arrow security, which promises to pay one unit of purchasing power, at date 2. For convenience, I will assume that agents trade only with the auctioneer. At date 2 the auctioneer announces commodity spot prices, relative to the unit of purchasing power. Agents then settle their debts associated with the Arrow security, and trade on the commodity spot markets. Further, firms distribute their profits to shareholders at date 2.

A theory of the future, for an agent, will now include beliefs about the profits of firms, at date 2, as well as beliefs about the commodity spot prices, to be announced by the auctioneer, at date 2. I will often refer to a theory about spot prices as a theory of price formation.

A temporary equilibrium in this world is given by a spot price vector, at date 1, a theory of the future for each agent, and a spot price vector, at date 2, such that:

(i) every agent honours their debts,

(ii) all commodity markets (at both dates 1 and 2) clear, and

(iii) the Arrow security market clears.

Condition (i) ensures that any agent who sells Arrow securities, at date 1, has sufficient endowments, at date 2, to honour their debts. For example, consumer i holds a theory about her preferences and endowments, as well a theory about future spot prices. Suppose this causes her to sell x units of Arrow securities. At date 2, the actual spot prices are announced by the auctioneer, and i discovers her actual endowments and preferences, as well as her share of the profits of all firms. In a temporary equilibrium, the actual value of i's endowments, plus her share of the profits of firms, at date 2, must be no less than x units of purchasing power.

Assuming the vector of marginal utilities, for every consumer, is always strictly positive, condition (iii) follows automatically from (ii). The important condition, therefore, is (ii). This says that at date 1, given the spot prices and the theories of the future held by agents, the N commodity markets clear. Further, at date 2, given the actual endowments, preferences, profits and technologies of agents, the auctioneer announces a spot price vector that clears the N commodity markets.

Now, it may be the case that all the commodity markets clear, at dates 1 and 2, even though some agents hold incorrect theories of the future. This is why I called the equilibrium "temporary". I will therefore define a rational expectations equilibrium to be a temporary equilibrium in which every agent's theory of the future is correct.

A famous proposition of Arrow's establishes that the set of rational expectations equilibria, in the world with Arrow securities, is equivalent to the set of equilibria, in the Arrow-Debreu world. That is, if agents correctly predict profits and spot prices (in addition to endowments, preferences and technologies, which are necessary for an equilibrium in the Arrow-Debreu world), then agents will act as though a full set of commodity futures markets exists, at date 1 (see Arrow, 1953).

The concept of a rational expectations equilibrium is of little interest if agents cannot justify holding the particular theory of price formation. First, it is difficult to explain how an equilibrium will ever occur. For an equilibrium can be interpreted as saying if the auctioneer calls out *these* spot prices, at date 1, every agent holds *this* theory of price formation (in addition to their theory of profits, etc) and, further, the spot prices announced by the auctioneer, at date 2, are consistent with this theory, then every market will clear. Unless it can be explained why agents hold this particular theory, there is no reason to suppose that a rational expectations equilibrium will ever occur. Put another way, a temporary equilibrium approach is more appropriate if agents cannot justify their theory of price formation. Second, there is the problem of what it means to say that an agent believes, for certain, *these* spot prices will occur, at date 2. Suppose an agent believes she knows that a certain proposition, **x**, is true. By definition, she has no doubt. I will assume that it is irrational for an agent to state that a proposition, **not-x**, is impossible without being able to provide a justification. This can be seen as an application of some principle of insufficient reason. That is, if an agent has no reason to believe a particular proposition (or theory) is impossible, then she will assign a positive probability to the proposition being true. An immediate consequence of this assumption is that a rational expectations equilibrium will exist only if agents can justify their theory of price formation.

