AUSTRALASIAN JOURNAL OF LOGIC

THE LIBERATION ARGUMENT FOR INCONSISTENT MATHEMATICS

Franci Mangraviti

Department of Philosophy I, Ruhr University Bochum

legallyfranci@gmail.com

Abstract

Val Plumwood charged classical logic not only with the invalidity of some of its laws, but also with the support of systemic oppression through naturalization of the logical structure of dualisms. In this paper I show that the latter charge - unlike the former - can be carried over to classical mathematics, and I propose a new conception of inconsistent mathematics - *queer incomaths* - as a liberatory activity meant to undermine said naturalization.

1 Introduction

Classically, there can be no mathematical theorem of the form "*A* and not-*A*" (where *A* is not ambiguous). The idea is that, if such a statement happens to be derived, then something must have gone wrong: one of the assumptions, definitions, or proof steps is to be rejected. Classical logic cements this attitude by including the Explosion rule: from a contradiction, everything follows.

Inconsistent mathematics rejects this perspective. Put very broadly, inconsistent mathematics is any sort of mathematics allowing for genuine inconsistencies without treating them as mistakes, ambiguities, approximations, etc. Inconsistent mathematics as a field of research has not been long of this world, and it has yet to prove its value to the mainstream mathematics community. Most of it fits quite naturally within research in nonclassical mathematical *logic*, to the effect that it provides a test for the expressive and deductive power of many nonclassical logics. Much has also been said of the power of inconsistent mathematics to solve classical philosophical paradoxes [Syl19][Pri06][Web21a]. Interaction with mainstream mathematical concerns, however, has been very limited.

My main goal in this paper is to argue that Val Plumwood's feminist critique of classical logic [Plu93] can be adapted to justify a certain view of inconsistent mathematics as a *corrective* to classical mathematics.[1] Roughly, Plumwood argues that classical logic best approximates the logical structure of *dualisms*, which are a particular kind of dichotomy underlying many forms of systemic oppression; because of this, the naturalization of classical logic contributes to the naturalization of dualisms, which is reason enough to *replace* classical logic. While there are difficulties trying to extend the charge that some classical logical arguments are invalid to classical mathematics, Plumwood's feminist perspective has important consequences for mathematics as well, in the sense that the adoption (implicit or not) of classical logic and the associated philosophical attitudes can be shown to be at the center of much criticism of mathematics on educational, social, and ethical grounds [Bur95][Wag17, ch.4][Ern18]. By featuring less problematic logics and attitudes, inconsistent mathematics can thus be presented as a possible solution to the damage classical mathematics can do, not just in terms of applications, but also as a way of seeing the world.

A rough outline of the argument, which I will call the *liberation argument*, is as follows:[2]

1. Classical mathematics contributes to the naturalization of dualisms, and therefore systemic oppression.

2. The naturalization of dualisms can be undermined by encouraging subversive practices which denormalize and denaturalize classical logic.

3. Thus, inconsistent maths - conceived as the activity of inconsistentizing accepted practices, and therefore creating subversive practices - is justified on grounds of positive social change.

An important upshot of this is a healthy recontextualization of the role of logic in inconsistent mathematics. No longer are inconsistent formalizations the very subject matter of inconsistent mathematics, with all the confusion about interpretation that perspective brings; rather, logical investigations can be an important

[1] To my knowledge, Plumwood never elaborated on the consequences of her feminist views for mathematics. I do not claim she would agree with my conclusions; in fact, I doubt she would, as I will explain later.

[2] A more precise outline can be found at the end of Section 7.

part of the context of discovery, as a way to suggest new inconsistent interpretations of classical work.

The outline of the paper is as follows. First, Section 2 provides a brief, non-technical introduction to the field of inconsistent mathematics and its relationship with classical mathematics. Section 3 introduces the standard relevantist attacks on the validity of classical logic, while Section 4 argues that no such arguments have the power to endanger current mathematical practice. Section 5 presents Plumwood's feminist critique of classical logic, and defends it from some common misunderstandings; most importantly, Section 6 argues that her critique can be carried over to classical *mathematics*. Section 7 then shows how inconsistent mathematics - conceived as a particular kind of activity, *queer incomaths* - could be an answer to Plumwood's worries while avoiding the pragmatic pitfalls of both full-scale replacement and classical recapture. Some possible objections are addressed in Section 8. Finally, Section 9 shows that queer incomaths intersects with other kinds of liberatory proposals in the literature.

2 The inconsistent revolution

First, let me say a few words on the field of studies I am going to drag into this. *Inconsistent mathematics* is a general term for any sort of mathematics taking contradictions seriously. A bit more specifically, we can say that a piece of mathematics is inconsistent if it is *explicitly* the case that it accepts inconsistent theorems, is about inconsistent concepts, or allows proofs to detour through inconsistency.[3] Throughout this paper, practitioners of inconsistent mathematics are called *inconsistent mathematicians*.

To my knowledge, the first programmatic suggestion towards a field of inconsistent mathematics goes back to Florencio G. Asenjo, who proposed the first

[3]Many definitions of inconsistent mathematics in the literature make explicit reference to the underlying logic. This is problematic for a variety of reasons: there is no sharply delimited class of appropriate logics, let alone any obvious privileged choice; the choice of logic does not (usually) force the presence of inconsistencies; and there are alleged examples of inconsistent mathematics relying solely on classical logic (together with some non-logical machinery) [BP04]. My definition is intended to be as inclusive as possible, while the reference to explicitness is meant to exclude cases of "accidental" inconsistency. Still, the exact details do not matter too much for the purposes of this paper.

logic specifically for the purpose of inconsistent mathematics [Ase54].[4] The field has been growing ever since, in many different directions, and by 1995 it was possible to provide a book-length collection of technical results from various areas of mathematics [Mor95]. Nowadays, the list includes so-called paraconsistent set theories trying to either capture the naive conception of set [Res92][Ist17], allow for the inclusion of particular inconsistent sets [CC13], or ground inconsistent analysis [dC00]; inconsistent nonstandard first-order models of arithmetic [MM84][PS08]; inconsistent theories of so-called "impossible pictures" [Mor10]; mathematical foundations for an inconsistent metaphysics [Web21a]; and more.[5]

More often than not, inconsistent mathematics comes with some nonclassical logic to underlie its theories. This is because, in classical logic, there is no distinction between formal theories from which a contradiction follows (*inconsistent*), and formal theories from which everything follows (*trivial*): classically there is only one (deductively closed) inconsistent theory, namely the trivial one. Logics that can draw this distinction are said to be *paraconsistent*: usually this involves at least dropping the classical inference rule of *Explosion*, namely "if A and not-A, then B".

Inconsistent mathematics is not always presented as *revisionary* with respect to current mathematical practice, and there are difficulties in attempting to frame it that way. It is usually open to classical mathematicians to dismiss, replace, or consistently treat any alleged inconsistent entities: this is a common move throughout the history of mathematics, from infinitesimals to sets. Even when it comes to more foundationalist projects, e.g. replacing classical ZFC set theory with some inconsistent naive set theory, the result is often a straightforward *extension* of the classical universe, so there is no pressure whatsoever on the classical mathematician to abandon their bubble. On the other hand, if the universe ends up being incompatible, the way e.g. the intuitionistic continuum is incompatible with the classical one, at least prima facie such inconsistent universes can unproblematically live side by side with their consistent counterparts. This is obvious from a

[4] It is sometimes claimed that there are historical examples of inconsistent mathematics, most notably the early calculus (see e.g. [PSN89]). This is a controversial claim - see [Vic13] for a rebuttal of many of the usual examples - but either way, historical examples are for the most part very different from the contemporary investigations, as the latter do not seem to treat contradictions as a mere accident or temporary step towards future consistentization. Since my interest is in inconsistent mathematics as a particular field of study distinct from mainstream mathematics, I will stick to recent work.

[5] For a proper first introduction to the field, see [Web22]. For a more detailed discussion, see [Man23].

formalist perspective; realists may either rely on the fact that classical set theory appears to be sufficiently expressive to interpret any nonstandard mathematics,[6] or more simply accept a really full-blooded platonist perspective [Bea99].[7] Such moves are supported by the orthodox cumulative view of mathematics, according to which no results are (or can ever be) truly rejected.[8]

Now, of course inconsistent mathematics does not *need* to be a revolution in any particular strong sense. It is easy enough to sell it as a substantial novelty in at least some weak sense: the acceptance of inconsistent mathematical truths, the existence of inconsistent mathematical objects, the nontriviality of naive set theory, etc. These can be significant changes, but in principle they need not affect *consistent* mathematics at all. Therefore, I think that the question of whether there could be more to it - of whether inconsistent mathematicians should in fact reject David Lewis's humble stance and *"march over to the mathematics department"* - is worth asking.[9]

3 Trivialization, irrelevance, suppression

One way to make inconsistent mathematics appear more revolutionary might be to insist that there is something *wrong* with classical mathematics, and inconsistent mathematics is the solution. For example, it could be argued that classical logic is *invalid*, i.e. that some of its laws or theorems fail to hold in every situation. Then the fact that classical logic appears to be ubiquitous in classical mathematics - at least, but not only, in virtue of constituting the accepted stan-

[6] This was the (mainstream) fate of e.g. non-well-founded set theory [Acz88] and nonstandard analysis [Rob16]. For an inconsistent example, see [Odd21]. It has been argued that some inconsistent mathematical concepts are simply untranslatable without *some* expressive loss [Pri14]; but even so, the loss may be acceptable for classical reductionist purposes.

[7] This is the paraconsistent-friendly version of Balaguer's full-blooded platonism [Bal01].

[8] This motivates the standard view on revolutions in mathematics, expressed by Crowe's Law 10: *"Revolutions never occur in mathematics"* [Cro75].

