In Support of Valerie Plumwood

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Abstract

This paper offers general support for what Valerie Plumwood's paper is trying to achieve by supporting the rejection of each of her four "false laws of logic": exportation, illegitimate replacement, commutation (aka. permutation) and disjunctive syllogism. We start by considering her general characterizations of entailment, beginning with her stated definition of entailment as the converse of deducibility. However, this applies to a wide range of relevant logics and so is not able to be used as a criterion for deciding what laws to include in a logic. In this context, we examine the two key differences between deduction from premises to conclusion and entailment from antecedent to consequent. We also consider her use of sufficiency as a general characterizing feature. We then discuss Plumwood's syntactic criteria used to reject the first three of her false laws of logic and add the Relevance Condition in this context. We next consider semantic characterizing criteria for a logic. After making a case against using truth, we introduce Brady's logic MC of meaning containment. We then examine the content semantics for MC and use it to reject all of Plumwood's false laws of logic together with some others. We follow with the related Depth Relevance Condition, which is a syntactic criterion satisfied by MC. This clearly rejects the first three of these laws and many others, but not the fourth law. We conclude by giving our overall support for her general enterprise.

1 Introduction.

Brady met Richard and Val Routley (previously Macrae) at the University of New England in 1966, as Richard was one of the lecturers in the one-year

MA Formal Logic course that Brady took in that year. (‘Valerie’ is shortened to ‘Val’ because this was always how she was called.) Val took this course herself in the previous year, subsequently marrying Richard. Brady not only enjoyed this course but was also inspired to continue his studies in formal logic, keeping in touch with Richard Routley along the way. Brady then moved to St Andrews to complete a PhD under Professor Len Goddard. Indeed, it was Goddard who set up the MA Formal Logic course over 1965-6, being the first course of lectures on formal logic in Australia and some of the very few logic subjects on offer in Australia at the time. He then went on to take the Chair of Logic and Metaphysics at the University of St Andrews early in 1967.

Val Routley gave a paper on suppression-less logic at the University of St Andrews in September 1967, which Brady attended. As far as Brady recollects, Richard Routley supported relevance as a criterion for a good logic, rather than one based on lack of suppression, and there would have been some discussion along these lines during the paper. It would also have been pointed out that lack of suppression does not always work as a criterion, as suppression of a provable antecedent can still leave the residual entailment valid in some particular cases. Val’s current paper “Some False Laws of Logic”, Brady believes, made a correction taking this into account, by making the distinction between suppression and omission, where omission was introduced to cover the cases where omission of a provable antecedent maintains the validity of the residual entailment. However, all this is based on distant memories.

After a period as a Research Reader at Monash University, Richard was appointed as a Senior Fellow at the Research School of Social Sciences, ANU, in Canberra. After a short stay in a Canberra suburb, Richard and Val then settled in Plumwood Mountain, a rainforest to the east of Braidwood in NSW, where the Routleys spent many years building a house, whilst living and doing their writing in a large on-site garage. (Indeed, the Routleys named the mountain, Plumwood Mountain, because of its very large stand of Plumwood trees with the absence of eucalypts.) Whilst there, they both discussed topics in logic, metaphysics and conservation, though it was Richard who was more heavily involved in the formal logical side of things. Many joint publications emerged, the key ones being the tome Exploring Meinong’s Jungle and Beyond in 1980 and the first volume of Relevant Logics and their Rivals in 1982. (See Routley [1980], which was reprinted in four volumes under the heading The Sylvan Jungle, with accompanying essays, as Routley

[2018] and [2019], and Routley and Routley [2019] and [2020]. See Routley, Meyer, Plumwood and Brady [1982] for the latter volume.) Although the Meinong volume was originally published in [1980] in Richard's name only, Val certainly contributed to it with some joint papers on Meinong previously written. (See Routley and Routley [1973], [1975] and [1975a].) Val also contributed to the important formal paper, Routley and Routley [1972], which preceded and influenced the Routley-Meyer semantics of relevant logics.

Sadly, the Routleys split up around 1982, at about the same time as the first volume of Relevant Logics and their Rivals was published, with Val changing her surname to Plumwood, after Plumwood Mountain, at which Val had then continued to stay for most of her remaining years. We will henceforth refer to her as Plumwood.

As much water has flowed under the bridge since Plumwood's paper was written, rather than consider the paper in some detail, we will rather pick up on some salient points that Brady sees as most relevant to the content of her paper. These points will be raised in accordance with the Brady's particular views on logic, which will differ considerably from those of the non-classical logical landscape in 1967-8, when the paper was conceived. Nevertheless, we see a lot of value in Plumwood's approach and we give general support for her enterprise.

We set out the structure of our paper, as follows. In §2, we start by considering general characterizations for entailment, but especially Plumwood's stated deducibility interpretation for entailments, making a case in natural deduction for this interpretation. However, we do need to distinguish between deduction within a subproof and deduction in the main proof, as the latter is usually formalized as a rule-arrow $\Rightarrow$, as opposed to the entailment $\to$ of a subproof. We go on to consider the two key differences between deductions using $\Rightarrow$ and entailments using $\to$. We then consider sufficiency as another general characterization for entailment, which we will argue is better captured using a rule $\Rightarrow$.