To make sense of a rational expectations equilibrium, therefore, I will introduce three new assumptions. First, I will assume every agent believes that the auctioneer will ensure all commodity markets clear at date 2, as well as date 1. Second, every agent believes all agents hold a correct theory of the future. Notice, this assumption was required, in the world with one trading date, to ensure every agent believed markets would not reopen. Third, every agent believes she knows the aggregate excess demand function of the economy, when all agents hold the same theory of price formation. That is, every agent believes she knows a function from spot prices, at date 1, and correctly held beliefs about spot prices, at date 2, to aggregate excess demand, at dates 1 and 2. Notice, this does not require agents to know the excess demand functions of other agents, merely the aggregate function. Agents know the "macro" behaviour of the economy, without necessarily knowing the "micro" behaviour of every agent. Notice, further, that I have not explained how agents come to know the functional form of the economy. In many ways it resembles the assumption of complete information in game theory.

It could be argued that the above assumptions should be expressed in terms of common knowledge; for example, i knows that j knows all markets will clear, i knows that j knows that k knows all markets will clear, and so on (see, for example, Brandenburger and Dekel, 1989). All that matters, however, is that i knows the set of rational expectations equilibria and, further, she believes that everyone else chooses a correct theory. It is the second assumption (above), therefore, that enables me to avoid common knowledge. It seems to me that any attempt to justify this assumption will require common knowledge assumptions. Presumably, i believes j holds a correct theory because i believes j reasons the same way she does. Hence, if the above three assumptions are common knowledge, and i believes she has calculated the correct theory of price formation, then she is justified in believing that every other agent has also calculated the correct theory.

From the aggregate excess demand function, agents can calculate the set of rational expectations equilibria. Given the spot prices announced by the auctioneer, at date 1, agents can calculate the set of spot prices, at date 2, that will ensure a rational expectations equilibrium. Of course, this set will be empty for most spot price vectors, at date 1. Further, for some price vectors, the set may contain more than one element. For a rational expectations equilibrium to make sense, therefore, given the spot prices at date 1, either the set is a singleton or agents have some way to co-ordinate beliefs about prices. For if there is more than one rational theory of price formation, associated with the given spot prices, at date 1, it is necessary to explain how every agent comes to hold the same theory, in equilibrium.

It is well known that for almost all Arrow-Debreu economies, the set of equilibria are finite and, hence, locally unique (see, for example, Debreu, 1970). From Arrow's equivalence proposition, mentioned above, it therefore follows that every rational expectations equilibrium is, in general, locally unique. Further, for most economies, no two equilibria have the same spot prices at date 1. To establish this proposition, firstly consider an economy where no two equilibria have the same spot prices, at date 1. By continuity of the excess demand function, this property will be preserved for all economies nearby. Secondly, consider an economy with at least two equilibria having the same spot prices, at date 1. As the space of economies can be associated with the space of excess demand functions (see, for example, Debreu, 1974), there exists an economy, arbitrarily close to the one being considered, for which the two equilibria now have different spot prices, at date 1. Hence, for most economies, a rational expectations equilibrium makes sense without a co-ordination story.

The assumptions mentioned so far are not sufficient, however, to ensure the common theory of price formation, held by agents, is identical to the market clearing price vector announced by the auctioneer, at date 2. Suppose the auctioneer does not know the aggregate excess demand function of the economy, and so a tatonnement process is used, at both dates 1 and 2, to find market clearing prices. For simplicity, suppose the spot prices, announced at date 1, are consistent with a rational expectations equilibrium. At date 2, consumers have given preferences, commodity and share endowments, as well as asset "endowments", which may be negative. Producers have given outputs and asset endowments, which also may be negative. Now, a set of spot price vectors exists that will ensure market clearing, at date 2. One of these vectors is given by the common theory of price formation held by all agents. If the auctioneer happens to find this particular one, then the rational expectations equilibrium is attained. If, however, there are other spot price vectors that ensure market clearing, at date 2, and the auctioneer finds one of these, then every agent will hold an incorrect theory of price formation. To avoid such a situation, I will assume the auctioneer knows the aggregate excess demand function of the economy, when every agent holds the same theory of price formation. Further, I will assume that it believes every agent will choose a correct theory of the future. At date 1, the auctioneer will, somehow, choose a particular equilibrium and announce the associated spot prices. At date 2, it simply allows the equilibrium to unfold by announcing the appropriate spot prices.

