Nurturing genius: the childhood and youth of Kelvin and Maxwell

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William Thomson and James Clerk Maxwell, nineteenth century natural philosophers, were friends and colleagues (Thomson was Maxwell's senior by seven years). This historical note gives a description of their early lives, with emphasis on the influence of their fathers and of Cambridge on their development.

Recent research on electrostatics got me into working contact with the early contributions of James Clerk Maxwell and William Thomson (later Baron Kelvin of Largs, and usually referred to as Kelvin). I read their biographies, and was struck by the remarkable similarities in their childhood and youth. Both were Scots, both lost their mothers at an early age, both had fathers who nurtured them intellectually and were ambitious for their career.

This note is mainly about William's and James' childhood and youth, and comes to a natural stop at their respective completions of the Cambridge Tripos examination. Only a brief catalogue of their later careers is given. Some of their electrostatic researches are discussed in my Author's Note at the end.

William Thomson, Lord Kelvin (1824–1907)

James Thomson, William's father, taught mathematics and geography at the Royal Belfast Academical Institution. William was born in Belfast. His mother Margaret (née Gardner) died in 1830 when William was six. His father became Professor of Mathematics at Glasgow in 1832, and the family of four boys and two girls moved there. An elder brother James (1822–1892, FRS) trained as an engineer, and became Professor of Engineering at Glasgow.

James Thomson senior was a man of wide interests, 'capable on emergency of teaching the University classes in classics'. His books cover an amazing range: *A treatise on arithmetic in theory and practice* went to seventy-two editions; other titles include *Introduction to modern geography*, *The romance of the heavens*, *Elements of plane and spherical geometry*, *Euclid's elements of geometry*, *Algebra*, and *Introduction to the differential and integral calculus*.[1, pp. 6, 7] And this from a farmer's son!

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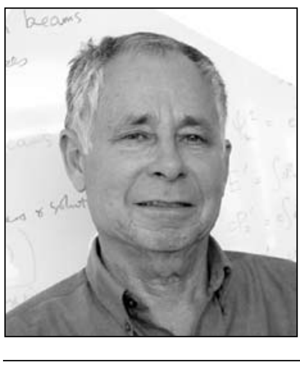
After Margaret died, the father taught James and William 'the use of the globes' and Latin [1, p. 6]. James and William were allowed to attend informally their father's lectures at the University. One of those present at the Junior Mathematics Class later recalled to Kelvin, 'As a mere child you startled the whole class, not one of whom could answer a certain question, by calling out: 'Do, papa, let me answer.' [4, p. 5] James and William matriculated at the University of Glasgow at ages 12 and 10, respectively, in October 1834. William '...carried off two prizes in the Humanity Class; this before he was eleven.' In the next session young William got prizes in Natural History and in Greek [1, pp. 8, 9]. And so on. Kelvin recalled (in 1907), 'A boy should have learned by the age of twelve to write his own language with accuracy and some elegance; he should have a reading knowledge of French, should be able to translate Latin and easy Greek authors, and should have some acquaintance with German. Having learned thus the meaning of words, a boy should study Logic'. In Natural Philosophy, under Professor Meikleham, William read *Mécanique analytique* of Lagrange and *Mécanique céleste* of Laplace [1, pp. 11, 12]. In 1839 he attended the Senior Natural Philosophy class taught by the professor of Astronomy, J.P. Nichol, who introduced William to Fourier's *Théorie analytique de la chaleur*. 'I asked Nichol if he thought I could read Fourier. He replied 'perhaps'. ... on the 1st May [1840] ... I took Fourier out of the University Library; and in a fortnight I had mastered it – gone right through it.' [1, p. 14]. William was fluent in French: in the summer of 1839 the family went to London, and then on to Paris, where the boys were left (in the charge of a trusted servant) for about two months to learn French. The father wished them to learn German also; for two months the whole family took lessons in German, and on 21 May 1840, Professor Thomson and his six children (William was 16, the youngest boy Robert was 11) left Glasgow for Liverpool, London, and then by steamer to Rotterdam. William's diary has the entry, 'Reached the bar at