In §3, we examine Plumwood's use of suppression and illegitimate replacement as a means of ruling out laws of logic, thus yielding potential syntactic characterizing features for entailment. We also consider the Relevance Condition, as another syntactic criterion for a good logic. We then see that such characterizations are deficient in that they are too specific to apply generally. However, they do much useful work in each case, except for illegitimate replacement, which we will argue as being useful in $\to$-form but not in rule-form.

In §4, we examine truth and meaning as suitable semantic concepts, making a case for meaning to be used as a way of characterizing entailment and its logic. We then argue for Brady's logic MC of meaning containment and its content semantics, which indeed can be used to reject all of Plumwood's "false laws of logic" using its own characteristics. We follow with the derived Depth Relevance Condition, a syntactic criterion which can be used to reject the first three of Plumwood's false laws, but not the fourth.

We conclude in §5 by providing overall support for her project, whilst picking out two points of difference.

2 General Characterizations for Entailment.

In the third paragraph on the first page, Plumwood asserts that p entails q, symbolized '$p \Rightarrow q$', holds iff q is deducible from p. Clearly, she uses the symbol $\Rightarrow$ as a connective within a formula, despite its conventional use for a rule taking premises to a conclusion. She goes on to say that the feature of this interpretation with which these laws (that is, her false laws of logic) are incompatible is that if q is deducible from p, *p must be sufficient for q*. (See also on the 3rd para., p.97.) In this section, we will consider such deducibility and sufficiency as attempts to provide general characterizations for entailment, taking into account Plumwood's statement that deducibility implies sufficiency.

We start with deducibility, but we apply this term to a natural deduction derivation within a subproof with hypothesis p and conclusion q, since Plumwood's $p \Rightarrow q$ is a formula and not a rule. We will continue to use the conventional symbol $\rightarrow$ for entailments and subsequently symbolize rules using the conventional $\Rightarrow$. Thinking back to the context of Plumwood's paper, it was written at a time when relevant logic was in its infancy with the focus on the logics E and R but with no semantics available for them. Indeed, at the time, ideas were being explored that might help one establish a sound and complete semantics. Nevertheless, the Fitch-style natural deduction system for relevant logics was available at the time, initiated by Fitch in [1952] for classical logic. This style suited relevant logics in that one can ensure that each hypothesis is being used, by indexing each hypothesis, and by eliminating each hypothesis in turn, in the process deleting its index. This prevents material implication paradoxes such as $A \rightarrow .B \rightarrow A$ from being derived, as the hypothesis B (and its index '2') cannot be eliminated upon proving $B \to A$. (See Anderson and Belnap [1975] for the use of this natural deduction system.)

We first show that $p \to q$ is uniquely determined by its natural deduction derivation of q from hypothesis p in the Fitch-style system used for relevant logics. Such a style of uniqueness was proved for conjunction and disjunction in Brady and Meinander [2013], to show that distribution does not follow from the meanings of these connectives. We let $p \to q$ and $p \to' q$ be two entailments determined by the deduction of q from p, using the standard introduction and elimination rules for relevant logics. We give the hypothesis $p \to q$ the index set $\{1\}$ in the first subproof. We then introduce a hypothesis p with index set $\{2\}$ in the next subproof. q is then proved using the $\to$-elimination rule, with index set $\{1,2\}$. $p \to' q$ with index set $\{1\}$ can then be proved using $\to'$ introduction, eliminating the index 2. Thus, $p \to q \to .p \to' q$, with the null index set $\varnothing$, is proved. Similarly, $p \to' q \to .p \to q$ with index $\varnothing$ can be proved using $\to$' elimination, yielding $p \to q \leftrightarrow .p \to' q$ with index $\varnothing$. This shows the two entailments, $p \to q$ and $p \to' q$, are inter-substitutable into any logical context and are thus unique.

Note that we do need the Substitution of Equivalents Rule to establish this inter-substitutability, but this would hold for any reputable relevant logic. This rule can be proved using the two Hypothetical Syllogism axioms, $A \to B \to .B \to C \to .A \to C$ and $A \to B \to .C \to A \to .C \to B$, or Richard Routley's Affixing Rule, $A \to B, C \to D \Rightarrow B \to C \to .A \to D$, included in logics without the two Hypothetical Syllogism axioms. In §3, we will be discussing such replacement principles, all of which Plumwood regards as illegitimate. However, we will be offering support for the Affixing Rule, which will then allow the derivation of the Substitution of Equivalents Rule.