Notice, it is difficult to see how agents could ever construct out-of-equilibrium beliefs. Suppose the auctioneer announces spot prices, at date 1, that are not associated with a rational expectations equilibrium. Every agent knows there is no theory that can be correctly held by everyone and ensure market clearing, at both dates 1 and 2. For example, consumer i knows that either some markets will not clear or some agents will hold incorrect beliefs. Hence, at least one of the assumptions, mentioned earlier, must be abandoned.

Uncertainty can be added to the model without changing any of the above results. As in the Arrow-Debreu world, suppose there is no uncertainty at date 1, and two possible states of nature at date 2. In the world of Arrow securities, there will be two assets, call them Arrow securities 1 and 2. One unit of Arrow security s promises to pay one unit of purchasing power, at date 2, if state s occurs, and nothing otherwise. A theory of price formation is now a function from the two states to spot prices. Further, a theory of profits is a function from the states to the profits of every firm. Notice the problem in an uncertain world, of verifying that a particular theory is true, discussed in the previous section, now applies to theories of prices and profits, as well as preferences, endowments and technology.

The above results also hold in worlds with assets other than Arrow securities. An asset can be thought of as a piece of paper that guarantees to pay (or collect) a certain number of units of purchasing power in each state of nature. Put another way, a unit of an asset is a function from states of nature to real numbers, representing the payoff of the asset. These payoffs are announced, by the auctioneer, at date 1. For example, consider an asset that promises to pay 3 units of purchasing power in state 1, and -1 units in state 2. If state 1 occurs, the agent rushes to the auctioneer, with her pieces of paper, and demands payment. The auctioneer pays the agent by crediting her trading account. Remember, commodity prices are expressed relative to a unit of purchasing power (sometimes called money prices, to create confusion), so everything is well-defined. If state 2 occurs, the auctioneer takes one unit of purchasing power away from the agent, for every unit she holds of the asset. Binding contract is, perhaps, a better description of the asset, than piece of paper.

Now, an asset's payoffs can be thought of as a vector with two components. If there are two assets, the four payoffs can be represented as a 2 x 2 matrix. In the case of Arrow securities, the payoff matrix is nothing more than the identity matrix. It is obvious that Arrow's proposition can be generalized to any nonsingular payoff matrix. Further, to ensure asset prices are always positive, in equilibrium, it is sufficient to assume the payoff matrix contains only non-negative elements and, for each asset, there is at least one positive payoff. Of course, as an "asset" is merely a contract, it is perfectly legitimate for the auctioneer to announce a payoff matrix that yields a negative price, for at least one asset, in at least one equilibrium. For purchasing an asset, with a negative price and a payoff vector, r, is equivalent to selling an asset, with a positive price and a payoff vector, -r (see, for example, Geanakopolos and Polemarchakis, 1986).

If the number of assets is less than the number of states of nature, the asset markets are called incomplete. Not surprisingly, Arrow's proposition does not generalize to economies with incomplete asset markets. In particular, if the unit of purchasing power is not tied to a specific commodity (or basket of commodities), sufficient conditions can be found to ensure a continuum of rational expectations equilibria exists (see, for example, Geanakopolos and Mas-Colell, 1989).

Consider now a world with two sunspots, at date 2, and no other uncertainty. Further, suppose the auctioneer decides to operate only one asset market. This is merely a special case of a world with incomplete asset markets (as there are less assets than states of nature). I will assume that consumers seek to maximise expected utility, firms seek to maximise expected profits, and every agent holds the same probability distribution over the states of nature (which can be thought of as an objective distribution).

