From Excluded Middle to Homogenization in Plumwood's Feminist Critique of Logic

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Abstract

A key facet of Valerie Plumwood's feminist critique of logic is her analysis of *classical negation.* On Plumwood's reading, the exclusionary features of classical negation generate *hierarchical dualisms*, *i.e.*, dichotomies in which dominant groups' primacy is reinforced while underprivileged groups are oppressed. For example, Plumwood identifies the *system collapse* following from *ex contradictione quodlibet*—that a theory including both φ and $\sim\varphi$ trivializes—as a primary source of many of these features. Although Plumwood considers the principle of excluded middle to be compatible with her goals, that she identifies *relevant logics* as systems lacking a hierarchical negation—whose first-degree fragments are both paraconsistent *and* paracomplete—suggests that excluded middle plays some role in hierarchical dualisms as well. In these notes, I examine the role of excluded middle in generating oppressive *homogenization* and try to clarify the relationship between Plumwood's critique and this principle from several contemporary perspectives. Finally, I examine the matter of whether Plumwood's critique requires *relevance* or whether a non-relevant logic could satisfy her criteria and serve as a liberatory logic of difference.

1 Introduction: Critiques of Logic

Given formal logic's central role in analytic philosophy, it is no wonder that a number of feminist critiques of logic have appeared. Among these, Val Plumwood's critique is unique for the benefit of her expertise as a philosophical logician. Plumwood's critique—presented in the papers Plumwood [1993a] and Plumwood [2002] and book Plumwood [1993b]—enjoys additional subtlety beyond those of her contemporaries. For example, two parallel shapes of critique of logic are far coarser in their prescriptions for respectively "taking *logic*... and/or *negation* to be automatically oppressive."[Plumwood, 2002, p. 61, emphasis added]

An exemplary critique that calls for a total abrogation of logic is due to Andrea Nye [1990]. After surveying the social and historical conditions under which formal logic was developed, Nye rejects the entire project of logic, arguing that its development reveals it to have been irrevocably molded by the hands of dominant groups and therefore unsalvageable. Nye's conclusion that "[t]he relations between speakers that logic structures are alien to feminist aims... it [is not] possible to argue that these are misuses of logic which a feminist logic can correct"[Nye, 1990, p. 179] is an unequivocal indictment of any revisionary program.

Less coarse-grained are revisionary critiques that Plumwood calls "negationist." While stopping short of indicting logic proper, Marilyn Frye [1996] and Nancy Jay [1981] target the very notion of negation, arguing that negation acts as a tool of oppression in virtue of its use by dominant groups to define underprivileged groups in negative—rather than positive—terms.

> In both frames, the existentialist and the Lacanian... to be a woman is to be not-self, not-man, not-the-phallus. Both are ways of constructing the social/ontological category of men as the **A** side of a universal exclusive dichotomy: **A/not-A**. Both Irigaray and Beauvoir want to to liberate women from the consignment to the **not-A** side of such a dichotomy.[Frye, 1996, p. 994]

According to such critiques, it is negation—in virtue of its power to impose dichotomies—that must be abandoned. Logic, on this reading, requires a revision in which all categories of otherness retain their distinctive and *positive* characteristics.

Plumwood convincingly argues that both forms of critique are too coarse and that this coarseness follows from the authors' identification of "logic" with *classical logic*. If one can envision no alternative to classical logic or Boolean negation, respectively, then the suggestion that one must abandon logic *in general* or negation *in general* is a reasonable and compelling response, made all the more understandable for the fact that most philosophers' introductions to logic will have failed to acknowledge any such alternatives. But alternatives do exist and Plumwood's familiarity with the landscape of alternative logics sets her critique apart.

2 Plumwood's Feminist Critique of Classical Logic

Plumwood's work appearing in Plumwood [1993a], Plumwood [1993b] and Plumwood [2002] presented a far more nuanced critique of logic. As a key contributor to the development of the Australasian school of relevant logics, Plumwood possessed a grasp of the subtle details of formal logic lacking in the progenitors of other feminist critiques. Maureen Eckert and Charlie Donahue describe her standpoint:

> For Plumwood, some logics, such as classical logic, are tools of the patriarchy and oppressors, but this does not mean that logic itself has nothing to offer feminist theorists... For feminists unfamiliar with logic at Plumwood's level and the insider view of the battles fought between logicians, this selectivity among logics may not make a great deal of sense, if any.[Eckert and Donahue, 2020, p. 427]

Plumwood does not accept that dichotomies in themselves—and thus the use of negation through which such contrasts are articulated—are responsible for the types of domination addressed by Nye, Frye, Jay, and others.

> When the qualities of one item are specified negatively in relation to another item conceived as norm or center, what is oppressive is not negation as such but the power relation, the hegemonic centrism that distributes both positive and negative attributions in ways that privilege the center.[Plumwood, 2002, p. 61]

Rather, Plumwood contrasts *dichotomy*—the drawing of distinctions, differences, or nonidentities between classes—with *hierarchical dualism*, a state that results when a dichotomy becomes hegemonic due to the governance of some logical features that construct and reinforce oppressive hierarchies.[1]

[1] For the reader not satisfied with the terseness of how I have offered this contrast, note that the initial pages of Plumwood [2002] provide a detailed and elegant statement of Plumwood's understanding of the concepts of dichotomy and dualism.

This logic of domination is due to a combination of factors: *centrism*—that positive, default notions will be determined by a dominant class—and the logical features whose weaponization gives this centrism teeth.

While centrism is a sociological problem, the reappraisal of those tools of classical logic through which this centrism expresses and enforces oppression is a logical problem. It is, for example, not the mere fact that classical negation *is a negation* that supports domination. Rather, it must be some property particular to the classical interpretation of negation; under Plumwood's analysis, feminist aims are advanced by identifying, analyzing, and rejecting such features.

2.1 The Role of Negation

A formalization of logic serves as a model for an assemblage of distinct components through which their individual features as well as their interrelationships receive an interpretation. A formal deductive calculus might include a number of extensional connectives, intensional operators, quantifiers, and so forth under the umbrella of its analysis.

Although I have so far mentioned only *negation*, *a priori*, no particular piece of logical vocabulary is readily identifiable as bearing particular culpability for the oppressive features of classical logic. Indeed, Plumwood identifies several features through which the classical analysis of *implication*—*i.e.* identifying implication with the material conditional—finds service as a tool in the oppression of underprivileged groups.

Plumwood's remarks on classical implication certainly form an important plank in her critique of logic; it is a topic to which we will return in Section 5. However, it is negation that will take center stage over the course of the next several sections. We will pause in order to discuss and justify this apparent incongruity.

For one, the degree to which negation will receive disproportionate attention is largely aligned with Plumwood's own topical priorities in her writing. In *e.g.* Plumwood [1993a], there are surprisingly few sections exclusively devoted to investigating the features of implication through which hierarchical dualisms are reinforced. The single paragraph in which Plumwood turns her attention exclusively to oppressive features of classical implication stands in contrast with the many pages dissecting features of negation.

Beyond the matter of frequency, the preeminent role played by negation in the analysis appears to be part of Plumwood's design. This is clearest in a passage in which Plumwood distills the ultimate recommendations of her project:

> the way to escape this structure is to replace dualistic negations with others expressing a non-hierarchical concept of difference which does not import dualistic structures into thinking about the other.[Plumwood, 1993a, p. 447]

The dissolution of negation's oppressive features is only possible upon their discovery. Because Plumwood takes this dissolution as the cornerstone of liberation from classical hegemony, it is necessary to prioritize the analyses of negation through which these discoveries will be earned.

It is true that a formal account of logic is in many ways a holistic tapestry, in which features of one syntactic item may exert a tangible influence over others. *E.g.*, if one's intuitions about a particular connective drive one to commit to a particular model-theoretical framework, this commitment has side-effects constraining the range of potential interpretations for other connectives. For example, the portrait of investigation implicit in Brouwer's intuitionistic conditional leads naturally to Kripke's model theory; even if, say, conjunction may be conservative over the implicational fragment, its truth conditions must still be described in a way that coheres with Kripke models.

Still, in such a web of mutually interdependent interpretations of distinct elements of vocabulary, Plumwood suggests that negation is naturally regarded as *prior* to other connectives.

> As work in relevant and paraconsistent logic... has shown, negation is the key axis of comparison among implicational systems.[Plumwood, 1993a, p. 454]

In other words, the merits of a particular interpretation of implication may be judged on the basis of its influence over the corresponding interpretation of negation. The present approach follows this precedent. One can acknowledge the importance of other elements of vocabulary while prioritizing a clear understanding of negation.