This uniqueness means that $p \to q$ is uniquely determined by the derivation of q from p. So, $p \to q$ holds iff q is deducible from p, as stated by Plumwood. However, this can be seen to be independent of the key relevant logics since this relationship holds for all the relevant logics from B through to R. (The axiomatizations of these two logics will appear in §4, together with that of MC.) In Brady [1984], the natural deduction systems for the key logics in this range are all given, where the logics weaker than R carry a proviso on the standard $\to$-elimination rule: from A_{a} and $A \to B_{\mathrm{b}}$, to infer $B_{\mathrm{a} \cup \mathrm{b}}$. However, the proviso for system B is 'if $b \neq \varnothing$ then a is a singleton set $\{m\}$ such that $\max(b) < m$' which is relaxed for each of the logics stronger than B, leading up to R which is without proviso. The point here is that the above deduction can be carried through, subject to the proviso for the logic B, and hence for all relevant logics stronger than B, right up to R. (This will also include the logic MC of meaning containment, which drops distribution.) So, the deducibility applies to whatever system one is working in and this characterization of entailment is "general" and does not allow one to differentiate relevant logics with their particular laws, what is what Plumwood was attempting to do in the overall paper. Nevertheless, in fairness, she did not try to do this with deducibility and was only trying, quite legitimately, to establish a meaning for entailment.

Further, such deducibility is entirely within a subproof which distinguishes it from the deduction which takes a set of premises to the conclusion of a valid deductive argument, such premises and conclusion being part of the main proof of a natural deduction system. Here, for valid deductive arguments, the conclusion is certain, given the premises, rather than being of high probability as is the case for inductive arguments. To achieve this certainty, the conclusion is analytic either by itself, with its analyticity determined subject to the satisfaction of restrictions applied by the premises, or with its analyticity entirely determined by the premises, as is often the case. (See Brady [2015], [2021a] and [2022], but with the focus in [2021a] being on meaning analysis.) As stated in Brady [2021a] and [2022], if T is an analytic truth, then 'if $A, T \Rightarrow B$ then $A \Rightarrow B$' is deductively equivalent to '$A \Rightarrow T$'. 'If $A, T \Rightarrow B$ then $A \Rightarrow B$' is just a case of the standard removal of analytic axioms and theorems from the premises of a deduction, and so '$A \Rightarrow T$' follows. Simple examples of analyticity being entirely determined by its premises would be the Adjunction Rule or the Modus Ponens Rule for $\rightarrow$. The other case of analyticity, which is determined subject to the satisfaction of restrictions applied by the premises, is exemplified in Brady [2015] by the non-emptiness restriction on restricted quantification, and by substitution rules for identity and equivalence.

The important point here is that the suppression of analytic truths from among the premises of such a derivation is entirely legitimate, in contrast to the suppression of truths in the antecedent of an entailment, to be dealt with in §3 below. Oddly, Plumwood makes scant reference to the deduction of conclusion from premises, only using its symbol '$\longrightarrow\!\!\triangleright$' in footnote 7 on p.105, to compose Rule Factor and Rule Syllogism.

This brings us to sufficiency, often used in argument by Plumwood, but related by inference from deducibility on p.97. However, the use of sufficiency suggests a relationship which is less tight than entailment as it does not suggest a meaning relationship between antecedent and consequent, just suggesting mere sufficiency for a conclusion to follow, much like the above satisfaction of premissed restrictions to establish an analytic conclusion. Moreover, it suggests truth-preservation, which is indeed implied by entailment. Truth-preservation and its logical form, implication, are discussed in Brady [2022], where the conclusion is drawn that it is best represented by the rule $\Rightarrow$ of a deduction from premises to conclusion. In this case, the premises would suffice for the conclusion to follow as a matter of certainty. The arguments for this in Brady [2022] are that the implicational candidates provided by strong relevant logics are inadequate due to their criterion of relevance failing in applications, and that the other obvious candidate of classical material implication incorporates negatives which are not a stated part of truth-preservation. Such negatives are specified in classical logic by the Law of Excluded Middle (LEM), $A \vee \sim A$, and Disjunctive Syllogism (DS), $\sim A, A \vee B \Rightarrow B$, together stating that exactly one of A and $\sim A$ hold. Note that neither of these laws are included in Brady's logic MC of meaning containment, to be introduced in §4.

As for deducibility within a subproof, sufficiency, being deducibility in the main proof, is also general in that it does not enable one to differentiate one logic from another. There are some standard rules that apply across logics, such as Adjunction, Modus Ponens and Affixing, and there are many rules which would be logic-specific in that they would follow from the entailments of the logic using Modus Ponens. So, sufficiency, as a concept, will not be able to pinpoint a logic. Again, Plumwood did not use sufficiency to determine that a law was false, except as an expression in an argument which used some other criterion to do the work. This takes us to the next section on syntactic criteria which will do such work.

3 Syntactic Criteria for Entailment.

Syntactic criteria for a good entailment include Plumwood's suppression failure together with her legitimate replacement, as well as the overarching Relevance Condition. By their very nature, these criteria are too constrained to cover all entailments. Nevertheless, Plumwood does use her two criteria to effectively argue against the first three of her four false laws of logic, which taken in order are: exportation, illegitimate replacement, commutation (aka. permutation), and disjunctive syllogism.