The first point to notice about this model is that the problem of firm f is rather strange. To maximise expected profits is meaningful only if profits are expressed relative to something. For example it may not be particularly intelligent to maximise expected money profits, if relative prices differ across states and state 1 is associated with higher "inflation" than state 2. For the firm will then give too much weight to the relative prices in state 1. It might make more sense for the firm to maximise expected profits relative to some basket of commodities. But which basket? As the world ends after date 2, and the firm has no intrinsic desire for commodities, there would seem to be no satisfactory answer. This problem has nothing to do with the sunspot; it is a result of more than one trading date, uncertainty and incomplete markets (see, for example, Dreze, 1974).

For simplicity, therefore, assume there is no production (see, for example, Geanakopolos and Polemarchakis, 1986; Geanakopolos and Mas-Colell, 1989; Werner, 1985; who all focus upon pure exchange economies). Even though every state of nature is a sunspot, and every agent holds the same probability distribution, it is possible to construct a rational expectations equilibrium in which the state of nature affects consumption (see Cass, 1989, for an example). The intuitive reason for this result is quite simple. With only one type of asset and two states of there nature, there are 3N - 1 relative prices. However, because each consumer has two budget constraints, there are only 3N - 2 independent equations that must be satisfied, in equilibrium. Consider any equilibrium in the world of certainty. This equilibrium can be transformed into an equilibrium in the world with sunspots, by taking money prices in both states to be equal to the certainty prices. Now, consider the matrix of partials of consumption, with respect to prices, at this transformed equilibrium. The matrix has 3N - 2 rows (the equations) and 3N - 1 columns (the prices). If the rank of the matrix is 3N - 2, then there is one dimension of indeterminacy (see Geanakopolos and Mas-Colell, 1989). This is sufficient to guarantee a continuum of rational expectations equilibria, and hence the existence of an equilibrium in which sunspots matter.

Now, given the auctioneer operates asset markets, rather than contingent commodity markets, sunspots can be used to explain why the asset markets may be incomplete. Given K assets, K "intrinsic" states of nature (that affect preferences and endowments) and H sunspots, if agents believe sunspots will matter, then the asset markets will necessarily be incomplete, if H is greater than one. Agents' beliefs, therefore, can turn a complete set of asset markets (with respect to the intrinsic states) into an incomplete set. This does not explain, however, why the auctioneer chooses to operate asset markets, rather than a complete set of contingent commodity markets (for delivery at both dates 1 and 2), at date 1.

For simplicity, I will assume that if asset markets are complete, then the auctioneer would rather operate complete contingent markets. The existence of asset markets will therefore depend upon their incompleteness. Geanakopolos (1990, p.2) gives three main reasons as to why asset markets may be incomplete; first, the existence of non-external (and possibly internal) states;

second, the fact that some agents may be unable to participate in trades at date 1 because they have yet to be born; and third, if asset markets are costly to set up, there may be an upper limit on the number of "profitable" markets. Only the first reason, however, is directly applicable to the model being discussed. The second reason is applicable, however, to the overlapping generations model.

IV Overlapping generations

In this section I will analyse an economy with an infinite number of trading dates. Consider a pure exchange, overlapping generations model with N perishable commodities, I agents per generation, one asset (called money) and no uncertainty. Each agent lives two periods and then dies. For each generation there are I utility functions (from 2N-space to 1-space), and I endowment vectors (of length 2N). The population is stationary, in the sense that the L utility functions and endowment vectors are the same for every generation.

At date t, there are I young agents, and I old agents, alive. The auctioneer announces a vector of spot prices, and agents observe their current endowments and preferences. Each young agent forms a theory about how will their world will look, at date t + 1. That is, consumer i forms beliefs about her preferences and endowments, when old, as well as a theory of price formation. At first sight, these beliefs are identical to those in the world with two trading dates. I will show, however, that important differences exist for the theory of price formation.

The world begins at date 1. At this date, there exists an old generation, called generation 0. To keep things simple, I will ignore their consumption problems and assume that "prehistory" determines an aggregate consumption vector for this generation, at date 1. Spot prices, at date 1, together with the aggregate endowment vector of the old, then determine the aggregate money stock (or money "endowment") of generation 0.