In light of this, for the next several sections we will assume that we work in a setting of *first-degree consequence*. This assumption permits us to examine the features of negation in an isolated environment, unimpeded by the possible interference of intensional connectives.

2.2 Exclusion and Homogenization

Plumwood identifies an assortment of the oppressive features that invariably accompany systems that generate hierarchical dualisms, taking care to discuss the methods through which these features oppress. Among these themes are two of particular importance, identified by Plumwood as *radical exclusion* and *homogenization*.

The feature of *radical exclusion* is described in harrowing terms; the degree of harmfulness of its consequences is underscored by its presentation in direct and immediate language:

> Because the other is to be treated as not merely different but as inferior, part of a lower, different order of being, differentiation from it demands not merely distinctness but radical exclusion, not merely separation but hyperseparation... [Thus, c]onceptual structures stressing polarisation allow the erection of rigid barriers to contact which protect and isolate dominant groups.[Plumwood, 1993a, p. 448]

Understanding negation as a type of "othering" operator—so that $\sim P$ is read as "other-than-P"—radical exclusion is a guarantee against the intrusion of the other into the cordoned-off domains of privileged groups. On Plumwood's reading, the barriers erected—and reinforced—by this feature are inherently oppressive insofar as they actively marginalize and alienate underprivileged groups.

The feature of *homogenization* complements radical exclusion by imposing *stereotypes* suppressing any differences or unique features of members of underprivileged groups.

> [A] dominated class must appear suitably homogeneous if it is to be able to conform to and confirm its 'nature.' In homogenisation, differences among the inferiorised group are disregarded... The colonised is reduced to a function, and the relationship of domination destroys the ability to perceive or appreciate characteristics of the other over and above those which serve this function.[Plumwood, 1993a, p. 451–453]

While less overt than radical exclusion, on Plumwood's reading, homogenization lays its foundation; in flattening the vibrancy of the other, a license is provided to dismiss the homogenized class *in toto* as aberrant. It is, after all, far easier to disregard the humanity of a group when its members are faceless and featureless.[2]

According to Plumwood's account, these two features act in concert. Radical exclusion actively denies participation and prominence to the nondominant groups while homogenization acts as a propagandist, portraying the dominated group as an amorphous, undifferentiated—and thus undeserving—underclass. While radical exclusion actively does violence, homogenization more insidiously sanitizes this violence, thereby safeguarding oppression against criticism or resistance. As Plumwood describes this partnership:

> Radical exclusion and homogenisation combine to naturalise domination.[Plumwood, 1993a, p. 452]

Importantly, Plumwood takes care to identify pathways between such features of dualism and particular features of classical logic from which they emerge. In the case of exclusion and homogenization, an additional demand can be justified, namely, that an oppressive feature should not only be traceable to *some* feature of classical logic, but it should be traceable to a particular feature of classical *negation*.

This expectation seems to be manifest in many of Plumwood's remarks. *E.g.*, we had noted earlier Plumwood's assertion that "replacing dualistic negations with others" is "the way to escape" oppressive consequences of classical logic; that dualistic negation should serve as *the* barrier to liberation is possible only in case all oppressive features of logic *ultimately* flow from a corresponding feature of negation.

But this demand can be justified more robustly. One might consider the task of eliminating homogeneity to be centrally concerned with restoring to other the individuality hitherto obscured by a stereotype. The resolution of dualisms is, in all cases, a matter of making progress towards an adequate account of the other that is free from oppression.

[2] As a referee has remarked, the stereotypes by which a nondominant class is homogenized are themselves features. Moreover, as marks of the failure to satisfy a dominant characteristic, such features play a functionally important role in a dominant group's justifying and perpetuating the subjugation of a nondominant class. In describing a dominated class's members as "featureless," I do not mean to discount this mechanism. Rather, I mean to emphasize the resulting atrophy of the *individuating* features needed if one is to distinguish one member from any other and, in turn, the eventual erasure of anything *but* the stereotype that follows.

> A dualism... should be understood as a particular way of dividing the world which results from a certain kind of denied dependency on a subordinated other.[Plumwood, 1993a, p. 443]

Moreover, as Plumwood suggests, negation plays a unique role in the semantics of otherness.

> If negation is interpreted as otherness, then how negation is treated in a system, together with other features of the system, provides an account of how otherness is conceived in that system.[Plumwood, 1993a, p. 454]

If there is a direct path between some syntactic or semantic commitment of classical logic and homogenization, it stands to reason that its locus should be discoverable in negation itself.

In some cases, the causal pathway from negation is easily recognized. For example, Plumwood argues that radical exclusion is an inescapable byproduct of the classically valid principle of explosion, *i.e.*, that for a theory T, if φ and its negation $\sim\varphi$ are both elements of T, then T includes the entire language.

Her argument that explosion leads to radical exclusion is relatively straightforward. In a theory T, that a term t should fall under $\sim P(x)$ precludes its participation in $P(x)$ on pain of *system collapse*. Should both $P(t)$ and $\sim P(t)$ be included in T, then in the presence of explosion T will be trivial. But a trivial theory is unworkable in permitting no discrimination between statements. As the merest hint of interpenetration between the dominant and subjugated classes leads to an incoherent picture, classical logic demands that the other is kept at a maximal distance from the central class. This, Plumwood argues, exposes a direct lineage from the logical principle of explosion—a property of *negation*—to the sociological consequences of radical exclusion.

Plumwood is more brief when investigating the logical genealogy of homogenization. She is very explicit in describing an *association* between classical logic's interpretation of propositional identity to homogenization:

> These homogenising properties of classical negation are associated with the failure of classical logic to make any finer discriminations in propositional identity than truth-functionality.[Plumwood, 1993a, p. 454–5]

Clearly, there is a compelling observation here concerning homogenization and the above truth-functional criterion of propositional identity. To cite truth-value as the unique feature by which propositions are identified or distinguished partitions propositional space into the true and the false. The partitioning of this space follows a principle of maximal homogeneity, in the sense that no coarser partition is possible without sacrificing the concept of difference itself.

Despite this, the above genealogy has limitations. A mere association is neither causal nor directional. Plumwood's remarks do not indicate whether she intends to indicate that properties of negation induce the coarseness of classical propositional identity or *vice versa*. This sets up a dilemma. If the former direction holds, then the task of identifying the properties of negation that support homogenization remains unsolved. If the latter, converse direction holds, it is hard to discern how this coarseness reveals anything about negation in particular. The homogenization following this criterion of propositional identity is *global*, indiscriminately erasing distinctions without regard to negation.

In either case, the association cited by Plumwood yields no clues concerning a lineage from features of negation to homogenization itself. Nevertheless, we should be able to discover such a genealogy to homogenization.

We are left in a curious position at this point. Among the many proof-theoretic and semantic properties of classical negation, Plumwood has identified explosion alone as a feature of negation that generates hierarchical dualisms (at least, at the first-degree case to which we have restricted our attention). In a notable contrast, Plumwood has declined to criticize any dual properties of negation like *bivalence*. Irrespective of the merits of these attributions, one might reasonably anticipate that a rejection of explosion alone—and not bivalence—would be followed by a prescription of *paraconsistency*.

However, Plumwood exclusively promotes *relevant logic* as an appropriate target. Under the present assumption of a first-degree setting, this choice amounts to identifying the negation of *first degree entailment* (FDE) as an acceptable account of negation.

> [T]he negation of relevant logic, relevant negation, can be interpreted as expressing a notion of otherness as non-hierarchical difference.[Plumwood, 1993a, p. 458]

Plumwood, of course, qualifies this prescription by *e.g.*, identifying relevant logic as but one among a number of "implicationally adequate rivals" [Plumwood, 1993a, p. 454] or "alternative systems." [Plumwood, 1993a, p. 458] Yet her gesture towards the negation of FDE cannot be disregarded.

FDE is not only paraconsistent but it is also *paracomplete*. Thus, a crucial matter that invites clarification is Plumwood's position on the principle of excluded middle. To default to a suggestion that the negation of FDE—for which both explosion *and* the principle of excluded middle fail—should be preferred if *e.g.* the paraconsistent negation characteristic of LP of Priest [1979] (treated as a first-degree logic) would suffice seems heavy-handed.[3] Relevance is a far stronger property than paraconsistency; Plumwood's prescription thus is correspondingly broader in scope than the malady appears to require.