Plumwood used suppression to reject Exportation. By her own admission, suppression does not always work in rejecting false laws, as the remaining formula after the suppression of an antecedent theorem may still be a theorem of the logic. She calls these cases, where suppression does not make the formula invalid, 'omission' rather than suppression, but this then means that suppression is used in just those cases where it works and becomes like a general characterization of §2, that does not distinguish a logic. Indeed, Plumwood states "Suppression is always illegitimate suppression." (3rd para, p.99.) However, we think that this distinction is just an artificial device to ensure that suppression always works in rejecting false laws and we think that suppression should be applied on the assumption that it does not always work, as with syntactic criteria generally. Furthermore, Plumwood shows that the suppression argument against Exportation, $A \,\&\, B \to C \to .A \to .B \to C$, is appropriate and does do useful work here in finding the key fault in Exportation. Thus, she rightly argues in the first para. of her section II, that A should not be dropped off the entailment $A \,\&\, B \to C$ to yield $B \to C$, even though A is true. (On this point, see §4 for a discussion of truth versus proof.) C is entailed by conjunctively combining both A and B, meaning that both are essential for this entailment to hold. However, as stated above, one must be careful with suppression as it can still hold in particular cases of Exportation, for example, if $A \to A$ is suppressed in the Conjunctive Syllogism instance, $(A \to A) \,\&\, (A \to B) \to .A \to B$, then this still leaves the residual entailment $A \to B \to .A \to B$ valid. So, the use of suppression is restricted to cases where it drops a key supporting antecedent component of an entailment.

However, if we replace each of the entailments by rules, that is by $(A \,\&\, B \Rightarrow C) \Rightarrow (A \Rightarrow (B \to C))$, then such suppression is warranted if A is provable, as this is just a case of the standard removal of previous theorems from the premises of a rule. This is a reasonable extension of the argument for the removal of analytic truths from the premises of a deduction, as was argued in §2 above. So, there may have been some confusion between entailments and rule-based implications or sufficiencies in using suppression in the first place. This would likely trace back to classical logic, where the Deduction Theorem, $A \Rightarrow B$ iff $A \supset B$ is a theorem, applies, which would then allow the transfer of rule-based properties into theorems of classical logic.

Illegitimate replacement is used by Plumwood to supplement suppression in cases of Hypothetical Syllogism, called by her Exported Syllogism, $A \to B \to .B \to C \to .A \to C$ and its permuted form, $A \to B \to .C \to A \to$

$.C \to B$. She compares these with Conjunctive Syllogism, $(A \to B)$ & $(B \to C) \to .A \to C$, the exported forms being obtained by iterating each of the two conjuncts into the entailments $B \to C \to .A \to C$ and $A \to B \to .A \to C$, the remaining conjunct being suppressed. (See the first para., section III.) So, it is really suppression that is being used here to reject Hypothetical Syllogism, rather than illegitimate replacement, which she goes ahead and uses to also reject the rule-forms of Hypothetical Syllogism. (See the last 2 paras on p.109 and the first 2 paras on p.110, where she concludes by using the detachment rule in her example.)

However, we need to carefully distinguish Richard Routley's Affixing Rule, $A \to B, C \to D \Rightarrow B \to C \to .A \to D$, which embraces the two rule-forms of replacement, from the corresponding entailment-forms of replacement. (See Routley, Meyer, Plumwood and Brady [1982] for the use of Affixing in axiomatizing a range of weaker relevant logics.) Plumwood argues against both entailment and rule forms, mistakenly as we will now argue. Affixing was introduced to ensure that the Substitution of Equivalents Rule, $A \leftrightarrow B \Rightarrow C(A) \leftrightarrow C(B)$, for logical context C, is derivable in weaker logics without Hypothetical Syllogism in entailment form. Moreover, Affixing ensures (and is ensured by) the Substitution of Entailment Rules, with substitution of A for B and of B for A from the entailment $A \to B$. That is, $A \to B \Rightarrow C(B) \to C(A)$, where B is in antecedent position in C, and $A \to B \Rightarrow C(A) \to C(B)$, where A is in consequent position in C. (See Anderson and Belnap [1975] for antecedent and consequent positions and see Plumwood on last para. of p.112 for examples.) Moreover, it is important that these substitution rules are derivable, as opposed to being primitive, as happened in the modal logic S1 for the Substitution of Equivalents Rule, where its awkwardness is displayed in its quite *ad hoc* semantics whose truth conditions were finally obtained by Cresswell in [1972]. These substitution rules show that no tighter concept than entailment is needed for the logic. So, Plumwood takes her usage of illegitimate replacements too far in rejecting the rule-forms of replacement, which are indeed quite legitimate, by the above arguments. This is our only significant criticism of Plumwood's paper.

Plumwood also used suppression to reject Permutation, $(A \to .B \to C) \to .B \to .A \to C$, or Commutation as Plumwood calls it. To see this, we can consider it in the rule-form, $B, A \to .B \to C \Rightarrow A \to C$, where the theorem B is suppressed from $A \to .B \to C$ to yield $A \to C$. This is like the case of Exportation, but with its conjunction being replaced by its exported form. Again, B is an essential component of the entailment $A \to .B \to C$

where $B \to C$ is entailed by A and so B cannot generally be removed, with the entailment collapsing to $A \to C$.

That suppression and illegitimate replacement do not reject all false entailments can be especially seen from her fourth false law, the entailment form of the DS, where she uses quite a different argument. Indeed, she shows that the reasoning of classical logic is let back in, with its associated irrelevance. This can be seen in the last para. on p.117 and the top of p.118, where Plumwood concludes that "we obtain the same effect as if $\supset$-methods were $\to$-methods".