[9] Paraphrased in [Web21a, p.104]. Lewis's point was a naturalist one: philosophers should probably not tell mathematicians how to do their job. Rather than derailing the paper with a conflicted rant on naturalism, I will merely note that there is little preventing inconsistent mathematicians from seeing themselves as mathematicians as well as philosophers (or logicians) - after all, they very much appear to be *doing mathematics*, often for straightforwardly mathematical - if niche - reasons. Also, I should maybe clarify that - even if we had the best argument ever - I do not know whether actually marching over would be socially advisable, or strategically wise; I leave that to better diplomats than me, although I will say that the contents of this paper have been bravely relayed to at least some classical mathematicians.

dard of adequate formalization - may be a reason to worry. From a paraconsistent perspective, there are three common lines of attack: classical logic is irrelevant [AB75], it unjustly suppresses premises [Plu23], and it trivializes inconsistent theories [SPMB82, Sect 1.6].[10]

The concern about inconsistent theories is quite straightforward, and is implicit in most work on paraconsistent logics: if a theory contains A and $\neg A$, then closure under classical consequence will trivialize it. But there are prima facie meaningful inconsistent theories all over the place, even in mathematics. The most famous example is naive set theory, which (in most formulations) contains the following naive comprehension axiom schema: $\exists y \forall x(x \in y \leftrightarrow \phi(x))$. Classical logic cannot handle this, as is well known from Russell's paradox. Explosion, together with a bunch of other things, has to go.[11]

The irrelevance charge is that $A \to B$ should not be valid unless the antecedent is in some sense *relevant* to the consequent. This appears to not be the case in classical logic: for example, if B is valid then it is classically implied by *every* sentence. A generally accepted *necessary* condition for relevance is Belnap's *variable sharing property*: if $A \to B$ is valid, then A and B need to have at least one propositional variable in common. This immediately invalidates many classical valid laws, e.g. $A \wedge \neg A \to B$ and $A \to (B \vee \neg B)$. Stronger conditions can be imposed to avoid other intuitively irrelevant classical laws, although there is little agreement among relevant logicians on which - if any - should be the correct one [Sta22].

What about suppression? One paradigmatic example is the classical Exportation law: $(p \wedge q \to r) \to (p \to (q \to r))$. By this law, $q \to r$ may be correctly asserted even if the contribution of p is necessary for r to actually follow from q. This is a problem because, according to Plumwood, *"Deductive logic has never been concerned with the willy-nilly churning out of true propositions implied by some true set of premisses, we care not which. A major function has always been the correct assigning of responsibility for those conclusions, and with the converse*

[10]These are not the *only* attempts to find invalidities in classical logic, even in the paraconsistent literature. For example, attempts have been made to link paraconsistent and constructivist concerns, denying the Law of Excluded Middle as well [Nel49]. I focus on these particular criticisms because they are the ones that reapper (in a different form) in Plumwood's later work, on which I will base my main argument. Furthermore, I think the objection in the next section can straightforwardly be generalized to *any* kind of invalidity claim in the literature.

[11]For a careful analysis of the logical requirements of a naive set theory, see [Web21a, ch.4]. Explosion can be saved by rejecting transitivity of deduction: these are the so-called non-transitive approaches to inconsistency [Rip15a][Ist17].

relation, assessing exactly what is involved in asserting some set of propositions. Hence its important traditional uses in criticising what someone says as insufficient for the conclusions he draws, and criticising what he says by looking at its consequences. [...] But suppression has a disastrous effect on all these functions". Importantly, this is the case regardless of whether we do actually accept the suppressed premises or not: *"even if the suppressed proposition were one we were not inclined to question [...] we might still want to know whether it was used to obtain a particular conclusion"* [Plu23, p. 102].[12]

Where are all these arguments leading to? By definition, paraconsistent logics should be able to deal with inconsistent theories. Relevance requirements take it a bit further, and point towards so-called *relevant logics*. However, both anti-suppression requirements and stronger relevance requirements end up excluding strong relevant logics like Anderson and Belnap's **R** and **E**, which validate Exported Syllogism: $(p \to q) \to ((q \to r) \to (p \to r))$.[13] Hence, if we take all these requirements seriously we end up with so-called *weak relevant logics* like **DJ** and **DK** (Brady's and Richard Sylvan's favorites, respectively) [SPMB82].[14]

4 The special case objection

Now, suppose we buy some (or all) of the invalidity charges given in the last section. One way of redirecting such criticism towards mathematics might be the following.

1. Classical logic supports invalid arguments.

[12] Suppression is also used by Plumwood to try and explicate the relevance requirement: *"q should be deducible from p only if there is a connection of being between p and q. But this connection may be destroyed if suppression is allowed; for the suppressed proposition, which although used no longer appears in the premiss set p, may be just what originally made the meaning connection between p and q"* [Plu23, p. 103].

[13] Exported Syllogism fails Ross Brady's *depth-relevance criterion*, asking that variables are shared at the same depth (i.e. under the same number of nested implications) [Bra06]. It is also said to fail the anti-suppression requirement, but this is harder to make precise, and attempted formalizations do not seem to suffice [Øga20]. It might look like dropping Exported Syllogism would endanger transitivity of entailment. However, as Plumwood points out, this is not so because we can still have the perfectly benign Conjunctive Syllogism: $(p \to q) \wedge (q \to r) \to (p \to r)$ [Plu23, p. 110].

[14] These are all propositional logics, but they can be extended with quantifiers, in which case a **Q** is appended at the end of the name (e.g. **DK** is extended to **DKQ**).

2. Classical mathematics uses classical logic.[15]
3. By 1 and 2, classical mathematics supports invalid arguments.
4. Paraconsistent logics support fewer invalid arguments.[16]
5. Inconsistent mathematics uses paraconsistent logics.[17]
6. By 4 and 5, inconsistent mathematics supports fewer invalid arguments.
7. By 3 and 6, we should march over to the maths department and tell them to do inconsistent mathematics instead.

Such an argument will only justify those variants of inconsistent mathematics which rely on paraconsistent logics avoiding the invalidities in question. In particular, it seems that taking irrelevance and suppression seriously might lead in the very specific direction of weak relevant logics, thus excluding (notwithstanding future reconstruction) an enormous amount of work in the field.[18] Someone buying into this argument may not have much of a problem with this: after all, by exactly the same argument they are already committed to sacrificing a bunch of classical mathematics. Maybe all those irrelevant investigations can rest happy in the context of discovery, a helpful - maybe even necessary - intermediate step on the way to the one true mathematics; and they can now be left behind, hopefully content with having done their part for the greater good.

More importantly, it is not at all obvious that Step 3 of the argument works. Just because classical logic supports some invalid arguments, it does not mean that such arguments are invalid in a classical mathematical context, nor that any bad arguments are ever actually carried out by mathematicians. Mathematics - or at least classical mathematics - could be a *special case*: a particular context where classical logic works just fine. Let me call this the *special case objection*.

Consider the issue of trivializing inconsistent theories. Classical mathematics is a special case to the extent that its objects of study are *consistent*; it does

[15]This does not need to be the case in any strong sense. It suffices that some typically classical logical moves are recognized within classical mathematics as valid.

[16]Mostly. Of course all sorts of paraconsistent contraclassical logics could be devised validating some uncontroversially invalid argument.

[17]Again, mostly. As already mentioned, some inconsistent mathematics gets by with classical logic.

[18]Virtually everything in Mortensen's books, for example [Mor95][Mor10].

not matter what it takes to follow from inconsistent theories, because - by definition - it does not care about them. Even if we accept that interesting inconsistent mathematical theories exist, the classical mathematician's use of Explosion is safe because it is never used to trivialize a theory.[19] Classical mathematicians and inconsistent mathematicians are just studying different things. Maybe the classical mathematician is wrong in calling every inconsistent theory trivial. But, since the classical mathematician is not (qua classical) interested in inconsistent theories anyway, this is really just a terminological point. We could simply replace every classical use of "trivial" with "inconsistent", or "trivial as a classical theory". Pedantry may push us to march over to the mathematics department to inform them of this, but an email will probably suffice.

A similar point can be made concerning irrelevance. Classical mathematicians are not (qua classical) interested in what is entailed (relevantly or otherwise) by, say, $0 = 2$; and again, this is not because classically $0 = 2$ implies everything, but rather $0 = 2$ might as well be taken to imply everything because to the classical mathematician the investigation stops there.[20] Anderson and Belnap claim that an editor would be very confused at a mathematician ending a paper with the following statement: *"if the conjecture is true, then the first order functional calculus is complete; whereas if it is false, then it implies that [a famous open conjecture] is correct"* [AB75, p.17]. Their point is that the editor would think that the inference is false rather than trivial; but either way, I think one moral is that classical logic is not inspiring any classical mathematician to write such statements in their papers.

Suppression also does not appear to be an issue: in fact, Plumwood specifically argues that the way in which mathematicians use lemmas does not involve any illegitimate suppression. *"Anderson and Belnap have claimed that 'the mathematician is involved in no ellipsis in arguing that "if the lemma is deducible from the axioms, then this entails that the deducibility of the theorem from the axioms*

[19]It could be objected that sometimes classical theories are found to be inconsistent, and Explosion prevents the classical mathematician from working *within* the theory while searching for an adequate replacement [Bat98]. While I do not deny that paraconsistent logics may be useful in dealing with some such situations, as a matter of fact classical mathematicians seem to have managed just fine until now with whatever other means they had at their disposal; after all, paraconsistent logics are a very recent discovery. Either way, this is a bit of a tangent: the question was whether the classical mathematician is "led astray" by Explosion, not whether it makes theory change a bit harder.

[20]As an aside, arguably classical mathematicians are interested in - and perfectly able to investigate - what happens when 0 and 2 are *identified*; this is consistently achieved by taking a quotient which puts 0 and 2 in the same congruence class. This is just modular arithmetic, no paraconsistent logic needed.

is entailed by the deducibility of the theorem from the lemma." '. But this sample mathematician's argument is not clearly a case of Exported rather than Conjunctive Syllogism. For precisely the difference between the two laws [lies] in our ability to drop from the premiss set, in the one case but not in the other, the implication $p \Rightarrow q$, in this case the information that the lemma is deducible from the axioms. But we are prevented from dropping this implication in the example by the use of the word "lemma", for "lemma" means something which is deducible from the axioms" [Plu23, p. 110].

More generally, the special case objection is supported by the vast majority of the literature on inconsistent mathematics, and paraconsistent logics more in general. The common line is that, as long as we have no reason to expect contradictions, it is perfectly rational to reason classically [Mar04, ch.10] [CC16]. In fact, inconsistent mathematicians often treat supporting the special case objection as an adequacy condition: this takes the form of *classical recapture*, i.e. the recovery of classical mathematics as a special case of the proposed new theory.[21] Given classical recapture, we may want to tell classical mathematicians that mathematics is broader than they think; but they are not really doing anything wrong, and on its own this revelation gives them no particular reason to change their ways.