Thus, two related observations seem to require clarification:

- Because homogenization is a feature of hierarchical dualism, one might expect that a formal property of classical negation serves as its source, but no such feature of negation is identified as responsible for homogenization

- Although paracompleteness of negation—or an analogous rejection of excluded middle—is not identified by Plumwood as a source of hierarchical dualisms, Plumwood's recommended negation exhibits this property

3 The Principle of Excluded Middle

I have argued that Plumwood's remark that "[t]he negation of classical logic... exhibit[s] other features which are characteristic of dualism" [Plumwood, 1993a, p. 455] suggests we should be able to trace the particular features of classical negation that support homogenization and other features of hierarchical dualisms.

Additionally, I have suggested that although the causal chain between the principle of explosion and radical exclusion is clear, an analogous path

[3] If one wishes to step outside the first-degree setting to consider logics with proper conditionals, one could reframe the matter by noting that requesting *relevant* logic when a merely paraconsistent implicational system like da Costa's C_1 of da Costa and Alves [1977] would suffice is similarly heavy-handed. (Although there are implicational features of C_1 that disqualify its utility as a feminist logic, as we will see in Section 5.)

from a particular feature of negation to homogenization remains to be determined. Although Plumwood acknowledged a parallel between homogeneity and classical logic's criterion of propositional identity, this association was insufficient to isolate any particular defect of classical negation as its source. There remains room, then, to investigate the source of homogenization.

Intuitively, it certainly appears that homogenization and the principle of excluded middle ought to share some common conceptual ground. This section will be devoted to exploring this conceptual ground.

3.1 Homogenization and Excluded Middle, Naively

Plumwood's language suggests that the homogenization of a dominated class is the product of its characterization in terms of a privileged oppressor class (*i.e.*, centrism) in conjunction with the default inclusion of any element deviating from the oppressor class into a homogenized class of the oppressed.

> It is [a] corollary that $\sim p$ cannot be independently identified, it is entirely dependent on p... compare what Simone de Beauvoir has to say to alienation of women where 'woman' is identified as 'other than man'; and is not positively identified, only introduced as alien to the primary notion, 'man.'[Routley and Routley, 1985, p. 217]

Recall that to Plumwood, the source of homogenization is not the fact that dominated groups are defined in negative terms. What is important is that the other is *entirely dependent* on the dominant class and that this dependence is regulated and enforced by dualistic negation.

On an intuitive—possibly naive—reading, the principle of excluded middle corresponds to the *exhaustivity of a domain*; the centrality of relevant negation's skepticism of exhaustivity is acknowledged by Plumwood in Routley and Routley [1985]: "The important point... is that one side does not somehow obliterate or wipe out or entirely exclude *or exhaust* its opposite. "[Routley and Routley, 1985, p. 220, emphasis added] Some reflection suggests that such exhaustivity can serve to support homogenization.

Exhaustivity *rules out* classification beyond the binary between P and its other. By *excluding* any option beyond classification under a dominant class P and its complement $\sim P$, all inferiorized elements reduce to their falling under a common category of $\sim P$. Those falling under $\sim P$ thereby are *stereotyped*, *i.e.* the characteristic feature of all such inferiorized elements is reduced to their satisfaction of $\sim P(x)$, their identities given meaning only as functions of the dominant $P(x)$. Elements of Plumwood's discussion align with the spirit of this link rather well. *E.g.*:

> The challenge to the polarities posed by challenging dualistic gender construction was important for gay as well as women's liberation because it opened up the space for the formation of *third or multiple terms* in place of a polarized binary gender structure.[Plumwood, 2002, p. 48, emphasis added]

Tertium non datur, of course, quite literally encapsulates a rejection of such *third* terms. Against Beauvoir's backdrop of a definition of "woman" as "other-than-man," exhaustivity leaves little recourse but to classify someone who identifies, say, as non-binary as "other-than-man." Under these conditions, the independent identification that allows one to differentiate between members of a non-binary gender and women is lost. Arguably, then, the principle of excluded middle—through its ties to exhaustivity—ensures that the non-binary individual is generically lumped within the homogeneous category of "other-than-man."

To illustrate, consider an informal example:

Example 3.1. Let a predicate `Male` collect individuals falling under a dominant group's criteria along the dimension of gender. As the dominant group, the predicate `Male` will take a prominent—*i.e.*, default—place in the language. Suppose that term `t` denote an individual who identifies as non-binary. Then the sentence $\texttt{Male}(\texttt{t}) \vee {\sim}\texttt{Male}(\texttt{t})$ is true as an instance of excluded middle. Insofar as `t`'s gender identity diverges from the dominant class `Male`, `t` must be classified as falling under $\sim$`Male`.

This stereotyping applies for any individual not deemed within the extent of `Male`, eliding all distinctions between any two genders not falling under the dominant group. Consequently, any features that individuate the classes of women and non-binary individuals are suppressed. This mass erasure of individuating characteristics is obviously a type of homogenization. Notably, such homogenization is *iterable* and can be reapplied to reinforce homogenization along dimensions beyond gender binaries—*e.g.*, race or ethnicity—

including dimensions of intersectional identities.[4]

It seems that the link between the exhaustivity characterized by excluded middle and homogenization is rather direct and clear. Plumwood's own words about "open[ing] up the space for the formation of third or multiple terms" seem to unambiguously identify exhaustivity as a wellspring of homogenization and, in turn, oppressive dualisms. Thus, it bears asking why excluded middle should not be explicitly identified by Plumwood as bearing responsibility for hierarchical dualisms of classical negation.

3.2 Plumwood on Excluded Middle

It may be surprising to find that Plumwood explicitly rejects the suggestion that excluded middle leads to the establishment of hierarchical dualisms. In her words:

> I offer a different analysis here of dualism which does not associate it with Excluded Middle... In terms of propositional logic, the dichotomising functions of negations which simply divide the universe and recognise a boundary between self and other *without*

[4]A referee has pointed out the following difficulty for the case described in Example 3.1: Both individuals who are genderfluid (*i.e.* whose gender identity undergoes some degree of variation over time) but presenting as male and trans men are groups that face oppression for their otherness. In case `Male` is understood as "recognized by society as male," then the genderfluid individual (who may have been assigned male at birth and presents as male) may satisfy the intension and be counted as a member of the dominant group. If `Male` is, on the other hand, is understood as "gender identity," then a trans man would be counted as a member of the dominant group in virtue of his identification. These are the two most natural interpretations one can give to `Male` but in each case some empirical fact of oppression is not reflected in the example. In order to account for both instances of oppression, a more artificial—perhaps disjunctive—interpretation would have to hold.

My sense is that the difficulty presupposes that a certain level of constancy is exhibited by the languages written by the dominant group. But the extensions of the terms authored by dominant groups are determined *by fiat*. Not only need there be no natural kind that determines the boundaries of *e.g.* `Male`, there need be no intelligible principle binding the members together beyond the assent and recognition of the groups in power. One cannot find a governing rule by which both the genderfluid individual and the trans man are oppressed, for the boundaries of the privileged class are not matters about which one can reason. Nevertheless, the observation that any two axes of gender-based or gender-adjacent oppression need not follow a common genealogy does lead one to question whether Plumwood's framework is sufficient as a uniform analysis of oppression.

> *importing a hierarchical structure* are associated with the Law of Non-Contradiction $\sim(A\&\sim A)$ and the Law of Excluded Middle $(A \vee \sim A)$.[Plumwood, 1993a, p. 446, emphasis added]

As Plumwood explicitly states that one can endorse excluded middle "without importing a hierarchical structure," it is clear that Plumwood does not intend for her critique to renounce excluded middle.

What might Plumwood say in response to our earlier naive analysis? We might look to the two-stage nature of Example 3.1, which proceeds by *initally* assuming a link between exhaustivity and excluded middle and *subsequently* tracing a path by which exhaustivity supports homogenization. Thus, one response available is simply to challenge the existence of a link between exhaustivity and excluded middle.

One clue suggesting that Plumwood takes such a position lies in the following observation: In the same remarks denying that excluded middle supports hierarchical negation, Plumwood also states that the *principle of noncontradiction*—$\sim(\varphi \wedge \sim\varphi)$—itself does not support hierarchical dualisms. Given the intepretative commonalities between noncontradiction and explosion, this might be surprising. Despite the similarities, the principles of noncontradiction and explosion are distinct, witnessed by Priest's LP for its satisfaction of the former and rejection of the latter. On similar grounds, Plumwood must have acknowledged that same distinction between the principle of noncontradiction and the principle of explosion.