Let us now consider Plumwood's final section V, where there are three final alternatives presented on p.134, which result from her investigations:

(1) Keeping all suppression and illegitimate replacement laws, while rejecting Addition laws.

(2) Keeping replacement laws but rejecting suppression laws; keeping Conjunctive Simplification but rejecting Factor.

(3) Rejecting all illegitimate replacement and suppression laws, while keeping Addition laws, Conjunctive Simplification and Factor.

We should rule out the first because of its keeping of illegitimate replacement laws and also its rejecting of Addition laws which we view as standard. (See Brady [2022] regarding Addition as a meaning containment.) We should also rule out the third because of its keeping of Factor, which is $B \to C \to .A \,\&\, B \to A \,\&\, C$ and $B \to C \to .A \vee B \to A \vee C$, in its respective conjunctive and disjunctive forms. Factor is problematic for relevant logic. We set out the following proof, as shown earlier in section V. By the conjunctive form of Factor, $B \to C \to .B \,\&\, B \to B \,\&\, C$ and hence $B \to C \to .B \to B$, using Conjunctive Simplification and applying Affixing repeatedly. By the disjunctive form, $C \to C \to .B \vee C \to B \vee C$ and hence $C \to C \to .B \to B \vee C$, by Addition and application of Affixing. By substituting $B \vee C$ for C into $B \to C \to .B \to B$ above, we obtain $B \to B \vee C \to .B \to B$ and by transitivity $C \to C \to .B \to B$, which fails the Relevance Condition. This leaves only the second alternative, which keeps the replacement laws and the obvious conjunctive simplification, whilst rejecting suppression laws and Factor, though one must be careful with rejecting suppression is it sometimes is quite legitimate, as seen above in this section. However, this alternative still leaves the preferred logic wide open, leaving further criteria to be applied. We would like to think that the logic MC of §4 below, based on meaning containment, would be such logic.

The Relevance Condition is used by Plumwood as background for her "false laws of logic" as they each do satisfy the Relevance Condition (if $A \to B$ is a theorem then A and B share a sentential variable). However, she realized that the strong relevant logics needed to be pared back, at least by her false laws.[1] This paring back is what we think is needed not only to maintain relevance but also to establish an alignment to a semantic concept, instead of further syntactic ones, which are generally insufficient to completely characterize a logic. Hence, we move on to the next section, which is devoted to semantic criteria for a suitable relevant logic.

4 Semantic Criteria for a Logic.

After the concerns raised in §3 regarding the inability of syntactic concepts to completely characterize entailments there is a need for an examination of semantic concepts with focus on their ability to capture not just entailments but also the logic as a whole, as the meanings of all the connectives will then be taken into account. The two semantic concepts that can and have determined logics are truth and meaning. (For some discussion of the concepts, information and necessity, see Brady [2016] and [2021a].) Truth generally refers to the truths of the world, but there is also analytic truth, which is truth according to the meanings of the words in the sentence concerned. Indeed, it is the meanings of the connectives that are essential in logical formalization just as we taught students to capture the meanings of all the words when putting arguments into symbolism. However, students are usually then taught that meanings are represented by truth conditions. This goes back to Tarski's semantic conception of truth (cf. Ch.8 of Tarski [1956]). This leads to the construction of truth tables for classical logic where, in particular, $v(A \vee B) = T$ iff $v(A) = T$ or $v(B) = T$. Here, atomic sentences are given a T or F valuation and formula-induction used for each of the connectives. When determining completeness, priming is then required to construct the canonical model M, that is, if $A \vee B$ is true in M then A is

[1] Another concern for strong relevant logics, with the relevance condition as their key characteristic, is that such a condition cannot be maintained in application. When formalizing relevant arithmetic, it is easy to prove $m = n \to l = l$, and similarly for relevant set theory, $x = y \to .p \leftrightarrow p$. (See Brady [1996] and [2006] for the details.)

true in M or B is true in M. This is a property that the Hilbert-style proof theory cannot explicitly have. This is because disjunction elimination in such proof theory is given by the rule, $A \supset C, B \supset C \Rightarrow A \vee B \supset C$, where the disjunction does not require a witness disjunct A or B to hold, as C is proved whichever of A or B holds. This explains why a priming property needs a special proof in establishing completeness, usually with the addition of extra witnesses into the canonical model to ensure that each disjunction $A \vee B$ has either A or B as a witness in the model. The upshot is that the so-called truth-conditional "semantics" of classical logic is not really a semantics at all, as it overreaches the proof-theoretic meaning of disjunction. (See Brady [2017] on this argument, as it applies to the Routley-Meyer semantics of relevant logics.) This is borne out by applications such as a disjunctive piece of information where there is no specified disjunct, but the disjunction is still informative by itself. Take the example of a car travelling toward a fork in the road with a lack of information as to which fork the car went down. This would require a non-logical axiom. (See also Brady [2022] where a similar argument is applied to Heisenberg's uncertainty principle, where electron spin is up or down, but neither can be determined, given the determination of its position.) Nevertheless, priming may still hold for the theorems of a (non-classical) logic. This can be seen from the metacompleteness of some logics of entailment, mentioned later in this section, and in note 3.