Sometimes it is claimed that, while there is nothing wrong with *informal* classical practices, there is a problem with the decision to formalize it in classical logic. If this is right, we may leave the maths department in peace, but require a few words with the mathematical logic chair. For example, here is Sylvan&co: *"Insofar as mathematics relies on valid argument, its proper formalisation is not in terms of classical logic. While it is true that much of classical mathematics can be reformulated using classical logic, such a formalisation is hardly unique, and alternative formalisations using strict systems or even relevant systems can undoubtedly be devised. In fact classical mathematics uses only comparatively weak logical inference principles, few highly nested implications and nothing like the power of classical logic or stronger Lewis systems"* [SPMB82, p.52].[22]

This does not do much to address the special case objection. For one thing, the first line is ambiguous. If "mathematics" stands for "classical informal mathematics", then this is exactly what the special case objection questions: just because classical logic can lead one astray, it does not follow that it leads classical mathematicians astray. If however it stands for "the discipline of mathematics", then it does not follow that classical mathematicians are doing anything wrong as long as

[21] See e.g. [CC13] [Ver13] [Pri06, ch.18].

[22] Robert K. Meyer makes a similar point in defence of relevant arithmetic [Mey21a].

we do not take the discipline of mathematics to reduce to classical mathematics.

It is certainly the case that informal mathematics underdetermines its formalization. Every classical formalization will entail a lot of stuff that informal mathematics will not consider at all: this includes any instance of $A \wedge \neg A \rightarrow B$, but also many of the classical "paradoxes" of material implication, e.g. $(A \rightarrow B) \vee (B \rightarrow A)$. However, this observation fails to speak against classical formalizations of classical practices because *from a classical mathematician's perspective* there is a lot of useless fluff in *any* formalization. For example, the distinction introduced by some relevantists between true conditionals with false antecedents and false conditionals with false antecedents is nowhere to be found in practice. In fact, it is often the case that material implication finds its way in informal formulations of theorems, most notably in universal statements of the form "for every x, if $A(x)$ then $B(x)$": to prove any such theorem, the only cases that a classical mathematician is going to check are those where $A(x)$ is true. More generally, claims to the effect that relevant logic provides a more *faithful* formalization of current mathematical practice seem to be quite unfounded [Web21b].[23]

A stronger answer to the special case objection might appear to be Zach Weber's. He argues that most of classical mathematics might be *false* - or at least that it has not been convincingly shown to be true yet - insofar as it diverges from our best theory of the world, i.e. a dialetheic metaphysics whose underlying laws are paraconsistent:[24] *"the intent is that, wherever a theorem is usually proved using disjunctive syllogism or other classically-only valid inference, there should be an alternative proof—perhaps still needing to be discovered—that leads to the same or similar result using only paraconsistently valid inferences. In the event that there is no such alternative proof, then the theorem is essentially classical and, depending on the case, may not be correct"* [Web21a, pp.105-106]. Here the invalidity of classical logic is taken to have directly led classical mathematicians to some false conclusions. Does this mean that classical mathematics should sooner or later be rejected in favor of true, dialetheic mathematics?

Weber appears hesitant to outright state that classical mathematicians *should* abandon their ways: *"One could take this as normative ('follow me!'), or simply*

[23]Neil Tennant might disagree, but his brand of relevance and paraconsistency is very different from anything I have been discussing, and does not in fact countenance inconsistent theories or models [Ten17]. Meyer also suggested that a relevant formalization might be better on meta-mathematical grounds - most notably, sidestepping Gödel's second incompleteness theorem - although whether this can work remains an open problem to this day [Mey21b].

[24]For the sake of argument I will set aside the controversial nature of Weber's metaphysics, which asks us to believe that true contradictions are *everywhere*.

as an alternative way of doing things ('...if you want to!')" [Web21a, pp.104-105]. In fact, I think the charge of falsity can only support a rejection of classical mathematics if we think of inconsistent and classical mathematics as trying to capture the same fixed, determined subject matter. Yet it is not clear why we should *reduce* mathematics to this goal. Mathematical theories usually do not come with a fixed worldly interpretation: the falsity of classical mathematics with respect to a certain interpretation hardly indicates its falsity with respect to all interpretations. A preference for, say, the naive conception of set - no matter how rational the grounds for such a preference are - does not invalidate the many uses of a mathematics grounded in the iterative conception.[25]

Even if some particular correspondence was deemed privileged enough to take over the meaning of truth, the resulting "false" mathematics would still be mathematics, and it may still be worth pursuing for any number of reasons; this is commonplace for science in general, which theorizes all the time with ideal approximations - not to mention the widespread acceptance of incompatible theories, which for most people entails that they cannot both be *entirely* true [Bat98][Col08].[26] Rejection remains, at least in principle, unwarranted; classical mathematicians could keep doing their thing unbothered by such developments. Falsity charges do not defeat the special case objection; at best, they show that classical mathematics is not all there is.

It could be objected that the normative power of truth should nevertheless affect classical practices to the extent that mathematicians should dedicate most of their time to the pursuit of true theories. However, this seems to be deeply at odds with the way mathematics is practised today. In a post-Hilbert world, it seems that any system could in principle be worth studying, and justification is usually provided on grounds of fruitfulness, not truth - or, to be more precise, it seems that if some notion of truth is at all involved then it is *dependent* on considerations of

[25] Nor is it at all clear that the naive conception can adequately replace such uses. In fact, without some way to import results from classical mathematics, the only way to determine this would be to rebuild several centuries worth of science from scratch. Quite the hard sell.

[26] To expand on the analogy with science, there is a distinction to be made between *false science* - science that is not true, whatever that means - and *pseudoscience* - something that is either "bad" science or not, strictly speaking, science. While the problem of drawing a clear demarcation is a famously difficult one, ignoring the distinction is hardly helpful from either a linguistic or practical perspective. The same holds for mathematics: while some hardcore ZFCers may want to claim that non-well-founded set theory is *false* - there are no non-well-founded-sets in the "real" universe - I have never heard someone say that it is *not mathematics*, nor that it should be thrown away on these grounds; and even if I had, I think we should resist such pointlessly exclusionary moves.

fruitfulness.[27] The current historical time is not one where mathematical theories are selected - let alone *rejected* - because of their direct correspondence with an external world. Maybe this is a bad attitude; but it will have to be challenged before falsehood can pull any weight in an attempted revolution.[28]

5 The logical structure of dualism

So, I have argued that the usual charges to the effect that classical logic is *invalid* - in particular, charges of trivialization, irrelevance, and premise suppression - succumb to the special case objection when it comes to taking issue with classical mathematics. If that is our only beef with classical logic, there is no marching to the maths department for us, except maybe to set some terminology straight and tell them about the many cool things other logics can do.

Let us wildly - if purposefully - change the subject. In her seminal 1993 paper, Plumwood takes *dualism* to be a particular kind of dichotomy underlying most forms of systemic oppression. Paradigmatic examples are what she takes to be the central dualisms of Western thought: man/woman, mind/body, civilized/primitive, and human/nature. *"The master perspectives expressed in dualistic forms of rationality are systematically distorted in ways which make them unable to recognise the other, to acknowledge dependency on the contribution of the other, who is constructed as part of a lower order alien to the centre. These forms of rationality are unable to acknowledge the other as one who is essential and unique, non-interchangeable and non-replaceable. The other cannot be recognised as an independent centre of needs and ends, and therefore as a centre of resistance and limitation which is not infinitely manipulable. This provides the cultural grounding for an ideological structure which justifies many different forms of oppression, including male-centredness, Euro-centredness, ethno-centredness, human-centredness, and many more"* [Plu93, p.453].

Plumwood identifies five central structural features of dualism:

1. Incorporation: the other is defined in relation to the master, as a lack or

[27] This can be seen very strongly in the debate concerning the "right" axioms of classical set theory [Mad11].

[28] As a constructivist, Errett Bishop recognized this, and explicitly attacked classical mathematics on the grounds of having lost track of the real world [Bis75]. But his views were not very influential. It is also worth noting that Bishop did offer some degree of classical recapture, although it could be objected that - from his own perspective - said recapture appears to leave classical mathematics either meaningless or false [Pou00].

negativity.

2. Hyperseparation: differences between master and other are maximized, while shared qualities are minimized.

3. Backgrounding: the other's essential contribution or reality is denied.

4. Instrumentalism: the other is objectified and conceived as means to the master's ends.

5. Homogenisation: differences among the dominated are disregarded, usually through stereotyping.

Plumwood was certainly not the first feminist theorist to recognize the oppressive upshot of the dominant readings of rationality, through which dualisms are made to look natural and inevitable. Most famously, Andrea Nye argued that the very idea of formal logic is intrinsically antithetical to feminist aims, and should be rejected in toto [Nye90].[29] Plumwood's original strategy is to counterargue that Nye is playing into the master's hands by incorrectly identifying formal logic with *classical* logic, ignoring the possibility that nonclassical logics may provide less oppressive forms of rationality which may then be adopted for feminist purposes. Nye's criticisms should therefore be redirected towards classical logic specifically.

Central to Plumwood's argument is the claim that the logical structure of the five central features of dualism is distinctly classical. This can be seen when $\sim p$ is interpreted as "the other of p":

1. Incorporation: *"$\sim p$ can then not be independently or positively identified, but is entirely dependent on p for its specification"* [Plu93, p.454].

2. Hyperseparation: Explosion ensures that p and its other are kept *"at a maximum distance, so that they can never be brought together (even in thought)"* (p.455).

3. Backgrounding: true premises can be suppressed, thus making it possible to hide the other's contribution to the conclusion.

4. Instrumentalism: *"any truth can be substituted for any other truth while preserving implicational properties"* (p.455).

[29] On the relationship between formal logic and feminist theory, see [FH02][Rus2x].

5. Homogenisation: truth-functionality is the only criterion of identity.[30]

Plumwood's thesis is that *"[d]ualisms are not universal features of human thought, but conceptual responses to and foundations for social domination"* (p.444). Other modes of thought - even *rational* thought - are possible, but the naturalization of classical logic as the standard for rationality has contributed (and continues to contribute) to the naturalization of dualisms through naturalization of their logical structure, which in turn makes domination look natural. *"The 'naturalness' of classical logic is the 'naturalness' of domination, of concepts of otherness framed in terms of the perspective of the master"* (p.454).

The recognition of nonclassical logics provides a conceptual way out of the grip of dualism. Desiderata on a non-dualistic logic will include some sort of non-explosive negation, in order to provide a non-hierarchical, non-exclusionary concept of difference; an implication that does not suppress true premises, in order to avoid backgrounding; and a more fine-grained notion of equivalence, to avoid instrumentalism and homogenisation. According to Plumwood, these requirements lead to precisely the weak relevant logics that the invalidity arguments of Section 3 were pointing to.[31] It is worth noting that *"[t]hese desiderata make good sense even if we were to view logic as neutral but [...] able to be weaponized (a less radical view than Plumwood's)"* [ED20, p.442].