A rational expectations (or perfect foresight) equilibrium is given by a sequence of spot price vectors, one for each date, an aggregate consumption vector, for generation 0 (and hence a money stock, which may be positive, negative or zero), and a theory of the future, for agent i, born at date t (for all i and t), such that:

(i) every agent honours her debts,
(ii) all commodity markets clear,
(iii) the money market always clears, and
(iv) every agent's theory of the future is correct.

As in the world with two trading dates, condition (ii) implies (iii). Hence, the aggregate demand for money, for each generation, is equal to the money stock.

Suppose every agent believes the population is stationary. If agent i, born at date t, can observe the endowments of the old, at t, then her theory of endowments, when old, will simply be based on her observations. As it is difficult to see how i could directly observe the preferences of others, her theory of preferences remains the same as in the world with two trading dates. If, however, i knows that every agent, born before t, has correctly predicted their future preferences, she may have confidence that she, too, will correctly predict her preferences. While this argument has some power, for t large enough, it is not particularly convincing when t is small. For example, at date 1, i has no information, whatsoever, about the predictions of previous generations. Further, the above argument raises questions concerning how much agent i, born at t, knows about the past. First, does she know the history of prices, announced by the auctioneer, from date 1 till t - 1? Second, does she know the actions of all agents, born before t, either at the micro or macro levels? Third, does she know the endowments and preferences of past agents, in order to confirm that the population is stationary? Perhaps a distinction should be made between endowments, which are tangible, and preferences, which cannot be directly observed. Finally, does she know the theories of the future held by past agents? The assumptions concerning what i knows about the past will be crucial for any discussion of i's theory of the future.

Now, define a state of the economy to be a spot price vector and an aggregate consumption vector for the young. Hence, a state of the economy is a point in 2N-space. The role of states of the economy, in an equilibrium, is straightforward. First, at date 1, given the aggregate consumption of generation 0, the state of the economy is the price vector announced by the auctioneer and the aggregate consumption vector of the young, that ensures market clearing. Similarly, at date t, spot prices and the consumption of the old are given by history. The aggregate consumption of the young is then found, so as to ensure market clearing. Hence an equilibrium can be thought of a sequence of states of the economy. I will say that a state of the economy generates a particular rational expectations equilibrium, if it is the state of the economy at date 1. Notice, for any equilibrium and date, t, the state of the economy, at t, generates a rational expectations equilibrium.

Let the set of rational states of the economy be given by those states of the economy that generate a rational expectations equilibrium. This set can be broken into three subsets; those associated with positive, negative and zero money stocks.

It is well-known that, locally, the set of rational states of the economy can be of any dimension, from 0 to 2N (see, for example, Kehoe and Levine, 1984 and 1985). Hence, the overlapping generations model can yield an indeterminacy of starting points, or initial conditions.

Suppose agents correctly predict endowments and preferences. This enables me to focus upon theories of price formation. For any rational state of the economy, there exists a non-empty set of associated rational theories of price formation. A rational theory is, simply, one associated with a rational expectations equilibrium. Given the state of the economy, at date 1, and the rational theory of price formation, the planned consumption of the old, at date 2, can be calculated, and hence the market clearing consumption of the young (at date 2) can be found. For each rational state and associated rational theory, therefore, a pair of rational states (one at date 1 and the other at date 2) can be constructed.

Now, consider a particular rational expectations equilibrium. Given the consumption of the old, at date 1, if the auctioneer calls out *these* spot prices, and every young agent holds *this* theory of price formation, then all markets will clear, at date 1. At date 2, if the auctioneer announces spot prices that are consistent with the theory (held by agents, at date 1), and every young agent holds *this* theory of price formation, then all markets will clear, and so on, for every date, t. To explain how this particular equilibrium unfolds, therefore, it must be explained how agents come to choose the correct theory of price formation, at every date, t.