This leaves us with meaning itself to compose a proper semantics for a logic and to drive the entailment connective of a suitable logic, bearing the point about priming in mind. Brady, over a long period of time and finally summing up in his [2022], introduced the logic MC of meaning containment, axiomatized below. It is a weak relevant logic, taking into account the sort of considerations Plumwood has made in her paper on "false laws". The main point about meaning containment is that it provides a concept which can then be applied as such. For example, it can be used to define meaning identity which is then applicable to definitions, as in Brady [2021], which then be used to solve the set-theoretic and semantic paradoxes. Meaning containment can also be used for applications outside of that of recursive sets, seen as the limit of application of classical logic in Brady [2022]. (See further in Brady [2018] for applications of the logic MC in dealing with cases of self-reference, which clearly go beyond recursive methods.) One can see that each of Plumwood's false laws are not included in MC, except for the Affixing Rule which is included as rule R3 below.

We present this logic as follows, using the bracketing convention from

Anderson and Belnap [1975]:
<u>MC</u>.

Primitives: $\sim$, &, $\vee$, $\rightarrow$.
Axioms.

1. $A \rightarrow A$.
2. $A \,\&\, B \rightarrow A$.
3. $A \,\&\, B \rightarrow B$.
4. $(A \rightarrow B) \,\&\, (A \rightarrow C) \rightarrow .A \rightarrow B \,\&\, C$.
5. $A \rightarrow A \vee B$.
6. $B \rightarrow A \vee B$.
7. $(A \rightarrow C) \,\&\, (B \rightarrow C) \rightarrow .A \vee B \rightarrow C$.
8. $\sim\sim A \rightarrow A$.
9. $A \rightarrow \sim B \rightarrow .B \rightarrow \sim A$.
10. $(A \rightarrow B) \,\&\, (B \rightarrow C) \rightarrow .A \rightarrow C$.

Rules.

1. $A, A \rightarrow B \Rightarrow B$.
2. $A, B \Rightarrow A \,\&\, B$.
3. $A \rightarrow B, C \rightarrow D \Rightarrow B \rightarrow C \rightarrow .A \rightarrow D$.

Meta-Rule.

1. If $A \Rightarrow B$ then $C \vee A \Rightarrow C \vee B$.

We note the tweaking of the logic of Brady [2006], that occurs in Brady and Meinander [2013], that is, the dropping of distribution in $\rightarrow$-form and its rule-form. We also note the step back in Brady [2022], where the single-premise meta-rule of Brady [2006] is re-instated as the only meta-rule of the logic, in place of the two-premise version of Brady [2015]. This latter tweak in Brady [2022] amounts to the reclassification of the rule-form of distribution as an admissible rule of the logic instead of a primitive rule. Furthermore, it is argued in [2022] that the rule-form of distribution, $A, B \vee C \Rightarrow (A \,\&\, B) \vee$

$(A \mathbin{\&} C)$, can still apply if the $B \vee C$ is prime or the A is an analytic truth, which together expand its usage as an admissible rule quite considerably.

The logics B and R mentioned in §2 above are axiomatized as follows, with reference to MC:

$$\mathrm{B} = \mathrm{MC} + A \mathbin{\&} (B \vee C) \to (A \mathbin{\&} B) \vee (A \mathbin{\&} C) - A9 + A \to {\sim} B \Rightarrow B \to {\sim} A.$$

$$\mathrm{R} = \mathrm{B} + \mathrm{A9} + A \to B \to .B \to C \to .A \to C + A \to .A \to B \to B + (A \to .A \to B) \to .A \to B.$$

Note that Plumwood uses the truth of the antecedent A to ensure that it can be dropped on application of Modus Ponens, $A, A \to B \Rightarrow B$, leaving just the consequent B. We think she should use the proof of A instead of its truth here, as this is just what is required for Modus Ponens to apply. This brings us to the issue of proof versus truth, which we now consider. Logic does not determine the truths of the world; that is left to observation and other disciplines. It is analytic truth, that is, truth by meaning analysis, that is in the domain of logic, as logic is a conceptual discipline based on the meanings of the logical words. Proof-theoretic meaning rather than truth-theoretic meaning is what is then critical to logic, as logic proceeds through proof-theoretic deduction, as in the development of arithmetic and set theory. (See Mendelson [2010] for these developments, though he does incorporate some natural deduction into his Hilbert-style proof method.) It is the case that the rules of logic preserve truth, but it is broader than this as logic equally applies outside the domain of truths, in particular within fictitious novels and elsewhere.[2]

The logic MC of meaning containment has a content semantics, given in Brady [1996], [2006] and [2016], which provides an intuitive sound and complete semantics where $c(A \to B) = c(c(A) \supseteq c(B))$, where '$\supseteq$' is set-theoretic containment and the content $c(A)$ is the analytic closure of A, that is the set $\{B : A \to B \text{ is a theorem of MC }\}$. That is, the content of a sentence A is the set of all sentences B analytically established from A, expressed by the closure of its entailments. Indeed, Plumwood states on l.20 of p.103, "If the meaning connection is taken specifically to be one of *inclusion of content*, any suppression will lead to misrepresentation of that relation." Earlier on

[2]In the usual quantificational extension of sentential relevant logics, the quantificational rule $A^a/x \Rightarrow \forall x A$ is added, distinguishing the free variable a from the bound variable x. However, as it stands, this rule does not preserve truth, though preserving validity instead. Nevertheless, this rule can be circumvented by universally closing each quantificational axiom with free variables and by proving this rule to be an admissible rule.

p.103, 2nd para, she explains "But a most important objection to the deductive suppression derives from deducibility as a *meaning* relation between propositions." Thus, Plumwood pre-empts the philosophical background to Brady's formal developments on meaning containment and contents.