While it is beyond the scope of this paper to defend Plumwood's view, let me say a few words to dispel what I take to be some common misunderstandings. Indeed Plumwood appears to believe that classical logic is not neutral, and that its choice was not merely mandated by some notion of "objective rationality", but rather serves the purposes of the master: in fact, the usual notions of objectivity and rationality are themselves deeply complicit in the Western history of oppression.[32] However, this does not entail that dichotomies expressed in classical logic are to be *identified* with dualisms, which involve a *concrete* relation of dominance; and it is not obvious to me that Plumwood ever claims otherwise. The point is that classical dichotomies and dualisms share an underlying logical structure, and ev-

[30] In connection with homogenisation, Plumwood also mentions the fact that *"the other of p, as receptacle, is indistinguishable from the rest of the universe"* (p.454). I am not entirely sure what to make of this last point; that being said, homogenization is directly connected with instrumentalism, so its presence is still a given, if only as a matter of implication rather than negation. See also Thomas Ferguson's contribution to this special issue.

[31] In fact, she defended both lines of argument [Plu23].

[32] On this last point, see [Nye90] and [Plu02]. Readers on the lower side of a dualism (or worse, essentially incompatible with one), may also think of all the times their lived experience was dismissed on "rational" and "objective" grounds.

ery such dichotomy *could* in principle give life to a dualism. I agree with Gillian Russell that there does not appear to be any dualism between, say, odd and even numbers [Rus20]; but I think Plumwood would agree as well.[33]

Similarly, I do not think it endangers Plumwood's view to point out that, formally speaking, $\neg A$ and A could be switched by double negation laws [Gar16]. Of course they could, much like in some society women could be the dominant gender, and everything masculine come to be seen as a lack. If anything, the symmetry between A and $\neg A$ represents the correct fact that dualisms would be damaging even if the roles were reversed, a fact which underlies Plumwood's rejection of feminist strategies of "uncritical reversal" [Plu02, ch.1]. Not every aspect of dualisms is formal; and besides, Plumwood is clear that *"classical logic is the closest **approximation** to the dualistic structure"* (emphasis mine), leaving the door open to some formal disconnect [Plu93, p.454].

Now, this kind of answer will certainly invite a different kind of objection. If classical logic does not automatically generate dualisms, why blame *it*? Why not just push towards an increased sensitivity to bad uses of classical logic, instead of throwing the baby out with the bathwater? This is a more complicated issue. One thing to note is that Plumwood's view may be compatible with not banning classical logic altogether. Rejection is only necessary to the extent that the image of classical logic as *universal* needs to be rejected: we should not reason about dichotomies under the default assumption that they work classically, and we should think hard about any case where classicality is alleged, especially if it is taken to be natural. But this leaves the possibility open that classical logic may be safely used in certain contexts.

Some contexts are more obviously dangerous than others. For example, Maureen Eckert and Charlie Donahue point out that *"classical logic may not be an ideal logic for LGBTQI theorizing, since we want to take seriously people's claims about their gender identity, which combine, adjust or altogether deny the gender binary. If debate and discussion of gender identity takes classical logic as default, the structure of argumentative space ends up (already) binary in character. Activists should be especially wary to give up their home ground of relevant default"*

[33] Although, for what is worth, I think we should always keep our minds open to the idea that any given classical dichotomy *might* underlie, or come to underlie, a dualism. Dualisms can be hard to spot: that is partly the point of naturalization. Besides, while the hyperseparation between odd and even may be harmless, it *is* imposed by us. For example, one could imagine a culture where it would make perfect sense to conclude, from what is to us a proof of the irrationality of $\sqrt{2}$, that some numbers are both odd and even [Blo91, ch.6]. If such a culture clashed with ours, then maybe the classical odd/even distinction could come to be seen as a genuine dualism.

[ED20, p.440]. Now, the classicist will object that the structure of argumentative space could always be rewritten so as to encompass any unexpected possibility that comes to the fore. Maybe so; but a possibility can only be added to the classical space of possibilities once it has been recognized and made intelligible, and the whole point of having to add a possibility is that it is *not* known and intelligible to most people (or even anyone at all). This is especially the case if the lack of that possibility has been fully naturalized. On these grounds, I take Plumwood's focus on logic to be far from misplaced.[34]

6 Against the special case objection: dualism and mathematics

So, let us agree with Plumwood that classical logic is problematic on *social* grounds. Can this new criticism of classical logic be turned into an argument against classical mathematics, and in favor of inconsistent mathematics? The first step would now be something like this:

1. Classical logic contributes to the naturalization of dualisms.

2. Classical mathematics uses classical logic.

3. Hence, classical mathematics contributes to the naturalization of dualisms.

Prima facie, the special case objection still hits its target. The standard view is that (pure) mathematics is at least in principle removed from social or practical concerns, concerned only with an abstract agent-independent universe whose rules are not for us to decide. "Man" and "woman" are not mathematical entities; as problematic as classical laws may be when applied to worldly concepts, they may just *be* the laws governing (classical) mathematics, or at the very least be inoffensive in that context. To go back to Russell's example, is the exclusionary relationship between "odd" and "even" really something worth revolutionizing our mathematics over?

[34] Relatedly, I think the naturalization of classical logic can be - and has been - a great source of *hermeneutical injustice* [Fri07, ch.7], insofar as a conceptual scheme locked within classical logic can prevent the very *conceivability* - let alone intelligibility - of certain situations and identities which escape the classical dualisms, even to the very people who experience them. The ones more affected from this conceptual gap will of course be precisely those which were already systemically marginalized in virtue of said dualisms. I leave an elaboration of this angle for future work.

In order for Plumwood's critique to trickle down to mathematics, it is then important to show directly that classical mathematics's use of classical logic does in fact contribute to the naturalization of dualisms. Fortunately (so to speak) some suggestions of this sort can already be found in the literature criticizing the alleged neutrality of mathematics, once we start thinking about it in Plumwoodian terms.

First, mathematics is generally presented as universal and necessary in just the same way logic is - and just as illegitimately. Here is Leone Burton making this point: *"Mathematics tends to be taught with a heavy reliance upon written texts which removes its conjectural nature, presenting it as inert information which should not be questioned. [...] Language is pre-digested in the text, assuming that meaning is communicated and is non-negotiable"* [Bur95, p.276]. In fact, *"the dominance of a Eurocentric (and male) mathematical hegemony [...] has created a judgmental situation within the discipline whereby, for example, deciding what constitutes powerful mathematics, or when a proof proves and what form a rigorous argument takes, is dictated and reinforced by those in influential positions"* (p.279).[35]

This is not only analogous to the naturalization of classical logic, but goes hand in hand with it: starting in the 20th century the hegemony of mainstream Western maths - the rational field par excellence - is directly connected with the hegemony of the classical logic which is said to provide its foundations and basic language.[36] Because of the commonplace cumulative view of mathematics, this retroactively identifies mathematics (qua collection of necessary truths) with classical mathematics in the contemporary sense. Nowadays, any piece of nonstandard mathematics (e.g. nonstandard analysis, or non-well-founded set theory) tends to be either reassimilated into canon by classical translation, or written away

[35] Burton's feminist epistemology is partly inspired by social constructivism, which takes objectivity to be inextricably tied to social factors: it is this feature that makes mathematics so susceptible to gatekeeping from the dominant class. For a full defense of social constructivism in mathematics, see [Ern98]. The argument in this paper does not depend on accepting full-blown social constructivism; what is needed is the weaker thesis that social factors can influence the development and formulation of mathematics, and that alternative (in a fairly weak sense) mathematics can exist.

[36] The reader who got their perspective skewed by spending too much time around nonclassical logicians is invited to consult, as a paradigmatic example, the Princeton Companion to Mathematics [GBGL08]. The *"language and grammar of mathematics"* is built out of classical connectives (Sect I.2); *"ZFC is currently accepted as the standard formal system in which to develop mathematics"* (Sect IV.22); "logic" refers to "classical logic" throughout (most notably, in Sect IV.23); and in over a thousand pages there is not a single mention of nonclassical logics, or of any piece of mathematics based on nonclassical logics (save for *historical* references to intuitionism).

as a mere formal system which can be accounted for by classical metamathematics.[37] Nonclassical logics may be recognized as a (classical) mathematical object of study, but they are by and large not intended as something we do mathematics *with*, and any suggestions to the contrary (e.g. constructive analysis [BB12]) have gained very little support. In logic as in mathematics, we have a naturalization of certain dominant perspectives, often to the extent that genuine alternatives disappear altogether. As David Bloor famously put it: *"One of the reasons why there appears to be no alternative to our mathematics is because we routinely disallow it. We push the possibility aside, rendering it invisible or defining it as error or as nonmathematics"* [Blo91, p.180].

Not only the necessity, but the *neutrality* of mathematics has been questioned as well. For example, Paul Ernest argued that ascribing ethics-freeness to mathematics is dangerous because of the way mathematics educates to binary, instrumental thinking. *"Thus a training in mathematics is also a training in accepting that complex problems can be solved unambiguously with clear-cut right or wrong answers, with solution methods that lead to unique correct solutions. Within the domain of pure mathematical reasoning, problems, methods and solutions may be value-free and ethically neutral. [...] But carrying these beliefs beyond mathematics to the more complex and ambiguous problems of the human world leads to a false sense of certainty, and encourages an instrumental and technical approach to daily problems"* [Ern18, p.197]. Note that this is not a problem restricted to applied mathematics; rather, *"mathematics through its actions on the mind is already implicated in some potentially harmful outcomes even before it is deliberately applied in social, scientific and technological applications"* (p.206).

Here Ernest is talking about mathematics in general, but once again we can Plumwood this up and note that the instrumental thinking associated with mathematics can be connected with the use of material implication, which in turn is connected with the overwhelmingly popular picture of mathematics being reducible to the extensional - to classical set theory and truth-functionality.[38] The focus on clear-cut right or wrong answers is also supported by classical mathematics both in virtue of its alleged necessity, *and* in virtue of its standardization of Boolean

[37]Nonstandard analysis involves a structure - the hyperreals - which can be classically construed as an *extension* of the classical continuum [Rob16]. Non-well-founded set theory has axioms that contradict ZFC, yet it can also be straightforwardly interpreted as the study of a substructure of the classical universe [Acz88, ch.3].