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the mouth of the Maas, near Brill, at about 4½ o'clock in the morning, where we had to lie till 10. The vessel rolled greatly from side to side, but the rolling was intermittent, as every two or three minutes it calmed down and then rose again with perfect regularity. This probably arose from two sets of waves of slightly different lengths coming in in the same direction from two different sources'. The family visited the Hague (the diary notes a visit to the Museum to see a stuffed mermaid!), Delft, Düsseldorf, Bonn, Cologne, Frankfurt am Main (where they stayed till 2 August), then on to Baden, from where the brothers James and William went on a walking tour of several days through the Black Forest. The family returned to Glasgow in early September. Certainly an educational trip, much to the credit of Professor Thomson. But young William did not spend all his time practising German: he had taken his Fourier with him, and surreptitiously read it in the cellar. 'When my father discovered it he was not very severe upon me'. [1, pp. 16–18]. A text by Kelland, *Theory of heat*, 1837, stated that the Fourier expansions were 'nearly all erroneous'. William found, while at Frankfurt, the cause of the misunderstanding. This resulted in his first publication *On Fourier's expansions of functions in trigonometrical series* [8, Vol. 1, pp. 1–9].

In April 1841, William entered Peterhouse in Cambridge. (He had purposely avoided taking a degree at Glasgow, so as to be able to enter Cambridge as an undergraduate.) The choice of Peterhouse had much to do with the presence there of Dr William Hopkins, a geophysicist and famous as a Mathematics Tripos tutor. The Maths Tripos was an examination conducted (in Thomson's day) over six days, each with 5½ hours of hard writing, covering mathematics and the mathematical aspects of physics. To be placed high on the list, especially to be Senior Wrangler or Second Wrangler, was the making of a career. Hence the three years of intense preparation and tutoring. Young William, 17 when he entered Cambridge, was mature enough to realise the importance of the Tripos, and organise his life accordingly. He soon saw that there was a separation at Peterhouse into the classes of 'rowing men' and 'reading men'. 'All my friends are among the latter class, and I am gradually dropping acquaintance with the former ... even to know them is a very troublesome thing if you want to read, as they are always going about troubling people in their rooms'. (Letter to his father, 12 December 1841 [1, pp. 32–33].) However, together with another undergraduate, William bought a single sculling boat for £7. His father was surprised at not having been consulted, and urged William to 'Use *all economy* consistent with respectability. Be most circumspect about your conduct and about what acquaintance you form. You are young: take care you be not led to what is wrong. A false step now, or the acquiring of an improper habit or propensity, might ruin your life.' [1, p. 37]. William made good use of the boat, and rowed on the river Cam with another 'reading' man, G.W. Hemming of St. Johns, Senior Wrangler in 1844. His sister Elizabeth wrote on 27 February 1842 that 'papa' was reconciled to the purchase of the boat, much to the relief of William, who wrote to his father on 14 April 1842 that, 'The sculling is going on with great vigour, and is keeping me in excellent preservation. ... I find that I can read with much greater vigour than I could when I had no exercise but walking in the inexpressibly dull country round Cambridge'. [William was used to a more varied topography than the flat land surrounding Cambridge.]

During the summer vacation of 1842 the family were at Knock Castle (three miles from Largs, on the Firth of Clyde). There William wrote a paper *On the linear motion of heat* [8, pp. 10–15] in which he discusses solutions of the one-dimensional equation for the flow of heat, namely $\partial_t T = \partial_x^2 T$, where $T(x,t)$ is the temperature, in the form:

$$T(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} d\alpha \, e^{-\alpha^2} f\left(x + 2\alpha\sqrt{t}\right), \qquad T(x,0) = f(x)$$

Another paper, *On the uniform motion of heat in homogeneous solid bodies, and its connection with the mathematical theory of electricity* [9, pp. 1–14] was written that summer. Not bad for an undergraduate of 18!

Back at Cambridge in October 1842, William began his training under the tutor Hopkins, with the aim focused on the Tripos examinations in the Senate House in January 1845. He won a mathematics prize of £5, which he proposed to spend on an *Illustrated Shakespeare*, but his father preferred him to buy Liouville's *Journal de Mathématiques*.