This makes sense given Plumwood's emphasis on *system collapse*. Plumwood, in short, is primarily concerned with *theories* and what can be done with them. On such a theory-driven plan, to sever negation from its role in supporting hierarchical dualisms consists in ensuring the utility of theories in which the other may at times overlap with the dominant group.

Frequently, axioms are formulated to take advantage of an analogy with a rule of inference, serving as *axiomatic reflections* of the rule. Even so, the presence of a particular axiom in a theory is not in itself a license to operate upon that theory. To illustrate, note that because it is a *rule*, when assuming the principle of explosion, one can not fathom *e.g.*, an individual t that satisfies both $P(x)$ and $\sim P(x)$. Including both $P(t)$ and $\sim P(t)$ in a theory T entirely eliminates the utility of T by making its texture indistinguishable from any other inconsistent theory. In contrast, the axiomatic principle of noncontradiction lacks the capacity to single-handedly induce system collapse, *i.e.*, the axiom is in a sense *inert* without further *rule-based* reinforcements under which the theory is closed. Without the addition of the dynamic, rule-based machinery of the principle of explosion, a theory including both $P(t)$ and $\sim P(t)$ will not collapse solely for the presence of $\sim(P(t)\&\sim P(t))$.

There is an obvious parallel with the relationship between syntactic exhaustivity and the principle of excluded middle. On similar grounds, one observes that the mere inclusion of excluded middle in a theory is insufficient to induce exhaustivity in a theory. Given an individual t, that $P(t) \vee \sim P(t)$ holds does not require that a theory include either $P(t)$ or $\sim P(t)$. t can appear in formulae in the theory without the theory weighing in on its classification.

What *would* be analogous would be a requirement of the *negation completeness* of theories or (in the presence of the principle of excluded middle) an insistence that theories must be prime.[5] But theories on standard definitions—classical or otherwise—are not generally required to be complete.

To return to Plumwood's example concerning gender, excluded middle does not require that any individual not falling under a dominant concept will be by default classified under its other. The principle that an individual either falls under "man" or falls under "other-than-man" does not require that the individual in fact receives one or other of these classifications. To return to our illustration:

Example 3.2. Recall the details of Example 3.1 and let T be a theory including excluded middle. Then in case T is not prime, the presence of term $\mathtt{t}$ in the theory requires neither that $\mathtt{Male(t)} \in T$ nor that $\sim\mathtt{Male(t)} \in T$.

In other words, internal to a theory T, space exists for *e.g.* a non-binary individual t to maintain their presence without being forced into the binary. At the level of theories, the link between excluded middle and exhaustivity fails. Despite the compelling informal narrative, Plumwood's focus on theories prevents the narrative from applying without further refinement.

[5] A theory T is described as *prime* when for every disjunction $\varphi \vee \psi$ in T, T includes also either φ or ψ. T is described as *negation complete* when for every formula φ, T includes either φ or $\sim\varphi$. Generally, if a theory is prime and includes excluded middle, this suffices to establish negation completeness.

4 Charting Pathways to Homogenization

Having reviewed Plumwood's stance concerning excluded middle, we are prepared to undertake a reappraisal of our prior naive reading with an eye towards revising the core narrative in ways that would make it more compelling to a reader with similar commitments to Plumwood. To prepare, we recall that the stakes seems to be as follows:

• *Homogenization*—the feature of hierarchical dualisms by which everything not satisfying P are lumped together and stereotyped by a single, homogenizing property of "other-than-P"—is one of the two most prominent features by which classical negation can be leveraged in the oppression of underprivileged groups.

• *Exhaustivity of a domain* requires that any t either falls under P or "other-than-P." The members of "other-than-P" become stereotyped and indistinguishable from one another, clearing the way for homogenization.

• The inclusion of *excluded middle* in a theory T is insufficient to guarantee syntactic exhaustivity in the sense that for all terms t, T need not classify t as either P or "other-than-P". The definition of theory in classical logic requires neither primeness nor negation completeness of T.

In short, despite an intuitively plausible case for a path from excluded middle to homogenization—and thus, hierarchical dualisms—Plumwood's interest in *theories* sidesteps this pathway by severing any apparent connection between excluded middle and exhaustivity. Sidestepped or not, our naive analysis continues to suggest the existence of such connections. Let us consider approaches through which new pathways between the two may be cleared.

Recall that we have assumed a setting of first-degree consequence to highlight negation. From contextual clues in Routley and Routley [1985], it is likely that Plumwood understands the negation of FDE to be the paradigm of non-exclusionary negation. To examine excluded middle, model-theoretic intuitions suggest that we select LP—interpreted as a first-degree logic in which FDE's "neither-true-nor-false" truth-value is dropped—as a foil in the following discussion. LP is paraconsistent but continues to support exhaustivity, making it an ideal setting in which to ask whether paraconsistency suffices to avoid homogenization in the first-degree setting.

4.1 Model- vs. Proof-Theoretic Homogenization

One could make an argument that Plumwood maintains the faultlessness of excluded middle only for maintaining an overly proof-theoretic—rather than model-theoretic—perspective. Again, excluded middle fails to ensure a sort of *syntactic exhaustivity* in theories, *i.e.*, excluded middle does not guarantee negation completeness. This failure renders arguments from excluded middle to exhaustivity inapplicable to Plumwood.

Of course, by reviewing the model theory for LP in Priest [1979], it is clear that this sort of syntactic exhaustivity is complemented by a *model-theoretic exhaustivity*. Moreover, not only is the path from excluded middle to this semantic exhaustivity immediate but the *subsequent* path to a type of *model-theoretic homogenization*, too, is immediate and philosophically compelling.

In LP, the absence of FDE's "neither-true-nor-false" truth-value ensures the *bivalence* of every LP model $\mathfrak{M}$ in the sense that either $\mathfrak{M} \vDash \varphi$ or $\mathfrak{M} \vDash {\sim}\varphi$ for any formula φ. LP is thus model-theoretically exhaustive in the sense that *from the perspective of a model*, any individual not falling under a dominant class P must fall into the common underprivileged class of "other-than-P." For example:

Example 4.1. Recall the details of Example 3.1 and let $\mathfrak{M}$ be an LP model. Then either $\mathfrak{M} \vDash \texttt{Male}(\texttt{t})$ or $\mathfrak{M} \vDash {\sim}\texttt{Male}(\texttt{t})$.

Importantly, this phenomenon of model-theoretic exhaustivity is directly tethered to the validity of excluded middle in LP (as well as in classical logic). Unlike a theory—which may be negation-incomplete—a model decides all sentences. Consequently, while a gender non-conforming t may escape binary classification in a theory T, semantic exhaustivity compels such classification in a model.

If models are pictures that represent reality, then avoiding proof-theoretic exhaustivity at the level of theories might be disregarded as a mere linguistic matter. That excluded middle does not induce this type of exhaustivity, in other words, merely sidesteps homogenization with respect to *the way in which we talk about the world*.

Emphasizing model-theoretic exhaustivity, in contrast, shows that to endorse a logic in which excluded middle holds is to make an assumption about models according to which homogeneity is the norm. One might object that this species of exhaustivity leads to an equivalent picture, a parlor trick that differs only for the technicality that semantics stand in place of syntax. I concede that models themselves are no different from theories in *one* sense, namely, that they are at best abstract and inadequate approximations of the world.

However, suppose that one takes a modestly realist stance, considering sentences to be made true or false in virtue of *the world*. Then this species of exhaustivity ranges over more than mere abstract representations of the world, that is, exhaustivity holds of the world itself. On this stance, the correctness of a negation like that of LP requires that reality—the world—*enforces* the stereotyping and homogenization that Plumwood aims to resolve. In short, while proof-theoretic exhaustivity is a constraint about how we *talk about* the world, model-theoretic exhaustivity is a condition on what we *must take the world to be*.