[38]The reduction was harshly criticized by the relevant school [MS77].

negation.[39]

Going beyond pure mathematics, obviously dualisms appear in mathematics whenever they are that to which mathematics is applied; and since dualisms are everywhere in our society, we can expect dualisms to appear in applied mathematics a lot. Consider for example Roi Wagner's discussion of the *marriage problem*, which involves finding an algorithm to match people according to their preferences in a stable way, i.e. such that in the end there is no pair of individuals preferring each other to their assigned spouses. The original solution to the problem - which, of course, took heterosexuality, monogamy, and a strict gender binary for granted - is the so-called Gale-Shapley algorithm: *"“[...] every boy proposes to his highest preference and every girl refuses all but her best proposal,” keeping her favorite suitor on hold. Each rejected boy continues to propose to his next highest preferences, and each girl continues refusing all but her highest preference among the boys who actually propose to her at any given time, possibly rejecting a boy whose proposal she had previously kept on hold. “This goes on until no changes [new proposals] occur; then every girl marries her only proposer she has not yet refused”"* [Wag17, p.114].[40] Besides being blatantly inspired by and reinforcing gender stereotypes, this solution is male-optimal, and was noted only several years later to be also female-pessimal [MW71].[41] Furthermore, this motivating interpretation of the problem was mostly abandoned once formal generalizations started contradicting any of the stereotyped assumptions.

Putting our Plumwoodian glasses on, we can see that the historical treatment of the marriage problem can be taken to be problematic in virtue of both initially reflecting the man-woman dualism, *and* refusing to question it even when the mathematics itself presented the opportunity. The formal presentation takes men and women to be hyperseparated: the group - which, again, is with false generality introduced as a group of *any* people - is divided into A and (classical) not-A. There is no situation in which such a division is not exclusive, or in which domain and range of the preference function intersect, etc. Yet the suppression of

---

[39] The issue is not one of having only two truth values, merely the way in which they are cashed out. First of all, the usual many-valued logics - even when paraconsistent - support backgrounding and instrumentalism in much the same way classical logic does, because of the truth-functional conditional. Furthermore, the classical binary is always lurking in the form of the dichotomy between designated and undesignated values.

[40] The in-quote citations are from [Bol98].

[41] This means that *"no stable matching exists, where any man marries a woman whom he prefers over the one assigned by the Gale-Shapley algorithm; on the other hand, no stable matching exists that marries any woman to a man less desirable to her than the one assigned by the Gale-Shapley algorithm"* [Wag17, p.117].

such situations is generated from the formal division only because of the classical negation involved. The decision to include any deviant situations out of a taste for generalizations only comes later; but by that time, rather than risk challenging the dualism, the interpretation is dropped altogether.

Conversely, the man/woman dualism is essentially used to express a certain abstract situation: as a mere dichotomy, it would fail to carve the possibility space in the intended way. The dualism also pervades the solution: women are homogenized through stereotyping, being all cast in the same passive role which eventually leads to engagement without consent, and instrumentalized by the male-optimality of the solution. This all suggests that dualisms do affect the choice of which mathematics is developed, and therefore they have a part in what is taken to be mainstream mathematics, namely *classical* mathematics. Since classical mathematics is itself naturalized, this leads to mathematics itself painting those originating dualisms as even more natural, and so on.[42]

To recap: classical mathematics is naturalized to the point of excluding all possible alternatives by fiat, it is inspired by and supportive of dualisms, and it educates to the very kind of thinking that makes dualisms look inevitable. Given all these considerations, I take the special case objection to be sufficiently answered: if classical logic is problematic on grounds of naturalizing dualisms, then so is classical mathematics.

7 Implementation: conservative, radical, queer incomaths

So, we have established a sense in which classical mathematics is indeed problematic; since this is connected to its use of classical logic, and in particular the dualistic features thereof, one might think that inconsistent mathematics can provide a better alternative through its use of paraconsistent logics, which are generally less dualistic. But is this really the case? And even if it is, can we say that inconsistent mathematics is a *feasible* solution?

[42] In fact, Wagner goes even further in arguing that not only do societal biases influence mathematics, but they occasionally do so by hindering creativity and progress [Wag17, ch.4]. Consider for example the *ménage problem*: the question asks for a formula to count the number of ways people can seat at a table so that noone is seated next to their partner. It took decades to find a straightforward proof: one conjectured explanation for such a late discovery is that it required contradicting the assumption that women be seated first [BD86].

According to Plumwood, the response strategy to the ubiquitousness of dualisms should involve *"challenging these oppressive forms of rationality and working for their replacement"* [Plu93, p.459]. Applying this to mathematics, we get something like this:

1. Classical mathematics contributes to the naturalization of dualisms.
2. Inconsistent mathematics contributes less to the naturalization of dualisms.
3. Hence, we should march over to the maths department and tell them to do inconsistent mathematics instead.

The obvious problem with this line of argument is that it risks being overruled by a different sort of concerns. Remember that, by Plumwood's lights, to fully avoid dualism we should look at weak relevant logics. But, contra original hopes, weak relevant logics are *really weak* for mathematical purposes, and it seems likely that the vast majority of current mathematics would have to be sacrificed if we just switched to those.[43] The issue is that such logics cannot really *express* many of the classical concepts, so successfully replacing classical logic with some weak relevant logic would entail enforcing a ban on what mathematicians can define. Historically, such proposals do not usually end well for the censoring side.[44]

Of course, the fact that the right thing to do is really hard and may involve some sacrifices is hardly a knockdown argument against doing it. Far too much human effort has gone into very horrible things, and sometimes ethical considerations may well justify throwing them away regardless of how much work is wasted by doing so. I do not believe that mathematics should in principle be exempt from this. The problem, I think, is rather that the arguments in the last section did not establish that any isolated piece of classical mathematics is particularly damaging on its own; rather, what is really at issue is the *status* of classical mathematics, how it both reinforces and is guided by dualisms, how it is taught and the effect it can have on thinking. It can be hard to shake the feeling that it should not be *necessary* to throw away the "content" of classical mathematics - the theorems, the proofs - in order to fix mathematics. And without that necessity, it is going to be a hell of a hard sell.

[43] In fact, this appears to be the case for *strong* relevant logics as well, already at the level of arithmetic [FM92].

[44] Constructivist schools are the most obvious example. Consider also that mathematicians hardly restrict themselves to first-order languages when defining concepts, regardless of whatever misgivings some logicians may have concerning higher-order logics.

Now, we *could* supplement weak relevant logics with classical recapture tools, of which we seem to have an endless supply: consistency operators [CC16], shrieking [Bea13], defeasible conditionals [Mar04], adaptive strategies [Str14], etc.[45] This would make classical assumptions more explicit, thus at least partially undermining dangerous naturalizations. However, Premise 2 would only hold in the lightest of ways. Such a replacement would be a mere extension of classical mathematics: in fact, since classical practices remain entirely justified within the revised discipline, mathematicians would not really have to *change* anything, which strongly undermines any liberating upshot. The slight conceptual improvement could be more or less ignored in practice, except for maybe adding a little disclaimer "under classical assumptions" at the start of every paper; and this is hardly satisfying as a solution to a problem that was first and foremost a problem *of the practice*.[46]

To sum it up, we seem to be stuck with the following dilemma:

- If we reject classical recapture, then we are undoubtedly dealing a significant blow to the naturalization of dualisms, but we are also asking the community to give up on a lot of mathematics merely in virtue of its potential harms. Call any such implementation *radical incomaths*.

- If we allow for classical recapture, then we are making classical assumptions more explicit, but failing to push for any change whatsoever in mathematical practice, thus falling short of the original liberatory intent. Call any such implementation *conservative incomaths*.

Whither liberatory mathematics? Personally, I think (some versions of) both radical and conservative incomaths could be sensibly defended. On one hand,

[45]The logicality status of most of these stratagems has been questioned. However, I do not take the distinction between logical and nonlogical to be of particular interest to the working mathematician, so from the perspective of practice any of these methods would be equally acceptable.

[46]One might try and invoke some sort of duty to explore outside of the classical universe once we know we can. But, as already discussed in Section 4, there seems to be no analogous duty in standard mathematical practice. Countless possible formal structures are left untouched because no use is known for them, or because they just do not sound that interesting. This hardly looks like a moral or epistemic failing of the community: clearly our limited resources have to be pointed in some directions rather than others. Of course, the extended universe *could* end up providing tools to solve, say, the Riemann Hypothesis, in which case there would be very standard reasons for classical mathematicians to check it out; but this is (as of yet) unsubstantiated, and a very different kind of argument from the ones discussed in this paper, which focus on perceived problems with classical logic.

classical recapture needs not be an all-or-nothing affair, so it may be implemented in such a way that current practices would still have to shift significantly to accommodate the replacement, maybe enough to have a serious positive effect. On the other hand, avoiding the pernicious influence of classical mathematics on society may well be worth a sacrifice even if it is technically overkill: besides, the very framing of it as a sacrifice, a ban, or a limitation of expressive power is unabashedly classical and quite possibly unfair.[47]

That being said, I am going to suggest a third option. Let us go back a bit. Given our diagnosis, what does mathematics need to get better, exactly? Here is Plumwood again: *"the development of alternative accounts of rationality, otherness and difference does have something to contribute to [...] the development of a world which truly 'changes the subject'* ***so that modes of reasoning which treat the other in terms of domination can no longer pass without question as normal and natural"*** (emphasis mine) [Plu93, p.459]. If we take this to be the central goal of liberation, then inconsistent mathematics will be liberatory as long as it manages to undermine the apparent normality and naturalness of classical maths, even without a full-scale replacement.

Following this line of thought, I claim that inconsistent mathematics is liberatory when conceived as the *activity* of inconsistentizing mathematical practices, i.e. of *reinterpreting* existing practices (and especially *dominant* practices) as inconsistent.[48] This can involve reinterpreting statements, theories, concepts, proofs, whatever; if you want an example, simply take your favorite piece of inconsistent mathematics, and think of it as an inconsistent reinterpretation of the closest piece of classical mathematics you know.[49]

Now, in order to be really effective, such an activity would have to be constant,

[47]For example, it has been argued that the whole debate on the undefinability of "just true" in LP is somewhat of a red herring: from a truly paraconsistent perspective, LP might be said to be just as expressive as classical logic [OW19]. See also [Web21b] on the alleged incompleteness of relevant arithmetic w.r.t. classical arithmetic.

[48]There are, I think, independent reasons for settling on this kind of characterization: namely, the unmanageable heterogeneity of paraconsistent logics [Rip15b], and the inescapable agent-dependency of inconsistency. For more on this, see [Man23, ch.4].