James Thomson's paternal care was ever focused on his son's long-term prospects: Dr Meikleham, the Professor of Natural Philosophy at Glasgow, was ill. If only he could last till William had completed the Tripos (and got the laurels of a Wrangler), William might succeed him – a natural wish for the father, to have his son join him as a professor at his University. On 9 April 1843, Professor Thomson writes to William that Dr Meikleham is better; he adds '...you must take care not only to *do* what is right, but to take equal care always to *appear* to do so. A certain [Professor of Moral Philosophy] here has of late been talking a good deal about the vice of the English Universities, and would no doubt be ready to make a handle of any report or gossip he might pick up.' [1, p. 53]. The next letter detailed the requirements of the chair of Natural Philosophy, which included skill in experiments. This he urges William to attain. William, ever cooperative, replies that in his spare time he is reading *Cours de Physique* by Lamé, 'which is an entirely experimental work'. James Thomson (4 May 1843) writes of the probable votes in an election of Dr Meikleham's successor, and adds 'Take care to give a *certain gentleman here* (who, as to private affairs, is more nearly omniscient than anyone I have known) no handle against you. Avoid boating parties of in any degree of a disorderly character ... as scarcely anything of the kind could take place, even at Cambridge, without him hearing of it.' [1, pp. 57, 58]. And William did avoid boating parties and any scandal, but he did row in the eights for Peterhouse, and won the single sculls [1, pp. 58–62]. He also played the cornet, and was one of the founding members of the Cambridge Musical Society.

The saga of the chair of Natural Philosophy continued, with Dr Meikleham becoming ill and recovering. On 20 April 1844 Professor Thomson urged William to 'Keep the matter in mind, therefore, and think on every way in which you might be able to get efficient testimonials ... Do not relax your preparation for your degree. I am always afraid some unknown or little heard of opponent may arise. Recollect, too, that you might be thrown back by illness, and that you ought therefore be *in advance* with your preparation. Above all, however, take care of your health.' William replied on the 22nd: 'I am very sorry to hear about Dr Meikleham's precarious state ... it is certainly very much to be wished that he should live till after the commencement of next session.'

Preparation for the Tripos was to continue during the long vacation, when Hopkins would go with a party of reading men to Cromer, Norfolk. William wished to go too, entailing extra expense for his supportive father, who agrees to the request. But soon William writes from Cromer (13 June 1844): ' My Dear Father – I have again to write to you on the same pleasant business that I had to write to you about so lately, which is to say that my money is again all gone.' (Details of his expenses follow.) [1, p. 80]. Later (12 October 1844), 'papa' sent his son the halves of bank notes for £100, noting that the three years' expenditure was now £774/6/7, and asked 'How is this to be accounted for? Have you lost money or been defrauded of it ...? ... you *must* exercise the *strictest* economy that shall be consistent with decency and comfort.' Lest the readers think 'papa' a cheapskate, let me remind them of inflation: the value of the pound has diminished by a factor of about 72 between 1844 and 2001 [10], so in present currency Dr Thomson's £774 is approximately £60,000.

The work of the 'reading party' entailed Dr Hopkins setting examination papers and discussing the students' answers with them. It went on for two months. After the reading party ended, Thomson and a fellow Scottish student 'took a boat and rowed out to sea, and intercepted the G. N. S. steamer *Trident*', which took them to Edinburgh! [1, p. 82] Railways were only just being established (the Edinburgh to Glasgow line opened in 1845), and travel was a major undertaking.

Let us fast-forward now to the ordeal of the Senate House examinations, set to begin on 1 January 1845. The 'Wrangler' contestants had trained like Olympic athletes for this six-day event. Nor was this the end, because the Smith's Prize (another week of examinations) followed soon after. And the results were: Parkinson of St. John's, Senior Wrangler, Thomson of Peterhouse, Second Wrangler. The disappointment of William's family and friends was mitigated by the fact that Thomson was judged clearly better in the two Smith's Prizes awards, Parkinson second.

Dr Thomson continued to advance his son's education (and the prospects of the Chair in Natural Philosophy at Glasgow) by funding a trip to Paris in early 1845. William went with introductions to Arago, Biot, Babinet, Cauchy and Liouville. He presented himself to Liouville, with whom he met often and became friends. He also met Sturm and Foucault, that is almost all of the living French scientists (Laplace, Legendre, Poisson, and Fresnel were no longer). Biot introduced him to Regnault, the professor of Natural Philosophy at the Collège de France, and researcher into the physics of heat engines. William worked with Regnault in his laboratory, met Liouville and Cauchy often, and in his spare time [1, p. 128], 'I have been reading Jacobi's *Nova Fundamenta* and Abel's 1st memoir on *Elliptic Functions*, but have been rather idle on the whole'. Indeed!