It strikes me as ill-conceived to consider excluded middle to be innocent for the absence of syntactic homogeneity in theories. We may retain excluded middle without having this oppression reflected in the language describing the world, but this leaves oppressive homogeneity in the world itself intact. Moreover, the implicit assumption that the resulting homogeneity is enforced by (*i.e.* emanates from) the world supports a further *naturalization* of oppression that entrenches dualisms more deeply by presenting oppression *e.g.* in the guise of natural law. The severity of the consequences of such a scientization of oppression is noted by Plumwood in Plumwood [1993b] while discussing Albert Memmi [1967]:

> What is actually a sociological point comes to be labelled as being biological or, preferably, metaphysical. It is attached to the colonized's basic nature.[Memmi, 1967, p. 71]

In short, if one is to eliminate homogeneity not only from the description of the world, but from the world in which those descriptions are made true, then excluded middle appears to act not only as a source supporting this feature of hierarchical dualisms, but a source that is particularly destructive.

4.2 Assertive vs. Rejective Homogenization

One need not look to model theory to discover species of exhaustivity that directly lead to homogenization. We can, instead, cite recent reappraisals of the notion of *theory* to diagnose the presence of oppressive homogenization at a proof-theoretic level, *i.e.*, discover oppression at the level of theories themselves.

Plumwood's critique is carried out against the received definition of theory employed by her contemporaries, in which theories are understood *unilaterally* as collections of commitments, assertions, *etc.*. It is on this *particular reading* that the failure of primeness serves as an *inoculation* against the emergence of homogeneity.

Although there were few—if any—substantially alternative definitions during the composition of Plumwood [1993a], Plumwood [1993b], and Plumwood [2002], subsequent analyses of the standard definition have led to the introduction of more nuanced accounts. This more recent work opens the possibility that Plumwood's emphasis on the explosive nature of negation—and her overlooking negation's dual feature of exhaustivity—are merely artifacts of the contemporary state of the art as Plumwood understood it.

In particular, let us turn to recent *bilateral* accounts of theory. In the wake of Ian Rumfitt's work in Rumfitt [2000] an acknowledgement has grown of the virtues of bilateral methods in logic, *i.e.*, approaches in which assertion and denial are treated on an equal footing. Greg Restall's Restall [2005] provides a notable example in its interpretation of sequents $[\Gamma \Rightarrow \Delta]$ as *positions* on which an agent may take a stand, interpreting Γ and Δ as collections of that agent's *assertions* and *rejections*, respectively.

The concept of position can be abstracted into a novel, bilateral definition of theory. Such "bitheories" have been investigated by Restall himself in Restall [2013] and have received a model-theoretic analysis by Carolina Blasio and collaborators in Blasio et al. [2021]. Blasio *et al.* describe the limitations of the unilateral definition:

> The standard notion of formal theory, in logic, is in general biased exclusively towards assertion: it commonly refers only to collections of assertions that any agent who accepts the generating axioms of the theory should also be committed to accept.Blasio et al. [2021]

Our informal inferential practice ranges over both assertions and denials. Affording inferential currency exclusively to assertions induces a Procrustean model of consequence for its failure to reflect important features of reasoning, *e.g.*, inferring rejections from prior rejections. Blasio *et al.* address this inadequacy by defining theories bilaterally as repositories in which both assertions and rejections are recorded. Importantly, there is a sense in which Plumwood herself anticipates such bilateralism in the *record cabinet model* of negation described in her joint paper with Sylvan:

> The cabinet, which can represent the files of the universe, is full of records, each record is an *issue*, or question, with p on one side and $\sim p$ on the other side... Relevant negation takes p as one side of the record and $\sim p$ as the other side of the *same* record, there being many many records in the cabinet.[Routley and Routley, 1985, p. 219]

On a bilateral view of theory, the failure of excluded middle to establish exhaustivity can be given a more refined gloss as the failure to establish *assertive exhaustivity*, *i.e.*, a requirement that one must classify each t under either P or $\sim P$ through assertions in the theory. The bilateral definition allows further contrast; treating rejections as first-class objects permits one to express a complementary notion of *rejective exhaustivity* that governs one's denials.

Recall once more the setting of first-degree consequence for the present and our interpretation of the negation of LP as a semantically complete correlate of FDE negation. Returning to LP in this setting permits us to examine negation in the bilateral context with precision while ensuring continuity with earlier sections.

We will sketch a picture of bilateral theories in LP that is inspired by the work in Blasio et al. [2021]. This development will be semantic in nature—defining features in terms of LP models—but there is no *a priori* barrier to defining such theories proof-theoretically. *E.g.*, it would be a straightforward exercise to introduce semantic tableaux that are sound and complete with respect to the following definitions. With luck, this promissory note will suffice to make the case compelling even to a proof-theory-centric view like Plumwood's.

In such bilateral notions of theory, we might take a theory to be a pair $\langle T_-, T_+\rangle$ with an intended interpretation in which T_+ and T_- are sets of assertions and denials, respectively. We take the notion of theory to be closed under valid inferences, motivating several definitions:

Definition 4.2. For a pair of sets of sentences $\langle \Gamma_-, \Gamma_+\rangle$, let $Cn^+_{\mathsf{LP}}(\langle \Gamma_-, \Gamma_+\rangle)$ be the set

$$\{\varphi \mid \text{ for all models } \mathfrak{M}\text{, if } \begin{cases} \mathfrak{M} \vDash \psi \text{ for all } \psi \in \Gamma_+, \ \& \\ \mathfrak{M} \nvDash \xi \text{ for all } \xi \in \Gamma_- \end{cases} \text{then } \mathfrak{M} \vDash \varphi\}.$$

Definition 4.3. For a pair of sets of sentences $\langle \Gamma_-, \Gamma_+\rangle$, let $Cn^-_{\mathsf{LP}}(\langle \Gamma_-, \Gamma_+\rangle)$ be the set

$$\{\varphi \mid \text{ for all models } \mathfrak{M}\text{, if } \begin{cases}\mathfrak{M} \models \psi \text{ for all } \psi \in \Gamma_+, \, \& \\ \mathfrak{M} \not\models \xi \text{ for all } \xi \in \Gamma_-\end{cases} \text{ then } \mathfrak{M} \not\models \varphi\}.$$

To paraphrase Blasio *et al.*, for a bilateral theory $\langle T_-, T_+\rangle$:

> The elements of $[Cn^+_{\mathsf{LP}}(\langle\Gamma_-, \Gamma_+\rangle)]$ might be thought of as the sentences that one is committed to assert once the sentences in $[\Gamma_+]$ are all asserted, in the context of the denial of all the sentences in $[\Gamma_-]$... Analogously, the elements of $[Cn^-_{\mathsf{LP}}(\langle\Gamma_-, \Gamma_+\rangle)]$ might be thought of as the sentences that one is committed to deny once the sentences in $[\Gamma_-]$ are all denied, in the context of the assertion of all the sentences in $[\Gamma_+]$.

This allows us to introduce a definition of deductive closure in LP of bilateral theories:

Definition 4.4. A bilateral LP theory must be deductively closed so that

$$\langle T_-, T_+\rangle = \langle Cn^-_{\mathsf{LP}}(\langle T_-, T_+\rangle), Cn^+_{\mathsf{LP}}(\langle T_-, T_+\rangle)\rangle.$$

Clearly, for any LP validity φ, if $\varphi \in T_-$, both the set $Cn^+_{\mathsf{LP}}(\langle T_-, T_+\rangle)$ and and the set $Cn^-_{\mathsf{LP}}(\langle T_-, T_+\rangle)$ will include the entire language (as there are no LP models for which $\mathfrak{M} \not\models \varphi$, the condition will be satisfied vacuously).

Consequently, the validity of excluded middle in LP translates to the trivialization of any bilateral theory $\langle T_-, T_+\rangle$ including a formula $\varphi \vee {\sim}\varphi \in T_-$. In turn, this translates to a proscription against the inclusion of both φ and ${\sim}\varphi$ in the set T_- on pain of trivialization.

This, too, translates to a sort of *rejective homogeneity* that whenever the inclusion of t within a dominant class P is *rejected*, its inclusion under the oppressed "other-than-P" *cannot be rejected*. To return a final time to our running illustration:

Example 4.5. Recall the details of Example 3.1 and let $\langle T_-, T_+\rangle$ be a bilateral theory in LP. Then if $\texttt{Male(t)} \in T_-$ and ${\sim}\texttt{Male(t)} \in T_-$, the theory is trivial.