[49]I do realize most people have never heard of inconsistent mathematics, let alone carry a favorite piece in their hearts, so here are some examples. A very general way to inconsistently reinterpret a classical theory is to weaken the logic so that it allows for inconsistent models [MM84]. Similarly, allowing for inconsistent sets in one's set theory [Ase96], or inconsistent numbers in one's arithmetic [Ase89], may be seen as inconsistentizing the concept of set/number. Sometimes it can be just a matter of changing perspective: what about reading classical monoids as paraconsistent groups [Web21a, ch.7], or topological spaces as inconsistent theories [Mor10, ch.2]?

because dualism always risks reasserting itself under the guise of simplification. Compare this with the liberatory practice of gender fucking from queer theory: the idea is that *"any community must be based on a principle of constant change to avoid the traps that the rules of gender dictate"* [Whi05, p.125].[50] Merely revising the classification of gender in a belated ad hoc attempt to be more inclusive is never going to be enough; rather, it is the constant activity of questioning, shifting, mixing, and reinterpreting gender categories that is hoped to prevent the settling in of new gender-based oppression. Similarly, the constant activity of questioning, shifting, mixing, and reinterpreting mathematical categories might have the effect of preventing dominant conceptions and interpretations, not to mention the direct influence of outside dualisms, from lingering unchallenged and appearing natural. In this sense, we could say that what mathematics needs is a community-level engagement in the liberatory activity of *mathfucking*. Let this conception of inconsistent mathematics be called *queer incomaths*.

Here, then, is the liberation argument for inconsistent mathematics:

1. Dualisms contribute to systemic oppression.

2. By 1, the naturalization of dualisms should be counteracted on grounds of positive social change.

3. Classical mathematics contributes to the naturalization of dualisms.

4. The naturalization of dualisms can be (partially) counteracted by encouraging subversive practices, i.e. practices that contradict some dominant assumption.

5. Queer incomaths generates subversive mathematical practices.

6. By 3, 4, and 5, queer incomaths counteracts the naturalization of dualisms from mathematics.

7. By 2 and 6, we should march over to the maths department and tell them to do some queer incomaths on the side.[51]

Note that paraconsistent logics are no longer directly involved in the argument. Reinterpretation is not bound to any particular logic; in fact, reinterpretation can

[50] This is in reference to Kate Bornstein's work [Bor94].

[51] Doing it ourselves is a good start, of course. But the proposal will be more effective the more mathematicians participate in it and support it in their teaching.

come *before* any choice of logic.[52] This tracks with Asenjo's suggestion that *"it should be the mathematics that eventually determines the logic, rather than the other way around"* [Ase96, p.55]. This is not to say that paraconsistent logics do not belong to queer incomaths; on the contrary, logical investigations can be quite useful in suggesting inconsistent interpretations, ways to reason with them, and ways to formalize them. The difference is simply that they are no longer central to the enterprise - paraconsistent formalizations are neither the goal nor a prerequisite.

Inconsistent mathematics as it appears in the literature, while differently motivated and presented, can in principle fit under queer incomaths and be seen as contributing to the same social cause.[53] How much it will contribute is a matter of degree: on Plumwoodian grounds, weak relevant reformulations might be preferable, but every denaturalization helps. Now, this may invite objections to the effect that *any* kind of nonstandard maths, not only inconsistent practices, should count towards the goal. In fact, I think they do. However, I also think inconsistent practices are generally going to be *better* at it, because keeping explicit contradictions around is specially subversive in at least two ways. First, it makes classical assimilation harder (conceptually, if not technically), since classical mathematics cannot even dream of accepting a contradiction. Second, it invites less dualistic logics: non-explosive logics will always be the first suggestion, and there are various technical scenarios in which weak relevant logics come naturally to the fore because most other paraconsistent logics are too strong to avoid triviality.[54]

8 Replies to some objections

Let me address some possible concerns. The lack of principled restrictions on logic - and, of course, the looming relativism of the whole proposal - often invites accusations to the effect that "then everything goes", hence epistemic anarchy,

[52]For example, I think we can intuitively grasp what it means to interpret the residue number class $\mathbb{Z}_n$ as an inconsistent finite model of arithmetic, even if it is not immediately clear what the underlying logic of such a model is or could be. More generally, I take it to be an obvious point that we can fruitfully reason with contradictions without having an explicit logic in mind. We could hardly function otherwise.

[53]The same could hardly be said for radical and conservative incomaths, since classical recapture is at least partly a technical matter: some inconsistent mathematics will satisfy it, and some will not.

[54]Most notably, Curry-like paradoxes [Web21a, ch.4].

hence society collapse.[55] So let me try pushing aside this scary strawman, with the caveat that yes, queer incomaths makes things harder, and it *should* make things harder, because things only ever look simple at the expense of what (or who) is made invisible.

Queer incomaths does not entail - classically or otherwise - that everything goes. At best it entails that everything *could*, in principle, go.[56] But meaning renegotiation - of which mathfucking is an instance - is always a two-way process. The goal of denaturalizing classical mathematics cannot be achieved through merely private reinterpretation acts: new meanings are to be shared with (some) community, which inevitably involves some assessment criteria.[57] The idea behind queer incomaths is that there is value in the *act* of looking for reinterpretations regardless of whether we have a "mathematical" reason to do so; this does not mean that every *outcome* is equally valuable. Similarly, to allow for any logic is simply to insure the practice against extremely nonstandard interpretations; it does not follow that every logic will work for any interpretation.[58] I am sure there are many logics that noone will ever find a use for. But this is no good reason to force an a priori restriction on the field.

Worries about truth might arise. Does the inconsistentization of established practices not take us away from truth? Should $2+2=4$ not be true in *every* practice? There are many ways to approach this kind of question, depending on how we understand truth. First of all, mathfucking is a tool to access *different* perspectives and possibilities, but these do not have to fight each other for supremacy. In particular, the dominant mathematical perspectives are not strictly speaking rejected, merely undermined: whatever truth they have remains privileged as truth relativized to a dominant practice, which any kind of rationalist account of the evolution of mathematics can try and justify as usual.[59] It is merely the absolutism that is rejected; hence, to some degree or another, the truth of classical mathematics is compatible with queer incomaths. This is not to say that some particular piece of queer incomaths could never end up becoming the new dominant; but from the perspective of queer incomaths any such "winner" should be challenged as well, lest it fosters new kinds of oppression.

[55] This is only barely exaggerating some of the comments I have received.

[56] A view not new to inconsistent mathematicians [Mor89].

[57] This is not to say that the private act is worthless. It may still contribute to one's own understanding.

[58] On the *existence* of said interpretations, see [Rus18].

[59] See e.g. [Kit84]. Stability of classical mathematical truth can also be explained through socially constructed notions of truth and objectivity [Ern98].

We can reframe the difference between queer, radical, and conservative incomaths in terms of shared truths:

- conservative incomaths goes for the largest possible common ground, and mostly gives up the idea that current practice should be reformed;
- radical incomaths tries to balance the choice of common ground with its liberatory intent;
- queer incomaths refuses to fix a common ground altogether, and gives up the idea of a privileged set of liberatory truths.[60]

Under certain epistemologies, we may also be able to see queer incomaths as contributing to a common search for truth. For example, according to a standpoint epistemology of mathematics, occupants of different social positions may have easier access to different sorts of mathematical practices, and letting them bloom may be the only way to achieve a more complete picture of mathematical truth. Queer incomaths is then a way to bring to the fore the kind of mathematical knowledge that cannot be accessed (or is particularly hard to access) from a dominant standpoint, to the extent that it actively encourages doing things differently from the norm.[61] Another option would be to adopt a queer epistemology and see mathfucking as a way to expose the inescapable fluidity and instability of mathematical truth.[62] Of course, mathematical truth is *the* canonical example of stable truth; but I take arguments to the effect that mathematics is not a "conceptual safe space" to be a promising first step in a new direction [Tan18].

What about constraints imposed by the world itself? If we say that $2+2=5$, are we not just counting *wrong*? Well, of course we are counting wrong with respect to the established practice. That's the point: to look at a *different* practice. But this says nothing about the applicability (let alone truth) of the new practice, because there is no unique way to relate a piece of mathematics to the outside

[60]Clearly every *particular* act of mathfucking will fix a common ground: when inconsistentizing a practice, *some* parts of the practice stay fixed. The point is just that there needs be no common ground shared across the community: what remains fixed can vary.

[61]This is the direction Burton's feminist epistemology seems to suggest [Bur95]. My understanding of standpoint epistemology is greatly influenced by Cat Saint-Croix's formal reconstruction [SC20]. Note that, in order for queer incomaths to contribute to such proposals, inconsistent mathematicians would have to occupy a large variety of subordinate standpoints.

[62]Suggestions to this effect can be found in the "queering mathematics" literature, on which I will say more in Section 9. For a general introduction to queer approaches to epistemology, see [BN10].

world. To say that $2 + 2 = 5$ cannot be true on pains of losing the ability to count correctly is assuming that all the ways and contexts in which we apply an inconsistent arithmetic would be *exactly* the same in which we apply classical arithmetic. But of course this needs not be the case: we are free to do whatever we want with the new practice. Different, incompatible theories can be correctly applied to the same world; and besides, nothing prevents us from finding meaning in different worlds - the point is to build one, after all.[63]

On this note, I should also mention that queer incomaths needs not entail *dialetheism*, i.e. the existence of true contradictions. At the level of pure mathematics, this simply depends on one's views on mathematical truth: fictionalists, for example, should have nothing to worry about. Applied mathematics may sound more dangerous, but in principle the applicability of an inconsistent practice is actually a quite mundane affair. As Diderik Batens puts it: *"A description presupposes a language and a correspondence relation that ties this language to the world. Whatever the world looks like, it is absolutely obvious that we may choose a language L and a correspondence relation R such that the true description of the world as determined by L and R is inconsistent"* [Bat98, p.267].

So, the relativism of queer incomaths is not a danger to truth or knowledge, nor does it automatically commit us to anything particularly controversial. In fact, from the completely opposite side, it could be objected that queer incomaths is quite *redundant* insofar as classical mathematicians are already perfectly willing to change definitions and methods when it serves their purposes. Something as apparently basic as the concept of number went through many iterations: consider the inclusion of 0 and 1 first, and then negative, irrational, complex and transfinite numbers, not to mention the great variance in metaphysical associations [Blo91, ch.6]. To this day, the history of mathematical concepts continues to be fluid, and meanings continue to shift across time, not always in a strictly cumulative way [Lak15][RS15][Tan18].