After four and a half months in Paris, William returned to Cambridge. At the British Association meeting he met Faraday. Soon after, he was elected Foundation Fellow of Peterhouse, this being worth about £200 per annum, with rooms in College. This post he held till his marriage in September 1852. In May 1846 the chair of Natural Philosophy at Glasgow became vacant by the death of Professor Meikleham. The timing was perfect. William and his father quickly gathered testimonials and information about other possible candidates. There were five other applicants. Among the testimonials supporting William Thomson were those from Arthur Cayley, George Boole, J.J. Sylvester, G.G. Stokes, M. Regnault, and M. Liouville. To the printed pamphlet of 28 pages containing the testimonials, given to the electors, Thomson added an appendix listing his published papers, twenty-six of them. William was 22 at the time of his appointment in October 1846, and kept the chair till his retirement in 1899.

Professor William Thomson, 1846

Our description of young William Thomson's nurture and development stops here. He was not just a mathematically gifted child – he had the great advantage of a highly intelligent and energetic father, dedicated to his son's advancement. In Cambridge, he had the support of the best tutor, working in possibly the best environment for mathematics and the natural sciences in Britain. In Paris, he met and worked with the foremost mathematicians and scientists of France. And he was sensible enough to make full advantage of these opportunities, through continuous and vigorous use of his exceptional brain.

James Clerk Maxwell (1831-1879)

James' father was born John Clerk, adding the name Maxwell upon inheriting the estate of Middlebie. He practised law in Edinburgh and seemed set on a quiet batchelorhood until he met and married Frances Cay. A child (Elizabeth) died in infancy, and James was born, when his mother was nearly forty, at 14 India Street, Edinburgh [11, pp. 2–3]. Frances was of a 'sanguine active temperament', and energised John to develop the estate of Middlebie and enlarge *Glenlair*, their home. John had a 'persistent practical interest in *all useful processes*'; he made a special last for shoes (square-toed) for himself and later for James, and planned the outbuildings of *Glenlair*, down to the working plans for the masons [11, pp. 7–9]. Even before he was three, little James likewise showed a practical interest in the world. A letter from Frances to her sister, Jane Cay, gives the picture: 'He is a very happy man ... has great work with doors, locks, keys, etc., and "Show me how it doos" is never out of his mouth. He also investigates the hidden course of streams and bell-wires ... he drags papa all over to show him the holes where the wires go through.' [11, p. 27]. Throughout his childhood the constant question was 'What's the go o' that? What does it do?' If not satisfied with an answer he would ask, 'But what's the *particular* go of it' [11, p. 28]. His great love was the outdoors, of streams and ponds and the frogs that inhabited them [11, pp. 33–34]. With his first cousin, Jemima Wedderburn, who was eight years older, he produced an animation of a tadpole wriggling from its egg and changing into a swimming frog [11, p. 37].

James was educated by his mother until she died of abdominal cancer when he was eight. After his mother's painful death in December 1839, Mr Maxwell hired a local lad to tutor James at home. 'The boy was reported slow at learning, and Miss Cay after a while discovered that the tutor was rough' [11, p. 41]. Just as well she did: his friend and biographer Lewis Campbell describes the 'roughness' (being hit on the head by a ruler, and having ears pulled till they bled), and the lifelong effect this had on James [11, p. 43].

So Mr Clerk Maxwell sent the boy of 10 to the Edinburgh Academy. He lived with his father's sister, Mrs Wedderburn, with occasional stays with his mother's sister, Miss Cay. His first day at school was tough: in his gray tweed jacket and square-toed shoes, he was a target for ridicule and worse. He returned home 'with his tunic in rags ... his neat frill [collar] rumpled and torn ...' [11, pp. 49–50]. His aunts made sure his dress conformed more to the norms, but his nickname 'Dafty' stuck with him. Places in class were allotted according to performance, and James was initially among the rowdy boys, who naturally made things worse for him. For the first two years or so, school was something to endure. Fortunately he had the warm refuge of his aunt's home at 31 Heriot Row, and its good library, plus the occasional visits of his father, when they would explore Edinburgh together. The love between father and son is clear in the letters reproduced in Lewis Campbell's biography. In a letter of 19 June 1844, addressed to 'My Dear Father', and signed 'Your most obt. servt. Jas. Alex. McMerkwell' (an anagram, decoded by numbers underneath), he remarks after news of swimming and other outings 'I have made a tetra hedron, a dodeca hedron and 2 more hedrons that I don't know the wright names for.' [11, p. 60]. Campbell notes that they had not yet begun geometry.