The additional expressivity licensed by bilateral theories reestablishes the causal pathway from excluded middle to homogeneity, as can be easily recognized. Because excluded middle is valid, for no model $\mathfrak{M}$ does $\varphi \vee {\sim}\varphi$ take a non-designated value. Thus, in any model, either φ or $\sim\varphi$ must take a designated value. So there are no models in which φ and $\sim\varphi$ are both found in the set of denials in a bilateral theory, for otherwise the bilateral theory would be trivial.[6]

In other words, for any term t, if one denies that a non-binary t falls under the dominant class `Male`, one *must not deny* that t falls under the class of other-than-`Male`. This amounts to a logical "gag order" disallowing non-binary individuals from maintaining bilateral theories in which the expression of their gender identity is respected. By shifting once more to a new framework, we identify a type of homogenization that follows from the validity of excluded middle.

5 Dualistic Features and Relevance

In this concluding section, I would like to examine whether Plumwood's preference for relevant logics is in fact a necessary consequence of her critique of logic.

In the foregoing—constrained to the first-degree case—I have tried to suggest that the spirit of Plumwood's critique may be incompatible with a logic whose negation resembles that LP insofar as its inclusion of excluded middle supports homogenization—and thus hierarchical dualisms—despite its paraconsistency. But while this gets us closer to a link between Plumwood's critique and relevance, it does not yet decisively answer the question.

If, *e.g.*, Plumwood's critique rules out the negations of Priest's LP (for its bivalence) and Kleene's K_3 (for its explosiveness), then one is not necessarily driven to the relevant negation of FDE in the first-degree case. The negation of the quasi-relevant logic R-Mingle (RM) and its first-degree fragment ($\mathsf{RM}_{\mathtt{fde}}$) is neither explosive nor bivalent. This leaves a question open concerning whether *relevance* is in fact necessitated by Plumwood's critique or whether a non-relevant notion of consequence like RM may itself suffice.

As Plumwood acknowledges, there are a host of rivals to classical logic whose negations do not support hierarchical dualisms. The preliminary nature of Plumwood's discussion leaves a great deal open about how to land on optimal solutions; relevant logics are identified as merely one possible family

[6] Again, *n.b.* that the semantic argument is merely an expediency. If a proof-theoretic argument would be more convincing, some back-of-the-napkin calculations will easily allow one to define a bilateral proof theory for such theories.

of solutions. But relevance is a feature characterized by the properties of *implication* whereas we have so far touched only lightly on oppressive features of the classical interpretation of vocabulary beyond negation. Although this variety of potential candidates provides fertile space inviting further development, it is a space that has so far been unapproachable to us for the stipulation that we would consider first-degree systems exclusively. As promised, we will now move past the first-degree setting to consider the more general case.

5.1 Dualistic Features of Implication

First, we will review what additional requirements Plumwood's critique imposes in the face of this new expressivity. Despite the priority that negation receives in Plumwood's discussion, we have acknowledged that Plumwood [1993a], Plumwood [1993b], and Plumwood [2002] include limited—but important—observations concerning oppressive features of classical *implication*. These constitute the few definitive constraints on implication Plumwood places on the class of suitable logics that can not be conscripted into logics of oppression.

Three such oppressive features of classical implication receive varying degrees of analysis. These are classical logic's feature of *truth suppression*, its *truth-functional criterion of propositional identity*, and its feature of *truth interchangeability*. We will consider these features in order, although we will pay special attention to the first of them.

The first feature is classical implication's support of *truth suppression* by which a premise's contribution to an entailment can be erased in case that premise is true. Plumwood describes how this feature of truth suppression leads to oppressive features in the following terms.

> The suppression of premises on condition of their truth gives formal expression to the dualistic condition of backgrounding, in which the contribution of the other to the outcome is relied upon but denied or ignored. If the major task of logic is about showing... a logic allowing truth suppression is about hiding.[Plumwood, 1993a, p. 455]

In contrast to her exhaustive work on suppression in Plumwood [2022] and Routley et al. [1982], Plumwood's account of truth suppression in her work on feminist critiques of logic is expressed cursorily. The succinct nature of her later remarks invites the risk of ambiguity. For our purposes, it will be to our benefit to produce a more formal representation (or, at least, several formal representations).

Getting a handle on the notion of truth suppression with any real detail, however, poses some surprising difficulties. Plumwood was the driving force behind investigations into an unqualified notion of "suppression," having written most of the relevant material in Routley et al. [1982] (drawn from her earlier 1967 paper Plumwood [2022]). That Plumwood in her Plumwood [1993a] describes this feature in a qualified form as "*truth* suppression" presents challenges as the precise relationship it bears to suppression *simpliciter* is unclear, namely, concerning how the qualifier "truth" is to be understood.

This qualification, on the one hand, might be understood to indicate that Plumwood means a *restricted* variety of a more general notion of suppression. Such restricted variants of suppression are natural; one might accept that some propositions are suppressible while others are not. As an example, for Sylvan *et al.* in Routley et al. [1982], particular degrees of suppression distinguish between the classical logician (*all truths are suppressible*), the advocate of strict implication (*only necessary truths are suppressible*), and the mainstream relevant logician (*only validities are suppressible*).

In light of the availability of these gradations, the most faithful choice of interpretation for the qualifier "truth" is not immediately obvious. That "truth suppression" should be understood *maximally* (*no truths of any type are suppressible*) or *minimally* (*no simple contingent truth is suppressible but all others are suppressible*) seem to be equally plausible readings. This leads to an interpretative dilemma. When considering the degree to which truth suppression runs counter to the aims of Plumwood's feminist critique, one could provide corresponding maximal and minimal interpretations.

Given the thoroughness of Plumwood's investigations into suppression in Plumwood [2022] and Routley et al. [1982], it is tempting to simply import these discussions as exegeses of the later "truth suppression." I suspect that considering the foregoing gradations warns against too hastily conflating the "truth suppression" of Plumwood [1993a] with "suppression" *simpliciter* of Plumwood [2022]. Although, as Eckert and Donahue [2020] convincingly argues, there is a great deal of continuity between Plumwood's feminist work and her earlier work with Sylvan, the individual critiques of suppression found in the 1967 paper Plumwood [2022] and the later Plumwood [1993a] differ significantly. The tone of Plumwood's remarks on suppression drawn from Plumwood [2022] reflects an author fully occupying the role of relevant logician; the continuity so immediately identifiable between Routley and Routley [1985] and Plumwood [1993a] is not so obvious here.

The analysis of suppression in Plumwood [2022] identifies a number of "false laws of logic" rejected for their being suppressive:

> These laws are *Exportation*, *Commutation*, (as well as various restricted forms of these), *Exported Syllogism* and *Disjunctive Syllogism*. All these laws are false for the same reason—that they license the suppression or replacement in some position of some class of propositions which cannot legitimately be suppressed or replaced.[Plumwood, 2022, p. 1]

Rather than considering each of these principles individually,[7] we can straightforwardly appeal to two formal constraints Plumwood outlines in this work to give succinct accounts of a wide interpretation of "truth suppression." First, we can cite a condition Plumwood offers as characteristic of suppression:

> [F]or every proposition p there is some proposition q such that the consequences of q are a proper subset of the joint consequences of p and q. There is no privileged class of propositions which are generally suppressible.[Plumwood, 2022, p. 6]

The maximal breadth of this condition is reflected in the requirement that no class of propositions is suppressible. In Øgaard [2020], Tore Fjetland Øgaard approaches the task of formalizing this principle, and we will follow his characterizationi:

Definition 5.1. A logic satisfies the Anti-Suppressive Principle if for every formula φ, there exist formulae ψ and ξ such that $\vdash (\varphi \,\&\, \psi) \rightarrow \xi$ but $\nvdash \psi \rightarrow \xi$

Alternately, Plumwood describes a property that entails the above characterization by means of a *Joint Force Principle* that

> tells us that the joint consequences of propositions may be more than the sum of the consequences of each.[Plumwood, 2022, p. 7]

[7]But *n.b.* that we will return to these briefly before concluding.

Again, we borrow from Øgaard's Øgaard [2020] to formally present this:

Definition 5.2. A logic satisfies the Joint Force Principle if for every formula φ, there exist formulae ψ and ξ such that $\vdash (\varphi \mathbin{\&} \psi) \to \xi$ while $\nvdash \varphi \to \xi$ and $\nvdash \psi \to \xi$.

Consequently, Plumwood's requirement that an acceptable logic does not validate "truth suppression" in the wide sense will be understood as that logic's satisfying either the Anti-Suppressive Principle or the Joint Force Principle.