As in the previous section, I will assume that every agent believes all commodity and asset markets will always clear, every agent believes all agents (no matter when they are born) will choose the correct theory of price formation and, further, every agent knows the aggregate demand function of any (and, hence, every) generation. More formally, there is a function, from spot prices, at date t, and correctly held beliefs about spot prices, at t + 1, to the aggregate excess demand of the young, at t, and of the old, at t + 1. Further, every agent knows this function. Notice, this function is the same for every generation.

From knowledge of the excess demand function, every agent can calculate the set of rational expectations equilibria, and hence the set of rational states of the economy. Given a rational state of the economy, agents can calculate the associated set of rational theories of price formation. Notice, agents may need infinite computational ability in order to calculate these sets.

The concept of a rational expectations equilibrium must be given a causal interpretation. At date t, the auctioneer announces a spot price vector and old agents announce their planned consumption. The young agents observe these prices, as well as the aggregate consumption of the old. Each young agent then chooses a theory of price formation and announces her planned consumption (at date t). The theory chosen by agent i is determined by what she believes the auctioneer will do, at date t + 1. Now, she knows there exists a theory that is consistent with a rational expectations equilibrium. Further, if this theory correctly predicts spot prices at t + 1, then there exists another theory (or, rather, a set of theories), such that if it is held by every young agent, at t + 1, then every market will clear, at t + 1. As the auctioneer will only allow trade to take place at market clearing prices, agent i must believe that every young agent, at t + 1, will choose the same rational theory. Hence, i recognizes that the young, at t + 1, will face an equivalent problem to the one she is facing. That is, the theory the young agents choose, at t + 1, will be determined by their beliefs about what the young will believe, at t + 2, and so on, forever. Notice the family resemblance between this infinite chain of beliefs and the concept of common knowledge, in game theory (see, for example, Brandenburger and Dekel, 1989).

The point being made here is not really as complex as it first seems. People, especially economists, are always trying to guess, today, what the world will look like, tomorrow. These predictions influence the people behave, today. The way the world actually is, tomorrow, will depend upon what people do, both today and tomorrow. Hence, the world, tomorrow, will be influenced by; (i) the beliefs of people, today, about tomorrow; (ii) the beliefs of people, tomorrow, about the day after tomorrow; and (iii) the beliefs of people, today, about the beliefs of people, tomorrow, and so on. Notice, (iii) is merely a special case of (i), that takes (ii) into account. Crudely, the present is influenced by the past behaviour, current beliefs about the future, and, therefore, past beliefs about the future.

All this makes sense if the state of the economy, at t, uniquely determines a rational expectations equilibrium. For there is only one theory all the young, at t, can hold that will ensure every future generation correctly predicts the future and every market clears, from today on. Because every agent knows this (and, presumably, every agent knows every agent knows, and so on), they have no choice. They have to hold the unique rational theory. Notice, this condition is stronger than the set of rational theories, at t, being a singleton. For example, suppose there is one rational theory, at t, but a continuum of rational theories, at t + 1, associated with the predicted state of the economy. Suppose, further, the young agents, at t + 1, do not have the "technology" to co-ordinate their theories, and the young agents, at t, (believe they) know this. It is difficult to see how the young, at t, can be certain that their predicted prices will ensure market clearing, at t + 1.

As the above example illustrates, if the state of the economy, at t, is associated with a "continuum" of rational expectations equilibria, a co-ordination story is needed, at some point, to explain why every young agent, at t, chooses the same rational theory. Examples of rational states of the economy, with a continuum of associated rational theories, can be easily found, as soon as uncertainty is added to this basic model.

Consider therefore, a world with no uncertainty, at date 1, and two possible states of nature, at every other date. At date t, there are 2^{t-1} possible histories of states of nature (up to, and including, t). For simplicity, suppose that these states do not affect the endowments or preferences of agents. That is, the states of nature are sunspots. Further, suppose the objective probability of state 1 occurring, at t, is independent of t and the history of states, at t - 1. That is, the random variable is identically and independently distributed. Every agent knows the probability distribution and seeks to maximise expected utility. Finally, suppose there is only one asset. Hence, the asset market is incomplete. Because there is no uncertainty, at date 1, a rational expectations equilibrium is given by a state of nature, at date 1, a sequence of spot price vectors, one for every possible history, at t, a consumption vector, for generation 0, and a theory of the future, for every agent, such that conditions (i) - (iv), above, hold. As in the previous section, every equilibrium, in the world without uncertainty, can be transformed into an equilibrium in the world with two sunspots.