Now, the easy reply is that both the acceptance of inconsistencies and the conscious use of paraconsistent logics are far from mainstream. So for all the open-mindedness of classical mathematicians, the direction suggested by queer incomaths remains mostly novel. However, I think there is a deeper answer, which concerns the *reasons* for change. Queer incomaths radically breaks from tradition by being socially - rather than empirically or intra-mathematically - motivated:

[63] See for example Alberto A. Martínez's discussion of how to make empirical sense of the idea that minus times minus is minus [Mar18], or Graham Priest's thought experiment about new scientific discoveries pushing towards an inconsistent arithmetic [Pri03].

mathfucking is not primarily a problem-solving tool, nor is it meant to only come into play when mathematics stumbles into an anomaly or an open question. Furthermore, queer incomaths insists on the inconsistency, instead of ignoring it or dissolving it into disambiguation. Now, such disambiguation may eventually be inevitable: the consistency pull is strong, and comes from the biggest side. But this is exactly why queer incomaths needs to be a permanent activity, rather than a single reconstruction project.

On this note, Plumwood scholars might have been raising their eyebrows at the turn this paper has taken. Indeed, although I appealed to Plumwood's work in developing the liberation argument, I should mention that - following the gender analogy - she would probably disagree with the idea of queer incomaths as a solution to systemic issues in mathematical practice. In fact, in several places she argues for a *restructuring* of current gender structures, as opposed to more deconstructive approaches, of which she speaks relatively harshly [Plu89] [Plu02, ch.2].

Still, for what is worth, I don't think our projects are necessarily at odds. One thing we agree on is that restructuring will always lead to problems as long as the dualistic framework is maintained - which at the very least means *as long as classical logic underlies the structure*. So queer incomaths may still be valuable as a way of *exposing* the influence of dualisms on classical (or other) formulations; the difference is that according to Plumwood the enterprise should be aimed at a full-scale replacement, after which presumably no more mathfucking would be necessary. The two positions become compatible under the extra assumption that - in one form or another - classical logic will always find a way of reasserting itself, partly because of its connection with domination and partly because of its alleged theoretical virtues, e.g. simplicity and precision.[64]

9 Related projects

Before I wrap this up, I want to quickly point out some parallels and overlaps between queer incomaths and other liberatory proposals in the literature.

After discussing the problematic history of the marriage problem, Wagner goes on to suggest that an openness to acceptance of unexpected interpretations in mathematical practice may not only *"help mathematicians think creatively"*, but

[64]These two causes are not unrelated. As Plumwood notes when talking about the connection between classical logic and dualism: *"The very features of simplicity which have helped to select classical logic over its rivals are implicated here"* [Plu93, p.454].

also *"shift some of our everyday biases"* [Wag17, p.123]. To this effect, *"we should encourage mathematicians to explore conceptions that are feminist or queer. Perhaps we should encourage social and exact scientists to carry their latent and explicit ideological commitments through mathematics' obscure transformations. Perhaps this would lead us to explore new semiotic possibilities for confronting the impossible impasses in our ways of speaking gender and/or science. Perhaps encouraging signifiers to cut across discursive systems where they do not, supposedly, "belong," does have some therapeutic potential for our contemporary social malaise"* (pp.126-127). Such explorations are bound to generate inconsistency with the way we take our everyday concepts to work, and in particular with whatever implicit bias affects their scope. So I take them to be a particularly meaningful instance of queer incomaths.

While the liberation argument presented in this paper took the Plumwoodian route towards queer incomaths, focusing on the issue of dualism, another possible route would be the Burton-Ernest one: subversive mathematical practices are justified because of the positive effect they can have on *learning* mathematics, and on the public image of mathematics. They can show that mathematics is fundamentally social, and that mathematical meaning - despite the way the subject is taught and presented in textbooks - is conjectural in nature and always open to renegotiation [Bur95][Ern91]. The connection with inconsistency is here a bit more tenuous, but still present. As Bloor puts it, a genuinely alternative mathematics - of the sort that can endanger the absolutist picture - *"would look like error or inadequacy [...] At least some of its methods and steps in reasoning would have to violate our sense of logical and cognitive propriety"* [Blo91, p.108]. And what violates our "sense of logical and cognitive propriety" more than a contradiction?

Given this line of argument, it is maybe not a coincidence that a similar proposal to queer incomaths can be found in the literature on mathematics education, under the label of *queering mathematics*.[65] First, the idea of *mathematical inqu[ee]ry* was introduced by Kai Rands as a step beyond the superficial "add queers and stir" approach to incorporate queer perspectives and problematics in education:[66] *"[q]ueering elementary mathematics education means pushing beyond binaries, questioning the (selective) tradition in the world of mathematics as well as using mathematics to pose questions about the world, and imagining new possibilities"* [Ran09, p.189].

[65]Believe it or not, I named my proposal *before* stumbling into this literature. For a general overview of the field, see [Dub16].

[66]The expression "add queers and stir" was coined by Catriona Sandilands to describe certain superficial approaches to addressing queer oppression within ecofeminism [San94].

In recent years, the project was extended towards queering mathematical content itself. *"Queering mathematics inquires about and questions boundaries, not only around social categories of gender and sexuality but also around mathematical categories. [...] Queering mathematics, as well as new kinds of logics and ways of thinking about gender and sexuality, implies creating space for new ways of mathematical thinking. In other words, queering mathematics will support us to interpret existing questions in new ways, ask altogether new questions, challenge premises that seem no longer self-evident, develop new kinds of representations and arguments, see patterns that may have been invisible before, and will ultimately support us in solving both new and heretofore unsolved problems"* [YR20, pp.238-239].

Of course, the subversive practices generated by queer incomaths nicely serve the purposes of the queering mathematics project. But does queering mathematics, conversely, lead to queer incomaths? Well, again, as long as reinterpretation is set against the power of dominant meanings, inconsistency is right behind the corner. But maybe there is more. In James Sheldon's proposal for a queer curriculum of infinity we have an attempt to recover an original queerness intrinsic to the infinite - a resistance to mathematical treatment, an irreducible lack of consistency - that was lost in post-Cantor approaches caging it in a rigid formal theory [She19].[67] The idea is that there can be value in the very *experience* of inconsistency which queer incomaths provides and classical education hides, an experience which can be therapeutic insofar as it *"remind[s] us of the ontological necessity of conflict"* [Vat14, p.179] and prevents us from taking *"the existent order [...] as the sole possible "reality""* (p.176). Thus we have one more way in which inconsistent practices may be said to be more subversive than other nonstandard practices.

10 Conclusion

Queer incomaths is not going to single-handedly purify mathematics from its dangers and bias, for the simple reason that mathematics is not their primary source. The analogous point for logic is made by Plumwood herself: *"I am not of course arguing that classical logic itself is the cause of women's oppression, and that if we just change the logical theory, all will be well. Challenging dualistic otherness at the level of formal logical theory is only part of what needs to be done*

[67] Note how the history of mathematics is also a constant source of inspiration for contemporary inconsistent mathematics, from infinitesimals to naive set theory.

to problematise the naturalness of domination, and this conceptual and cultural challenge in turn is only part of a wider strategy for change" [Plu93, pp.455-456].

Queer incomaths, as a way of doing inconsistent mathematics, is a tool to fight the naturalization of oppressive ways of thinking. It has therapeutic potential; it promises educational advantages in both countenancing meaning renegotiation and promoting queer standpoints; and it serves as a constant reminder that things can be put differently. It has a chance to make the world a better place. Inconsistent mathematics could not have a nobler goal.

Acknowledgements

I would like to thank Andrew Tedder for much discussion and encouragement. Thanks also to Leo Lobski, Fenner Tanswell, Zach Weber, and audiences at the 5th SILFS Postgraduate Conference and the AAL 2022 Conference for their feedback. Finally, thanks to an anonymous referee for making me smile. This research was supported by the Alexander von Humboldt Foundation in the context of a Sofja Kovalevskaja Award.

References

[AB75] Alan Ross Anderson and Nuel D Belnap. *Entailment, Vol. I: The logic of relevance and necessity*. Princeton University Press, 1975.

[Acz88] Peter Aczel. *Non-well-founded sets*. Center for the Study of Language and Information, 1988.

[Ase54] Florencio G Asenjo. La idea de un cálculo de antinomias. *Seminario Mathematico, Universidad Nacional de La Plata*, 1954.

[Ase89] Florencio G Asenjo. Towards an antinomic mathematics. In *Paraconsistent Logic: Essays on the Inconsistent*, pages 394–414. Philosophia Verlag Gmbh, 1989.

[Ase96] Florencio G Asenjo. Antinomicity and the axiom of choice. a chapter in antinomic mathematics. *Logic and Logical Philosophy*, 4(4):53–95, 1996.

[Bal01] Mark Balaguer. *Platonism and anti-platonism in mathematics*. Oxford University Press on Demand, 2001.

[Bat98] Diderik Batens. Paraconsistency and its relation to worldviews. *Foundations of Science*, 3(2):259–283, 1998.

[BB12] Errett Bishop and Douglas Bridges. *Constructive analysis*, volume 279. Springer Science & Business Media, 2012.

[BD86] Kenneth P Bogart and Peter G Doyle. Non-sexist solution of the ménage problem. *The American mathematical monthly*, 93(7):514–518, 1986.

[Bea99] Jc Beall. From full blooded platonism to really full blooded platonism. *Philosophia Mathematica*, 7(3):322–325, 1999.

[Bea13] Jc Beall. Shrieking against gluts: the solution to the 'just true'problem. *Analysis*, 73(3):438–445, 2013.

[Bis75] Errett Bishop. The crisis in contemporary mathematics. *Historia Mathematica*, 2(4):507–517, 1975.

[Blo91] David Bloor. *Knowledge and social imagery*. University of Chicago Press, 1991.

[BN10] Kath Browne and Catherine J Nash. *Queer methods and methodologies: Intersecting queer theories and social science research*. Taylor & Francis, 2010.

[Bol98] Béla Bollobás. *Modern graph theory*, volume 184. Springer Science & Business Media, 1998.

[Bor94] Kate Bornstein. *Gender outlaw*. Vintage Books New York, 1994.

[BP04] Bryson Brown and Graham Priest. Chunk and permeate, a paraconsistent inference strategy. part i: The infinitesimal calculus. *Journal of Philosophical Logic*, 33(4):379–388, 2004.