The narrow interpretation, on the other hand, may be driven by noting that the feminist interest in truth suppression differs *a priori* from that of *e.g.* the relevant logicians. The present goal is not a logic that exhibits no paradoxical features in general but a logic that exhibits no *oppressive* features. This suggests a need for a more targeted interpretation of truth suppression, *i.e.*, a definition that is tailored to Plumwood's aims in *e.g.* Plumwood [1993a]. Whereas Sylvan (and Plumwood, in the mode of relevant logician) is concerned with suppression for its deductively paradoxical character, from the perspective of a feminist critique the importance of suppression begins and ends with its role in supporting oppressive dualisms. If backgrounding is supported only by special cases of suppression, then there are no grounds to reject suppression as a whole.

Importantly, Plumwood accompanies her discussion of truth suppression with a formula described as *Exploitation*:[8]

$$(p \mathbin{\&} ((p \mathbin{\&} q) \to r)) \to (q \to r)$$

Intuitively, one may read *Exploitation* as the principle that: Whenever p and q *jointly* entail r, the truth of p provides grounds to draw a further inference that q *alone* entails r. No matter how crucial the role p may play in the entailment $(p \mathbin{\&} q) \to r$, its contribution can be erased by the lights of classical implication.

The language through which Plumwood introduces *Exploitation*—asserting that the link between suppression and backgrounding "is most clearly expressed" [Plumwood, 1993a, p. 455] by *Exploitation*—yields a clue as to the degree to which a feminist logic may tolerate suppression in general. As a

[8]The name is chosen as an allusion to the related principle of *Exportation* that is more commonly encountered as a concern over the development of relevant logics.

maximally perspicuous encapsulation of the conditions under which truth suppression engenders backgrounding, *Exploitation* draws a characteristic boundary between the senses of truth suppression that support hierarchical dualisms and the senses that do not.

We will let this form the basis of a narrow criterion as regards truth suppression, namely, that a condition on a logic's being acceptable by the lights of Plumwood's feminist critique is that it does not validate *Exploitation*.

The second feature involves a logic's maintaining a *truth-functional criterion of propositional identity* by which any two propositions assigned the same truth-value must be identified with one another. In order to achieve an acceptable logic, then, we must find a deductive system for which the following fails:

> material equivalence as a criterion of propositional identity yields just one true and one false proposition.[Plumwood, 1993a, p. 455]

We have already encountered this feature in Section 2.2, devoting some space to discussing connections between the criterion and the feature of homogenization. As we have acknowledged, this feature leads to a sort of *maximal* or *global* homogenization that collapses all distinctions between propositions to the True and the False. This is an even stronger feature than the more fine-grained homogenization cited by Plumwood, which homogenizes along particular dimensions individually. While the homogenization we have primarily discussed collapses distinctions internally to the other, this stronger homogenization goes further by conflating even $P(s)$ and $\sim P(t)$ in many cases.

Although such a feature can be understood proof-theoretically, it may be most natural to interpret the feature model-theoretically so that $\varphi \leftrightarrow \psi$ is true in a model precisely when the model assigns φ and ψ the same truth-value. So understood, that classical implication enforces such a criterion of propositional identity can be immediately read from a truth-table for the biconditional.

Consequently, the second criterion for a logic may be characterized as a requirement that its implication on a model theoretic interpretation eschews this classical notion of propositional identity in favor of a more nuanced definition.

Plumwood describes a final dualistic feature of *truth interchangeability*. Although described as closely related to the feature of truth suppression, Plumwood offers a far more terse explication, merely describing this feature as the principle that

> any truth can be substituted for any other truth while preserving implicational properties.[Plumwood, 1993a, p. 455]

The description is silent with respect to the character of the "implicational properties" preserved in a system with truth interchangeability.

It seems reasonable, however, to interpret the basis of this condition model-theoretically so that whenever two formulae φ and ψ are true:

- $\varphi \to \xi$ is true if and only if $\psi \to \xi$ is true, and
- $\xi \to \varphi$ is true if and only if $\xi \to \psi$ is true

for any formula ξ. A simple recursion suggests that between these two points, truths can be substituted *salva veritate* when nested arbitrarily deeply within the scope of $\to$. This condition, surely, is as least as strong as Plumwood's "preservation of implicational properties."

This leads to our characterization of the final criterion for a logic to satisfy Plumwood's critique of implication: Its models must avoid truth interchangeability in the above sense.

5.2 Is Relevance Required?

Having provided formal counterparts to the three requirements on implication described by Plumwood, we are equipped to start investigating whether relevance is indeed required. In particular, we will consider the "quasi-relevant" logic RM (or R-Mingle), which is not relevant in virtue of *e.g.*, validity of the *Safety* principle $(\varphi \wedge {\sim}\varphi) \to (\psi \vee {\sim}\psi)$.

First, let us examine the criteria—both wide and narrow—by which an acceptable logic can be said to avoid truth suppression. First, consider the narrow interpretation. On this more targeted interpretation, it can be seen that RM avoids truth suppression. We assume the model theory of Dunn from Dunn [1976]; details can be immediately found in that paper.

Observation 5.3. $(p \mathbin{\&} ((p \mathbin{\&} q) \to r)) \to (q \to r)$ *is not a theorem of* RM.

Proof. We provide a countermodel. Let $\langle G, \{G\}, R\rangle$ be the model where GRG and $\varphi(p, G) = \varphi(r, G) = \{T, F\}$ and $\varphi(q, G) = \{T\}$ (*i.e.*, p and r are both true and false at G while q is simply true).

• That $\varphi(p \mathbin{\&} q, G) = \{T, F\}$ follows because T is a member of both $\varphi(p, G)$ and $\varphi(q, G)$ (*i.e.* $p \mathbin{\&} q$ is true) while $F \in \varphi(p, G)$ (*i.e.* $p \mathbin{\&} q$ is also false).

• That $\varphi((p \mathbin{\&} q) \to r, G) = \{T, F\}$ is straightforward. To see that T is an element, note that $\varphi(p \mathbin{\&} q, G) = \varphi(r, G)$, whence T (respectively, F) is a member of one precisely when T (respectively, F) is a member of the other. On the other hand, $T \in \varphi(p \mathbin{\&} q, G)$ while $F \in \varphi(r, G)$, so F is an element.

• That $\varphi(p \mathbin{\&} ((p \mathbin{\&} q) \to r), G) = \{T, F\}$ follows from the prior two points.

• To see that $\varphi(q \to r, G) = \{F\}$, note that $F \in \varphi(r, G)$ while $F \notin \varphi(q, G)$, whence the requirement that F is a member of the consequent only if F is a member of the antecedent fails. Thus, $T \notin \varphi(q \to r, G)$, whence $\varphi(q \to r, G) = \{F\}$.

• Finally, because GRG and $T \in \varphi(p \mathbin{\&} ((p \mathbin{\&} q) \to r), G)$ while $T \notin \varphi(q \to r, G)$, $\varphi((p \mathbin{\&} ((p \mathbin{\&} q) \to r)) \to (q \to r), G) = \{F\}$, whence the formula is not a theorem.

□

Although RM is not a relevant logic, the existence of countermodels for *Exploitation* shows that RM satisfies Plumwood's first criterion, understood on the narrow reading of truth suppression. This is not surprising, as *Exploitation* is (in Anderson and Belnap's terminology) a *modal fallacy* and is resolved in strict implication formulations of non-relevant modal logics like S4.

What of the *wide* readings of Plumwood's first criterion according to which any candidate logic must satisfy either the Anti-Suppressive Principle or the Joint Force Principle? While it would be desirable to continue working with RM, I lack an answer concerning whether RM is non-suppressive in the wide sense. But as an alternative, we can cite a different non-relevant logic—and some results concerning this logic due to Øgaard—in order to provide a proxy.

We had described two ways to understand the question of whether a logic is free from truth suppression in the wide sense. We offer the following result of Øgaard from Øgaard [2020] to show the following:

Observation 5.4. *There are non-relevant logics that satisfy the* Joint Force Principle*.*

Proof. The proof of Theorem 2 in Øgaard [2020] introduces a logic SIE extending Anderson and Belnap's E with new axioms, among which is the irrelevant *Safety* axiom, noting that SIE satisfies the Joint Force Principle. □

As a corollary, we can note that the other formalization of a wide reading of truth suppression is avoided by SIE as well:

Corollary 5.5. *There are non-relevant logics that satisfy the* Anti-Suppressive Principle*.*

Proof. Consulting the definitions immediately establishes that any logic satisfying the Joint Force Principle also satisfies the Anti-Suppressive Principle. Thus, Øgaard's SIE also satisfies the Anti-Suppressive Principle as a corollary of Observation 5.4. □

Øgaard's SIE thus witnesses that wide senses of "truth suppression" can be avoided in systems that are not relevant.