At school he excelled in Scripture, Biography, and English, and discovered that Latin and Greek were worth learning. At about this time, Lewis Campbell joined the school, and began a lifelong friendship. Lewis lived at 27 Heriot Row, and the two boys were continually together for about three years. 'We always walked home together, and the talk was incessant, chiefly on Maxwell's side. Some new train of ideas would generally begin just when we reached my mother's door. He would stand there holding the door handle, half in, half out ... till voices from within complained of the cold draught, and warned us that we must part.' [11, p. 68].

By July 1845 young James was coming into his own, with prizes for English and English Verse, and the Mathematical Medal. His father now 'became more assiduous than ever in his attendance at meetings of the Edinburgh Society of Arts and Royal Society, and took James with him repeatedly to both.' [11, p. 73]. A member of the Society of Arts, D.R. Hay, had written a book on *First principles of symmetrical beauty*; one of the problems in it was how to draw a perfect oval. James generalised the equation of an ellipse, $r_1 + r_2 = 2a$ (r_1 and r_2 are distances from the two focal points to a point on the ellipse, $2a$ is the length of the major axis), to curves which satisfy $mr_1 + nr_2 = \text{constant}$. With Mr Maxwell's skilled promotion of this work, the result was James' first paper, *On the description of oval curves* [12, pp. 1–3], which was communicated to the Royal Society of Edinburgh by Professor J.D. Forbes in 1846. Professor Forbes took Maxwell under his wing, and they became lifelong friends. As it happened, the curves were not new, having been described by Descartes, and their optical properties considered by Newton and Huygens, but Maxwell's practical construction by means of pins and string was new. And what illustrious company for a schoolboy of fifteen!

This paper and his other manuscripts on ovals can be found in the *Scientific letters and papers*, [14, pp. 35–67]. Maxwell was now launched into mathematical and scientific inquiry. His second published paper (1849) was *On the theory of rolling curves* [12, pp. 4–29], in which he already shows a mastery of plane differential geometry. Next, in 1850, came *On the equilibrium of elastic solids* [12, pp. 30–73], 'an astonishing achievement for a 19-year-old working almost entirely on his own. The mathematics went hand-in-glove with his experiments on polarised light ... He set out for the first time the general mathematical theory of photoelasticity...' [15, p. 32]. By this time, James was at Edinburgh University, which he had entered at seventeen. P.G. Tait, who was a school friend of Maxwell's and later a collaborator with Kelvin on their *Treatise on natural philosophy*, was one of James' chief associates at Edinburgh University, but stayed for only one session, going on to Peterhouse, Cambridge, in 1848.

Maxwell went to Cambridge also, but not till 1850. Campbell remarks [11, p. 114] '... it is perhaps to be regretted that he did not go to Cambridge at least one year earlier. His truly sociable spirit would have been less isolated, he would have gained more command over his own genius ...'. Eventually his father was persuaded, and James went to Peterhouse, but transferred to Trinity College to improve his chances of a fellowship. Maxwell's tutor in preparation for the Tripos was the same William Hopkins whom we had met earlier as William Thomson's tutor. Here is Hopkins' view of Maxwell, as recorded by a Cambridge contemporary: '... he is unquestionably the most extraordinary man [Hopkins] has met with in the whole range of his experience; ... it appears impossible for Maxwell to think incorrectly on physical subjects; that in his analysis, however, he is far more deficient; ... a great genius, with all its eccentricities ... one day he will shine as a light in physical science ...' [11, p. 133].

Unfortunately the letters James wrote as an undergraduate to his father from Cambridge are lost. His father's letters naturally seek his son's advancement: 'Have you called on Profs. Sedgwick at Trin., and Stokes at Pembroke? If not, you should do both. ... Provide yourself with cards.' [11, p. 150] James got a scholarship from Trinity College in April 1852. At the scholars' table he was in his element, with free debate on almost any topic. He was elected to the Select Essay Club, a discussion group of twelve students who were known as the Apostles. Maxwell's essays delivered to the Apostles (Chapter VIII of [11]) have titles such as *What is the nature of evidence* *of design*, which begins 'Design! The very word ... disturbs our quiet discussions about *how* things happen with restless questionings about the *why* of them all.' Another essay *Idiotic imps* is about pseudo-science (then called Dark Science), which Maxwell exposes and analyses. Yet another has the intriguing title, *Has everything beautiful in Art its original in Nature*? A serious late essay, from February 1856, is on analogies: *Are there real analogies in nature?* We need both data and theory to make sense of the world: 'The dimmed outlines of phenomenal things all merge ... unless we put on the focussing glass of theory and screw it up sometimes to one pitch of definition, and sometimes to another, so as to see down into different depths ...' In the same essay, Maxwell remarks on space and time: '... space has triple extension, but is the same in all directions, without behind or before, whereas time extends only back and forward, and always goes forward.' The arrow of time, which Maxwell's statistical physics was later to clarify!