[Bra06] Ross Brady. *Universal logic*. Center for the Study of Language and Information, 2006.

[Bur95] Leone Burton. Moving towards a feminist epistemology of mathematics. *Educational Studies in Mathematics*, 28(3):275–291, 1995.

[CC13] Walter Carnielli and Marcelo E Coniglio. Paraconsistent set theory by predicating on consistency. *Journal of Logic and Computation*, 26(1):97–116, 2013.

[CC16] Walter Alexandre Carnielli and Marcelo Esteban Coniglio. *Paraconsistent logic: Consistency, contradiction and negation*. Springer, 2016.

[Col08] Mark Colyvan. The ontological commitments of inconsistent theories. *Philosophical Studies*, 141(1):115–123, 2008.

[Cro75] Michael J Crowe. Ten "laws" concerning patterns of change in the history of mathematics. *Historia Mathematica*, 2(2):161–166, 1975.

[dC00] Newton CA da Costa. Paraconsistent mathematics. In *Frontiers of paraconsistent logic*, pages 166–179. Research Studies Press Ltd., Baldock, Hartfordshire, England, 2000.

[Dub16] Christopher Dubbs. A queer turn in mathematics education research: Centering the experience of marginalized queer students. *North American Chapter of the International Group for the Psychology of Mathematics Education*, 2016.

[ED20] Maureen Eckert and Charlie Donahue. Towards a feminist logic: Val plumwood's legacy and beyond. In *Ultralogic as universal?* Springer, 2020.

[Ern91] Paul Ernest. *The philosophy of mathematics education*. Routledge, 1991.

[Ern98] Paul Ernest. *Social constructivism as a philosophy of mathematics*. State University of New York Press, 1998.

[Ern18] Paul Ernest. The ethics of mathematics: is mathematics harmful? In *The philosophy of mathematics education today*, pages 187–216. Springer, 2018.

[FH02] Rachel Josse Falmagne and Marjorie Hass. *Representing reason: Feminist theory and formal logic*. Rowman & Littlefield Publishers, 2002.

[FM92] Harvey Friedman and Robert K Meyer. Whither relevant arithmetic? *Journal of Symbolic Logic*, pages 824–831, 1992.

[Fri07] Miranda Fricker. *Epistemic injustice: Power and the ethics of knowing*. Oxford University Press, 2007.

[Gar16] Pieranna Garavaso. The woman of reason: On the re-appropriation of rationality and the enjoyment of philosophy. In *Meta-Philosophical Reflection on Feminist Philosophies of Science*, pages 185–202. Springer, 2016.

[GBGL08] Timothy Gowers, June Barrow-Green, and Imre Leader. *The Princeton companion to mathematics*. Princeton University Press, 2008.

[Ist17] Erik Istre. *Normalized naive set theory.* PhD thesis, University of Canterbury, 2017.

[Kit84] Philip Kitcher. *The nature of mathematical knowledge*. Oxford University Press on Demand, 1984.

[Lak15] Imre Lakatos. *Proofs and refutations: The logic of mathematical discovery*. Cambridge university press, 2015.

[Mad11] Penelope Maddy. *Defending the axioms: On the philosophical foundations of set theory*. Oxford University Press, 2011.

[Man23] Franci Mangraviti. Rethinking inconsistent mathematics. Ruhr University Bochum, Accepted, 2023.

[Mar04] Edwin D Mares. *Relevant logic: A philosophical interpretation*. Cambridge University Press, 2004.

[Mar18] Alberto A Martínez. *Negative math: How mathematical rules can be positively bent*. Princeton University Press, 2018.

[Mey21a] Robert Meyer. Arithmetic formulated relevantly. *The Australasian Journal of Logic*, 18(5):154–288, 2021.

[Mey21b] Robert Meyer. The consistency of arithmetic. *The Australasian Journal of Logic*, 18(5):289–379, 2021.

[MM84] Robert K. Meyer and Chris Mortensen. Inconsistent models for relevant arithmetics. *The Journal of Symbolic Logic*, 49(3):917–929, 1984.

[Mor89] Chris Mortensen. Anything is possible. *Erkenntnis*, 30(3):319–337, 1989.

[Mor95] Chris Mortensen. *Inconsistent mathematics*. Dordrecht: Kluwer, 1995.

[Mor10] Chris Mortensen. *Inconsistent geometry*. College Publications, 2010.

[MS77] Robert K Meyer and Richard Sylvan. Extensional reduction—i. *The Monist*, pages 355–369, 1977.

[MW71] David G McVitie and Leslie B Wilson. The stable marriage problem. *Communications of the ACM*, 14(7):486–490, 1971.

[Nel49] David Nelson. Constructible falsity. *The Journal of Symbolic Logic*, 14(1):16–26, 1949.

[Nye90] Andrea Nye. *Words of power: A feminist reading of the history of logic*. Routledge, 1990.

[Odd21] Hrafn Valtỳr Oddsson. Paradefinite zermelo-fraenkel set theory: A theory of inconsistent and incomplete sets. Master's thesis, University of Amsterdam, 2021.

[Øga20] Tore Fjetland Øgaard. Farewell to suppression-freedom. *Logica Universalis*, 14(3):297–330, 2020.

[OW19] Hitoshi Omori and Zach Weber. Just true? on the metatheory for paraconsistent truth. *Logique et Analyse, forthcoming*, 2019.

[Plu89] Val Plumwood. Do we need a sex/gender distinction? *Radical Philosophy*, 51(2):2–11, 1989.

[Plu93] Val Plumwood. The politics of reason: Towards a feminist logic. *Australasian Journal of Philosophy*, 71(4):436–462, 1993.

[Plu02] Val Plumwood. *Feminism and the Mastery of Nature*. Routledge, 2002.

[Plu23] Val Plumwood. Some false laws of logic. *Australasian Journal of Logic*, 2023.

[Pou00] Bruce Pourciau. Intuitionism as a (failed) kuhnian revolution in mathematics. *Studies in History and Philosophy of Science Part A*, 31(2):297–329, 2000.

[Pri03] Graham Priest. On alternative geometries, arithmetics, and logics; a tribute to łukasiewicz. *Studia Logica*, 74(3):441–468, 2003.

[Pri06] Graham Priest. *In contradiction*. Oxford University Press, 2006.

[Pri14] Graham Priest. Contradictory concepts. In *Logic, Reasoning, and Rationality*, pages 197–215. Springer, 2014.

[PS08] Jeff B Paris and A Sirokofskich. On lp-models of arithmetic. *The Journal of Symbolic Logic*, 73(1):212–226, 2008.

[PSN89] Graham Priest, Richard Sylvan, and Jean Norman. *Paraconsistent logic: Essays on the inconsistent*. Philosophia Verlag GmbH, 1989.

[Ran09] Kai Rands. Mathematical inqu [ee] ry: Beyond 'add-queers-and-stir'elementary mathematics education. *Sex Education*, 9(2):181–191, 2009.

[Res92] Greg Restall. A note on naive set theory in LP. *Notre Dame Journal of Formal Logic*, 33(3):422–432, 1992.

[Rip15a] David Ripley. Naive set theory and nontransitive logic. *Review of Symbolic Logic*, 8(3), 2015.

[Rip15b] David Ripley. Paraconsistent logic. *Journal of Philosophical Logic*, 44(6):771–780, 2015.

[Rob16] Abraham Robinson. *Non-standard analysis*. Princeton University Press, 2016.

[RS15] Manya Raman-Sundström. A pedagogical history of compactness. *The American mathematical monthly*, 122(7):619–635, 2015.

[Rus18] Gillian Russell. Logical nihilism: Could there be no logic? *Philosophical Issues*, 28(1), 2018.

[Rus20] Gillian Russell. Logic: A feminist approach. In *Philosophy for Girls: An Invitation to the Life of Thought*, pages 79–98. Oxford University Press, 2020.

[Rus2x] Gillian Russell. From anti-exceptionalism to feminist logic. *Hypatia: A Journal of Feminist Philosophy*, 202x. To appear.

[San94] Catriona Sandilands. Lavender's green? some thoughts on queer (y) ing environmental politics. *UnderCurrents: Journal of Critical Environmental Studies*, 6:20–25, 1994.

[SC20] Catharine Saint-Croix. Privilege and position: Formal tools for standpoint epistemology. *Res Philosophica*, 97(4), 2020.

[She19] James Sheldon. Towards a queer curriculum of infinity. In *STEM of Desire*, pages 307–318. Brill, 2019.

[SPMB82] Richard Sylvan, Val Plumwood, Robert K Meyer, and Ross T Brady. *Relevant logics and their rivals*. Ridgeview Pub Co, 1982.

[Sta22] Shawn Standefer. What is a relevant connective? *Journal of Philosophical Logic*, 2022.

[Str14] Christian Straßer. *Adaptive logics for defeasible reasoning*. Springer, 2014.

[Syl19] Richard Sylvan. *Ultralogic as universal?* Springer, 2019.

[Tan18] Fenner Stanley Tanswell. Conceptual engineering for mathematical concepts. *Inquiry*, 61(8):881–913, 2018.

[Ten17] Neil Tennant. *Core logic*. Oxford University Press, 2017.

[Vat14] Gianni Vattimo. Insuperable contradictions. In *Contradictions: Logic, History, Actuality*, volume 6, page 173. Walter de Gruyter GmbH & Co KG, 2014.

[Ver13] Peter Verdée. Non-monotonic set theory as a pragmatic foundation of mathematics. *Foundations of science*, 18(4):655–680, 2013.

[Vic13] Peter Vickers. *Understanding inconsistent science*. OUP Oxford, 2013.

[Wag17] Roi Wagner. *Making and breaking mathematical sense: Histories and philosophies of mathematical practice*. Princeton University Press, 2017.

[Web21a] Zach Weber. *Paradoxes and inconsistent mathematics*. Cambridge University Press, 2021.

[Web21b] Zach Weber. Relevant arithmetic and mathematical pluralism. *The Australasian Journal of Logic*, 18(5):569–596, 2021.

[Web22] Zach Weber. *Paraconsistency in mathematics*. Cambridge University Press, 2022.

[Whi05] Stephen Whittle. Gender fucking or fucking gender? In *Queer theory*, pages 115–129. Catherine Belsey. New York: Palgrave Macmillan, 2005.

[YR20] Cathery Yeh and Laurie Rubel. Queering mathematics: Disrupting binary oppositions in mathematics pre-service teacher education. In *Borders in mathematics pre-service teacher education*, pages 227–243. Springer, 2020.