At least as the first requirement for a logic free of dualistic implication is concerned, we so far have a guarantee that *some* non-relevant logic satisfies the condition of freedom from truth suppression, whether on a wide or narrow reading of Plumwood's Plumwood [1993a].

To examine the final two criteria—avoidance of a classical criterion of propositional identity and truth interchangeability—we return to RM. As our interpretation has been semantic, we again borrow from Dunn's Dunn [1976]. First, note that RM models avoid the classical notion of propositional identity criticized by Plumwood as an oppressive feature:

Observation 5.6. *In* RM*, that p and q are assigned the same truth values does not entail the truth of $p \leftrightarrow q$.*

Proof. Consider a model $\langle G, \{G, H\}, R\rangle$ with GRH such that $\varphi(p, G) = \varphi(p, H) = \varphi(q, G) = \{T\}$ and $\varphi(q, H) = \{T, F\}$. Then $\varphi(p \to q, G) = \{F\}$ because $F \in \varphi(q, H)$ while $F \notin \varphi(p, H)$. Consequently, $T \notin \varphi((p \to q)\ \&\ (q \to p), G)$ despite p and q being truth-functionally equivalent at G. □

Likewise, we can show that RM is free of the feature of truth interchangeability as we have formalized it:

Observation 5.7. *In* RM*, that p and q are both true at a world H does not guarantee the truth of $q \to r$ at H whenever $p \to r$ is true at H.*

Proof. We provide a countermodel. Let $\langle G, \{G, H\}, R\rangle$ be a model where GRH. Define φ so that $\varphi(p, G)$, $\varphi(q, G)$, $\varphi(q, H)$, and $\varphi(r, H)$ are assigned $\{T\}$ while $\varphi(p, H)$ and $\varphi(r, H)$ are assigned $\{T, F\}$. Note that p and q are both true—indeed, both are *only* true—at G.

Note also that $T \in \varphi(p \to r, G)$; that $\varphi(p, G) = \varphi(r, G)$ and $\varphi(p, H) = \varphi(r, H)$ ensures that each of T and F is an element of the value of the antecedent precisely when it is a value of the consequent. But there exists an accessible world H such that $F \in \varphi(r, H)$ although $F \notin \varphi(q, H)$, whence $\varphi(q \to r, G) = \{F\}$. □

We have also the complementary observation:

Observation 5.8. *In* RM*, that p and q are both true at a world H does not guarantee the truth of $r \to q$ at H whenever $r \to p$ is true at H.*

Proof. For a frame with single world G, let $\varphi(p, G) = \varphi(r, G) = \{T\}$ and $\varphi(q, G) = \{T, F\}$. Then despite p and q both being true in G, $T \in \varphi(r \to p, G)$ (as $\varphi(p, G) = \varphi(r, G)$) while $T \notin \varphi(r \to q)$ (because $F \in \varphi(q, G)$ while $F \notin \varphi(r, G)$). □

So RM model theory shows that the semantic interpretation of the latter two dualistic features are avoided. Unfortunately, we have no corresponding model theory for SIE at this point, so are unable to meaningfully articulate the question of whether Øgaard's system is free of these two features. (Although, as we will see soon, this fact does not present too much of a drawback.)

To summarize, we know that on the narrow interpretation of suppression, RM is a non-relevant logic that avoids the three dualistic features of implication described in Plumwood [1993a]. On the wide interpretation of suppression, we know at least that Øgaard's non-relevant SIE avoids suppression in both senses introduced by Plumwood in Plumwood [2022] but without a model theory, cannot directly analyze the latter two criteria of its *criterion of propositional identity* and *truth interchangeability*, which both seem best analyzed semantically.

But we can nevertheless provide a witness for a non-relevant logic that avoids all three features of dualistic implication: The intersection logic RM ∩ SIE. As *Safety* is valid in both, this logic is non-relevant. Likewise, on any interpretation of "truth suppression," $\mathsf{RM} \cap \mathsf{SIE}$ is suppression-free. Finally, even without a model theory with respect to which the intersection logic is sound and complete, it follows from the properties of intersection that every RM model is a $\mathsf{RM} \cap \mathsf{SIE}$ model. Consequently, $\mathsf{RM} \cap \mathsf{SIE}$ both has a non-dualistic criterion of propositional identity and avoids truth interchangeability.

Before concluding, let us return again to the initial lines of Plumwood [2022], in which Plumwood proffered a list of particular "false laws" that were suppressive. As a referee has pointed out, the joint rejection of these principles evinces a condition even stronger that what I have described as the "wide" reading of suppression for the following reason: Although Øgaard's SIE satisfies both the Anti-Suppressive and Joint Force Principles, as an extension of E, it nevertheless licenses *Exported Syllogism*

$$(p \to q) \to ((q \to r) \to (p \to r))$$

which had been rejected as suppressive. This suggests the potential of an even stronger interpretation beyond the Anti-Suppressive Principle alone.

Given Plumwood's remarks on *Exploitation*, I suspect that the narrow interpretation best fits her feminist critique. Furthermore, the degree to which *Exported Syllogism* is in fact suppressive is obscure. Sylvan *et al.* describe it as merely a "very limited form of suppression and... not one which produces spectacular effects, like more wholesale suppression"[Routley et al., 1982, p. 271] and Øgaard argues in Øgaard [2020] that its rejection as suppressive is simply confused. Essentially, what I have described as the wide interpretation appears to be the maximal *plausible* reading of suppression.

Nevertheless, it is of interest to investigate whether this yet stronger notion of suppression compels one to adopt a relevant logic. Providing an answer requires enough technical work to demand its own paper. But we can conclude by giving a this future investigation a small kick-start with a brief observation.

As Plumwood observes, *Exported Syllogism* in *axiom form* fails in C.I. Lewis's $\mathsf{S2}$ system of strict implication. Consequently, *Exported Syllogism* will not be a theorem of the further intersection $\mathsf{RM} \cap \mathsf{SIE} \cap \mathsf{S2}$.[9] Moreover, it is also easy to confirm that *Safety* is valid in $\mathsf{S2}$, whence $\mathsf{RM} \cap \mathsf{SIE} \cap \mathsf{S2}$ remains irrelevant. Of course, considering *Exported Syllogism* in a *rule form*

[9] Upon identifying Lewis's $\prec$ with the relevant $\to$, of course.

requires more work.[10] Nevertheless, this observation suggests the possibility that one can incrementally work one's way through stronger interpretations of suppression while finding non-relevant deductive calculi satisfying the increasingly strong criteria.

6 Conclusions

I will not posture as though I have conclusively demonstrated that Plumwood's critique demands a rejection of excluded middle nor that it is in fact compatible with a non-relevant logic. What I believe has been established, however, is that there remains room for discussion of and elaboration on Plumwood's critique and where its borders lie. Hopefully, the foregoing remarks suffice to spark some thought concerning how to best respect and implement Plumwood's feminist critique, especially in the higher-degree case.

I will conclude with the following two suggestions:

- While RM seems to avoid the oppressive features of implication cited by Plumwood, this is not to say that no further oppressive features lie in waiting, left undiagnosed by Plumwood. Andrew Tedder's recent Tedder [2022] provides some compelling critiques of RM and $\mathsf{RM}_{\mathtt{fde}}$ (including Bob Meyer's critique of RM in Anderson and Belnap [1975]). Revisiting Tedder's observations through a Plumwoodian lens will likely help to clarify the relationship between hierarchical dualisms and relevance itself and I plan to return to this in a sequel to this piece.

- That a link should exist between Plumwood's program and relevance opens a space for a radical feminist critique of *mathematics*. While feminist critiques of mathematical practice have almost universally stopped at critiques of mathematical pedagogy or hiring and tenure procedures, Plumwood's critique of deduction induces a feminist reading of Robert K. Meyer's program of *relevant arithmetic* outlined in Meyer [2021a] and Meyer [2021b] that is more radical in its conclusions. As documented in Ferguson and Priest [2021], Meyer's system $\mathsf{R}^\sharp$ witnesses that arithmetical practice under a less oppressive deductive regime leads to radical shifts in the fabric of mathematical truth. This topic will be addressed in a sequel to this piece as well.

[10] As Øgaard points out in Øgaard [2020], the rule form of *Exported Syllogism* is valid even in the extremely weak relevant logics favored by Sylvan and Plumwood, such as DK and DL.

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