In the midst of preparations for the Tripos exams, James took a few days of the 1854 Easter vacation, to stay at Birmingham with a friend. His father wrote [11, pp. 7, 168] 'View, if you can armourers, gunmaking and gunproving – swordmaking and proving – *Papier-mâchée* and japanning – silverplating by cementation and rolling – ditto, electrotype – Elkington's works – Brazier's works, by founding and by striking out dies – turning – spinning teapot bodies in white metal, etc – making buttons of sorts, steel pens, needles, pins and any sorts of small articles which are curiously done by subdivision of labour and by ingenious tools ... foundry works, engine-making ... If you have had enough of the town lots of Birmingham, you could vary the recreation by viewing Kenilworth, Warwick, Leamington, Stratford-on-Avon, or such like.' James began with the glassworks.

Maxwell now faced the trial of the Senate House examinations – in his year, five days of 5½ hours each. Ever solicitous and practical, his father wrote 'You will need to get muffettees for the Senate-Room. Take your plaid or rug to wrap round your feet and legs.' James was Second Wrangler, E.J. Routh of Peterhouse Senior Wrangler. They were declared equal as Smith's Prizemen.

Maxwell with his colour wheel, circa 1855

In October 1855, James Clerk Maxwell was elected Fellow of Trinity College. He had supported himself by taking private pupils, but this could now stop. Apart from teaching third-year hydrostatics and optics, he was free to do research. He was now 24. He left Cambridge in 1856 to take up the chair of Natural Philosophy at Aberdeen, then was Professor at King's College, London, from 1860 to 1865, when he resigned to live and work at *Glenlair*. After Kelvin and Helmholtz declined the offer, Maxwell became the first Cavendish Professor of Physics at Cambridge in 1871. He had but eight years to live. He died in 1879 of abdominal cancer, aged 48, at nearly the same age that his mother had died of the same type of cancer.

We are fortunate in having a warm and affectionate biography by his friend Lewis Campbell. Especially moving are his depictions of James' childhood and adolescence, and of his early death. We admire his works, and with this biography we can also love him.

Epilogue

William Thomson and James Clerk Maxwell both achieved greatness; it was certainly not thrust upon them. However, both were fortunate in their fathers, in more than their genetics. And their fathers were fortunate in them: in a letter anticipating James' 21st, Mr Maxwell says 'I trust you will be as discreet when Major as you have been while Minor', quoting Proverbs x.1 [A wise son maketh a glad father.] Both sons showed remarkable good will and cooperated fully with their fathers' guidance and instruction. This in contrast to much modern behaviour, and also to that of the musical genius Wolfgang Amadeus Mozart, who eventually rebelled against his father Leopold. Thomson and Maxwell senior never had to face Leopold's tragedy of having a cherished child spurn them.

In the addition to the wonderful love, instruction and support from their fathers, they each had the support of family, in Maxwell's case particularly the comfort of the Aunts. In the wider sphere, we should also note that Scotland had been important in the European enlightenment and that the rates of literacy were exceptionally high. William and James grew up in a culture with a strong work ethic and widespread respect for knowledge, a powerful combination.

Finally, they both had the great advantage of their Cambridge experience. This environment suited both, matured them, and gave them lifelong connections with some of the brightest minds then living.

Author's Note

Victoria University physicists Pablo Etchegoin and Eric Le Ru have refined surface-enhanced Raman scattering to such an extent that they are able to detect single molecules [16]. This remarkable feat is accomplished by using the enhancement of an external electric field (provided by an intense laser beam) in the gap between two close conducting particles. The simplest applicable model is that of two conducting spheres in a steady (DC) external field, which had been solved by Maxwell and others [17–19]. The solution is exact, and in the form of infinite series which converge rapidly when the sphere separation *s* is comparable to or larger than the radii of the spheres. However, the field enhancement is large when the sphere separation *s* is small compared to the sphere radii, and there the series converge more and more slowly as *s* decreases. This is precisely the physically interesting limit, that utilised by Pablo and Eric to such good effect. So we have the unhappy situation where an exact theory fails to deliver just where it is needed.