We present the content semantics for MC as follows:

The *content semantics for the logic MC* is set out as follows, as in Brady [2016]:

A *content model structure (c.m.s)* consists of the following 4 concepts: $T, C, *, c$, where C is a set of sets (called contents), $T \neq \varnothing$, $T \subseteq C$ (the non-empty set of all true contents), $*$ is a 1-place function on C (the $*$-function on contents), and c is a 1-place function from containment sentences $c_1 \supseteq c_2$ between contents c_1 and c_2 of C or from unions $c_1 \cup c_2$ of c_1 and c_2 of C, to members of C, subject to the semantic postulates p1-p15, below. The concepts $\cap$, $\cup$, $=$ and $\supseteq$, are taken from the background set theory, $\cap$ being a 2-place function on C (the intersection of contents), $\cup$ being a 2-place function on C (the union of contents), $=$ being a 2-place relation on C (identity), and $\supseteq$ being a 2-place relation on C (content containment). We follow the good idea of Mares in his book [2004], replacing Brady's use of a bar over the union to indicate the content of the union, as occurs in his earlier [1996] and [2006].

The *semantic postulates* are:

- p1. $c(c_1 \cup c_2) \supseteq c_1, c(c_1 \cup c_2) \supseteq c_2$.
- p2. If $c_1 \supseteq c_2$ and $c_1 \supseteq c_3$ then $c_1 \supseteq c(c_2 \cup c_3)$.
- p3. $c_1 \supseteq c_1 \cap c_2, c_2 \supseteq c_1 \cap c_2$.
- p4. If $c_1 \supseteq c_3$ and $c_2 \supseteq c_3$ then $c_1 \cap c_2 \supseteq c_3$.
- p5. ${c_1}^{**} = c_1$.
- p6. If $c_1 \supseteq c_2$ then ${c_2}^* \supseteq {c_1}^*$.
- p7. If $c_1 \supseteq c_2$ and $c_1 \in T$ then $c_2 \in T$.
- p8. If $c_1 \in T$ and $c_2 \in T$ then $c(c_1 \cup c_2) \in T$.
- p9. If $c_1 \cap c_2 \in T$ then $c_1 \in T$ or $c_2 \in T$. (See Note 3.)
- p10. $c(c(c_1 \supseteq c_2) \cup c(c_2 \supseteq c_3)) \supseteq c(c_1 \supseteq c_3)$.
- p11. $\mathrm{c}(\mathrm{c}(\mathrm{c}_1 \supseteq \mathrm{c}_2) \cup \mathrm{c}(\mathrm{c}_1 \supseteq \mathrm{c}_3)) \supseteq \mathrm{c}(\mathrm{c}_1 \supseteq \mathrm{c}(\mathrm{c}_2 \cup \mathrm{c}_3))$.
- p12. $c(c(c_1 \supseteq c_3) \cup c(c_2 \supseteq c_3)) \supseteq c(c_1 \cap c_2 \supseteq c_3)$.

p13. $c(c_1 \supseteq c_2) \supseteq c(c_2{}^* \supseteq c_1{}^*)$.

p14. $c(c_1 \supseteq c_2) \in T$ iff $c_1 \supseteq c_2$.

p15. If $c_1 \supseteq c_2$ then $c(c_3 \supseteq c_1) \supseteq c(c_3 \supseteq c_2)$ and $c(c_2 \supseteq c_3) \supseteq c(c_1 \supseteq c_3)$.

An interpretation I on a c.m.s. is an assignment, to each sentential variable, of an element of C. An interpretation I is extended to all formulae, inductively as follows:

i. $I(\sim A) = I(A)^*$.

ii. $I(A \mathbin{\&} B) = c(I(A) \cup I(B))$.

iii. $I(A \vee B) = I(A) \cap I(B)$.

iv. $I(A \rightarrow B) = c(I(A) \supseteq I(B))$.

A formula A is true under an interpretation I on a c.m.s. M iff $I(A) \in T$.
A formula A is valid in a c.m.s. M iff A is true under all interpretations I on M.
A formula A is valid in the content semantics iff A is valid in all c.m.s.

Soundness (if A is a theorem of MC then A is valid in the content semantics) follows readily and completeness (if A is valid in the content semantics then A is a theorem of MC) follows by the usual Lindenbaum method for algebraic-style semantics, but here there is a slight difference. In constructing the canonical models, instead of taking equivalence classes of formulae as the contents, we put the content $[A]$ of A as $\{C : A \rightarrow C \in T'\}$, where T' is constructed as a prime extension of the set of theorems which does not include a particular non-theorem B. This essentially means that these canonical contents are closed under entailment, that is, they are *analytic closures* of the sentence (or sentences) involved, since the set T of theorems is already prime, due to the logic MC being metacomplete.[3] Since entailments here are understood as meaning containments, closure under entailment is closure under meaning containment and hence closure under the analysis of the meanings of words.