A state of the economy is now defined at a state of nature. Hence, there are two sets of rational states of the economy, one for each state of nature. Because the states of nature are sunspots and, further, identically and independently distributed, these two sets will be identical. That is, if a state of the economy, at the state of nature, s = 1, generates a rational expectations equilibrium, then so too will the same state of the economy, at s = 2.

In the world without uncertainty, take a rational state of the economy, at date 1, and its associated rational theory of price formation. As shown above, from the planned consumption of the old, at date 2, a rational state of the economy, at date 2, can be constructed. Take a neighbourhood, V (in the 2N-space of prices and consumption), of this new rational state. As an extreme example, I will assume that every element of this neighbourhood is also a rational state. Further, suppose that the matrix of partials, of consumption when young, at date 1, with respect to the commonly held theory of price formation, is nonsingular, for the chosen pair of rational states. Notice, this ensures that the rational theory is locally unique in the world with certainty. In the world with two sunspots, let the initial state of nature be s = 1. Further, let the state of the economy, at date 1, and the state of the economy at date 2, state of nature s, be as in the world of certainty (for both states of nature, s). As mentioned, above, this generates a rational expectations equilibrium. Now, suppose the commonly held theory of price formation, at date 1, is slightly modified. In particular, suppose the price vector, at date 2, state 1, is arbitrarily altered. Because the matrix of partials is nonsingular, there exists a price vector, for date 2, state 2, that will ensure the aggregate consumption of the young, at date 1, remains unaltered (see, for example, Burnell, 1990). Further, the price vector, at state 2, will not equal the price vector, at state 1. Hence, two new states of the economy, at date 2, can be found. As these two new states will both be elements of V, a new rational theory of price formation, for which sunspots matter, has been found, associated with the given state of the economy, at date 1. In fact, because the price vector, at date 2, state 1, can be arbitrarily altered, the set of rational theories has dimension N. Hence, in a world of uncertainty and incomplete asset markets, sufficient conditions can be found to ensure a continuum of rational theories.

The assumption that the set of rational states, in V, has dimension 2N is not necessary to establish the existence of a continuum of rational theories. Suppose, instead, the dimension is δ + 1. With two states of nature, the set of rational states, at date 2, has dimension $2\delta + 2$. Hence, the set of rational states, with a given money stock, has dimension 2δ, while the set of all states, in V, with a given money stock, has dimension 4N - 2. Now, the set of theories that ensure market clearing, at date 1, has dimension N. It therefore follows that the set of states of the economy, at date 2, associated with market clearing, at date 1, also has dimension N. In general, a continuum of rational theories will exist if $2\delta + N > 4N - 2$ (see Burnell, 1990).

It should be noted that this result has nothing to do with sunspots. If the states of nature affect endowments or utility functions, the result still holds. Hence, in a world of uncertainty and incomplete asset markets, a co-ordination story may be necessary to explain why every young agent chooses to hold the same rational theory.

V Summary.

In the basic Arrow-Debreu model with more than one delivery date, the equilibrium concept requires agents to believe they can correctly predict their future preferences (as a function of their consumption today), endowments and technology. To ensure mutual gains cannot be made by reopening commodity markets in the future, every participant in the economy - includung the auctioneer - must believe that every other agent is rational and holds a correct theory of the future. Finally, to ensure market clearing in the future, the auctioneer must be able to punish agents who fail to honour their contracts.