I got interested, and spent considerable time investigating the exact series, their integral equivalents and especially the logarithmic terms which appear at small *s*. What started as an exploration of field-enhancement in the limit of close approach of the two spheres [20a, d] grew to encompass the capacitance of two spheres (at the same potential, or with equal and opposite charges) [20b], and the polarisabilities (longitudinal and transverse) of a two-sphere system [20c]. In all cases, terms logarithmic in the sphere separation *s* appear in the formulae.

Maxwell had approached the problem from the other end: he obtained, for quantities related to the capacitance coefficients C_{aa}, C_{ab} and C_{bb} of two spheres of radii *a* and *b* and separation of centres *c* (with *c* and *s* related by $c = a + b + s$), expansions in reciprocal powers of *c*. There is the remarkable Section 146 of his *Treatise on Electricity and Magnetism* [17], in which he matches spherical harmonic expansions about the two sphere centres to obtain ℓ, *m* and *n* coefficients (defined below) as series in reciprocal powers of *c*. Section 146 is seven pages of formulae, in which the calculation is carried to the twenty-second reciprocal power of *c*! As is well-known, series expansions of this type get more complex the higher the order. Maxwell had no computing aids, not even a mechanical calculating machine. I checked all the coefficients in his formulae (using computer algebra, of course) and found all were correct. This attests to Maxwell's amazing ability to carry through very long and intricate calculations, but also raises the question: why did Maxwell do this enormous amount of work? His coefficients ℓ, *m* and *n* give the total electrostatic energy of the two spheres, carrying charges Q_a and Q_b, as

$$W = \tfrac{1}{2}\ell Q_a^2 + mQ_aQ_b + \tfrac{1}{2}nQ_b^2 \qquad (1)$$

The coefficients ℓ, *m* and *n* are related to the capacitance coefficients C_{aa}, C_{ab} and C_{bb}:

$$\ell = \frac{C_{bb}}{C_{aa}C_{bb} - C_{ab}^2}, \qquad m = \frac{-C_{ab}}{C_{aa}C_{bb} - C_{ab}^2}, \qquad n = \frac{C_{aa}}{C_{aa}C_{bb} - C_{ab}^2} \qquad (2)$$

The total energy expanded in reciprocal powers of the distance between sphere centres *c* begins [21]

$$W = \frac{Q_a^2}{2a} + \frac{Q_b^2}{2b} + \frac{Q_aQ_b}{c} - \frac{Q_a^2b^3 + Q_b^2a^3}{2c^4} - \frac{Q_a^2b^5 + Q_b^2a^5}{2c^6} + \ldots \qquad (3)$$

The first two terms are the self-energies of the two charged spheres, the third is the Coulomb energy, the fourth and fifth are due to mutual polarisation of the two spheres. Maxwell had the information to give the energy up to terms of order c^{-22}, but he did not do that. Why not? And, why do all that work and give the results in his *Treatise*? My guess is that (i) Maxwell was looking for a pattern in the series, and hoped to sum them completely if he found the pattern; and (ii) he wanted to compare experimental results on the force between two charged spheres with theory, and needed all these terms to do so. There is no hint in Section 146 as to his reasons. Perhaps neither of (i) or (ii) came to fruition, but he wanted the results of his labours to be available to others.

Preceding Maxwell's work were the Kelvin papers of 1845 and 1853 [9]. William Thomson was 21 when the earlier of these was published. It deals with the force between an earthed sphere and a charged sphere, and uses the method of images that he invented. He obtained an infinite series for the force $F(c)$, in which successive numerators and denominators of terms in the series are related by recurrence relations. It is now easy to write down the complete expression for the energy [21]: if sphere *a* carries charge Q_a, and sphere *b* is earthed, the electrostatic energy, and the force between the spheres, are given by

$$W(c) = \frac{Q_a^2}{2C_{aa}(c)}, \qquad F(c) = -\partial_c W(c) \qquad (4)$$

So, if we know the capacitance coefficient C_{aa}, a simple differentiation will give us the force. Incidentally, the inverses of the relations (2) are

$$C_{aa} = \frac{n}{\ell n - m^2}, \qquad C_{ab} = \frac{-m}{\ell n - m^2}, \qquad C_{bb} = \frac{\ell}{\ell n - m^2} \qquad (5)$$

so the Maxwell coefficients ℓ, *m* and *n* could be used directly to give the force as

$$F(c) = -\tfrac{1}{2}Q_a^2\,\partial_c(\ell - m^2/n) \qquad (6)$$

The force is always attractive, as is to be expected since the charge induced on the earthed sphere *b* has opposite sign to Q_a. The force increases as the separation *s* between the spheres decreases, and in fact diverges as *s* tends to zero.