In a world with two trading dates, a rational expectations equilibrium requires agents, at date 1, to believe they can correctly predict future prices and the future profits of firms. Ultimately, this would seem to require every participant - including the auctioneer - to believe they know the aggregate excess demand function for the economy, when all agents hold a common theory of price formation. Further, every participant must believe all other participants know the same function. If asset markets are incomplete, sufficient conditions can be found to ensure a continuum of rational expectations equilibria. However, given commodity and asset prices at date 1, each rational theory of price formation will, in general, be locally unique. Finally, even if the number of assets is equal to the number of intrinsic states, the addition of sunspots (or extrinsic states of nature) enables sufficient conditions to be found that ensure sunspots matter, for almost every equilibrium.

In pure exchange, overlapping generations models with uncertainty and asset markets, a rational expectations equilibrium requires every agent, born at date t, to believe they can correctly predict prices at date t + 1, for every possible state of nature. Suppose the asset markets are incomplete. Given the state of nature, prices and consumption of the old at date t, sufficient conditions can be found to ensure the existence of a continuum of rational theories of price formation. As a rational expectations equilibrium requires every agent to believe one, and only one, rational theory of price formation is correct, a serious co-ordination problem emerges. A theory becomes true only if agents believe in it alone and attatch zero weight to every other potentially true theory. The benevolent auctioneer can now influence the welfare of society by co-ordinating the beliefs of agents.

BIBLIOGRAPHY

Arrow, K. J., Le role des valeurs boursieres pour la repartition la meilleure des risques, *Econometrica*, Colloques Internationaux du C.N.R.S., **11** (1953), 41 - 47.

__________, Rationality of self and others in an economic system, in R. M. Hogarth and M. W. Reder, *Rational choice*. Chicago: The University of Chicago Press, 1987.

Arrow, K. J. and G. Debreu, Existence of an equilibrium for a competitive economy, *Econometrica*. **22** (1954), 265 - 290.

Brandenburger, A. and E. Dekel, The role of common knowledge assumptions in game theory, in F. H. Hahn (ed), *The economics of missing markets, information and games*. Oxford: Oxford University Press, 1989.

Burnell, S. J., Sunspots and rationality, *mimeo*, 1990.

Cass, D., Sunspots and incomplete financial markets: The leading example, in G. Feiwel (ed.), *The economics of imperfect competition and employment*. London: Macmillan, 1989.

Debreu, G., *Theory of value*. New York: Wiley, 1959.

________, Economies with a finite set of equilibria, *Econometrica*. **38** (1970), 387 - 392.

________, Excess demand functions, *J. of Mathematical Economics*. **1** (1974), 15 - 21.

J. Dreze, Investment under private ownership: Optimality, equilibrium and stability, in J. Dreze (ed), *Allocation under uncertainty: Equilibria and optimality*. New York: Wiley, 1974.

J. D. Geanakopolos, An introduction to general equilibrium with incomplete asset markets, *J. of Mathematical Economics*. **19** (1990) 1-38.

Geanakopolos J. D. and A. Mas-Colell, Real indeterminacy with financial assets, *J. of Economic Theory*. **47** (1989), 22 - 38.

Geanakopolos J. D. and H. M. Polemarchakis, Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete, in W. P. Heller, R. M. Starr and D. A. Starrett (eds) *Uncertainty, information and communication*. Cambridge: Cambridge University Press, 1986.

Kehoe, T. J. and D. K. Levine, Regularity in overlapping generations exchange economies, *J. of Mathematical Economics*. **13** (1984), 69 - 93.

____________________________, Comparative statics and perfect foresight in infinite horizon economies, *Econometrica*. **53** (1985), 433 - 453.

Makowski, L., A characterisation of perfectly competitive economies with production, *J. of Economic Theory*. **22** (1980), 208 - 221.

Ostroy, J., The no-surplus condition as a characterisation of perfectly competitive equilibrium, *J. of Economic Theory*. **22** (1980) 183 - 208.

Radner, R., Competitive equilibrium under uncertainty, *Econometrica*. **36** (1968), 31 - 58.

Werner, J., Equilibrium in economies with incomplete financial markets, *J. of Economic Theory*. **36** (1985), 110 - 119.