A more interesting but more difficult problem is that of the force between two charged spheres (Kelvin 1853 [9]). The Maxwell expansion in reciprocal powers of *c* fails at close approach, and in particular at contact, when the spheres are at a common potential. They share the charge; the force is clearly repulsive, whatever the sign of this charge. Again Kelvin used his method of images, and again obtained an infinite series for the force. For spheres of equal radii, in contact, his expression for the force is proportional to a double series,

$$\begin{array}{ccccccccccc}
\frac{1}{2^2} & - & \frac{1.2}{3^2} & + & \frac{1.3}{4^2} & - & \frac{1.4}{5^2} & + & \frac{1.5}{6^2} & - & \ldots \\
 & - & \frac{2.1}{3^2} & + & \frac{2.2}{4^2} & - & \frac{2.3}{5^2} & + & \frac{2.4}{6^2} & - & \ldots \\
 & & & & \frac{3.1}{4^2} & - & \frac{3.2}{5^2} & + & \frac{3.3}{6^2} & - & \ldots \\
 & & & & & - & \frac{4.1}{5^2} & + & \frac{4.2}{6^2} & - & \ldots \\
 & & & & & & & & \frac{5.1}{6^2} & - & \ldots
\end{array} \qquad (7)$$

Kelvin notes that adding by vertical columns gives diverging series, while adding by horizontal rows gives a convergent series, which he sums to $\frac{1}{6}(\ell n 2 - \frac{1}{4})$.

The evaluation of the double sum demonstrates young William's mathematical skill. He expresses the sums of the first, second and third rows respectively as

$$\int_0^1 d\theta \frac{\theta\,\ell n\frac{1}{\theta}}{(1+\theta)^2}, \qquad -2\int_0^1 d\theta \frac{\theta^2\,\ell n\frac{1}{\theta}}{(1+\theta)^2}, \qquad 3\int_0^1 d\theta \frac{\theta^3\,\ell n\frac{1}{\theta}}{(1+\theta)^2} \qquad (8)$$

[For those interested in the mathematics: set θ = e^{-x} to convert $\int_0^1 d\theta \frac{\theta \ell n \frac{1}{\theta}}{(1+\theta)^2}$ to the more familiar $\int_0^\infty dx \frac{x}{(e^x+1)^2}$; then expand in powers of e^{-x} to obtain the sum of the first row.] Noting that $(1 + \theta)^2 = 1 - 2\theta + 3\theta^2 - ...$, William writes the sum of the row sums as the integral

$$\int_0^1 d\theta \frac{\theta \ell n \frac{1}{\theta}}{(1+\theta)^4} \quad (9)$$

which he evaluates without further comment as

$$\frac{1}{6}\left[\frac{\ell n \frac{1}{\theta}}{(1+\theta)^3}(3\theta^2+\theta^3) + \ell n(1+\theta) - \frac{\theta}{(1+\theta)^2}\right]_0^1 = \frac{1}{6}\left(\ell n 2 - \frac{1}{4}\right) \quad (10)$$

A reader who verifies each of these steps will appreciate what is involved, but perhaps not the difficulty of its formulation, and certainly not the complexity of the infinite sets of electrical image charges that it is based on.

Without further discussion, William takes the convergent result as correct! When I first saw this, I wondered how it was that the (mathematically extremely able) young Thomson could be ignorant of Riemann's theorem about conditionally convergent series, namely that they can be summed to any desired result by suitable re-arrangement of terms. The answer lay in chronology of course: Riemann (1826–1866) was a student at Göttingen under Gauss (with a spell at Berlin) from 1846 to 1849, and did not teach till 1854. His paper on the re-arrangement of series was completed in 1853, but not published until after his death in 1866.

In fact the Kelvin result is correct. I have obtained it directly from the properties of the capacitance coefficients, and have generalised the result to spheres of arbitrary radii, at arbitrary separation [21]. But young Thomson's choice of one result from the infinity of possible sums of that double series is the boldest move I have seen in theoretical physics.

P.S. From 1854 there was much correspondence between Maxwell and Thomson, who became friends. The Maxwell letters relevant to electromagnetism are reprinted in [22].

Annotated Bibliography

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