[3] Metacompleteness was introduced by Meyer in [1976] for positive logics and extended to logics with negation by Slaney in [1984] and [1987] for so-called metacomplete logics. Priming is the key property that can then be established for metacomplete logics such as MC. We include the semantic postulate p9 on this basis.

We proceed to use this content semantics to reject each of Plumwood's false laws of logic. However, it should be pointed out that the content semantics is an algebraic-style semantics which closely follows the axioms and rules of the logic MC. This means that the arguments used to reject formulae will be taken from our understanding of the above content semantics, which is in turn dependent on the axiomatization, rather than the more precise arguments used in truth-theoretic semantics, which are based on formula induction in terms of truth and falsity. Truth-theoretic semantics is ruled out anyway because it would validate the $\rightarrow$-form of distribution through its formula-inductive presentation, this form not being included in MC. (See Brady [2022] for a discussion of this point.)

For Exportation, $A \,\&\, B \rightarrow C \rightarrow .A \rightarrow .B \rightarrow C$, discussed in section II of Plumwood's paper, we consider its rule-form, $A, A \,\&\, B \rightarrow C \Rightarrow B \rightarrow C$, putting $c(A) \in T$ and $c(A \,\&\, B \rightarrow C) \in T$. Thus, $c(A \,\&\, B) \supseteq c(C)$, with c being the interpretation, without loss of generality. However, both A and B may well be needed to establish C, whether A is true or not, and the containment of C in B is thus not guaranteed. This indeed follows the style of Plumwood's arguments. A counterexample would be Factor, $B \rightarrow C \rightarrow .A \,\&\, B \rightarrow A \,\&\, C$, easily seen by its failure to follow the standard Fitch-style natural deduction rules in Brady [1984], as the index set on $A \,\&\, C$ needs to be $\{1,2\}$, whilst the index on A is just $\{2\}$. The point here is that $(A \rightarrow A) \,\&\, (B \rightarrow C) \rightarrow .A \,\&\, B \rightarrow A \,\&\, C$ is clearly valid, using this same natural deduction system, as the addition of $A \rightarrow A$ adds the extra index 1 to the A in $A \,\&\, C$.

Nevertheless, we can strengthen this argument by considering the containment statements themselves. We consider the rule-form, $A \,\&\, B \rightarrow C \Rightarrow A \rightarrow .B \rightarrow C$, comparing $A \,\&\, B \rightarrow C$ with $A \rightarrow .B \rightarrow C$. If $c(A \,\&\, B \rightarrow C) \in T$ then $c(A \,\&\, B) \supseteq c(C)$, where the content of C could be included in that of A, but even this is not necessary. However, $c(A \rightarrow .B \rightarrow C) \in T$ iff $c(A) \supseteq c(c(B) \supseteq c(C))$, that is the containment statement $c(B) \supseteq c(C)$ is contained in the content of A, which is not ensured by the much looser conditions on $c(A)$ from the antecedent, $c(A \,\&\, B) \supseteq c(C)$. So, $c(A \rightarrow .B \rightarrow C) \in T$ does not follow from $c(A \,\&\, B \rightarrow C) \in T$, and hence Exportation fails in rule-form and hence in $\rightarrow$-form.

For Hypothetical Syllogism (or Exported Syllogism, as Plumwood calls it in section III), $A \rightarrow B \rightarrow .B \rightarrow C \rightarrow .A \rightarrow C$ and its permuted form, $A \rightarrow B \rightarrow .C \rightarrow A \rightarrow .C \rightarrow B$, we must consider their $\rightarrow$-forms, as their corresponding rule-forms are valid, as argued above in §3. This means that we need to consider $c(A \to B \to .B \to C \to .A \to C) \in T$ and hence $c(A \to B) \supseteq c(c(B \to C) \supseteq c(A \to C))$, that is, the containment statement $c(B \to C) \supseteq c(A \to C)$ is contained in the content of $A \to B$. However, this containment statement is a containment of containments involving the individual contents of A, B and C. Such a containment statement, as a containment of containments of that of A, B and C cannot be contained in a simple containment of the content of B in that of A, as it has an extra layer of complexity.

Before moving on, we show that the rule-forms, $A \to B \Rightarrow B \to C \to .A \to C$ and $A \to B \Rightarrow C \to A \to .C \to B$ are valid in the content semantics. We let $c(A \to B) \in T$, that is $c(A) \supseteq c(B)$. We need to show that $c(B \to C) \supseteq c(A \to C)$, that is $c(c(B) \supseteq c(C)) \supseteq c(c(A)) \supseteq c(C))$, and similarly for $c(C \to A) \supseteq c(C \to B), c(c(C) \supseteq c(A)) \supseteq c(c(C) \supseteq c(B))$. For the first one, since $c(A) \supseteq c(B)$, we can substitute $c(A)$ for $c(B)$ into $c(c(B) \supseteq c(C))$, which would then contain $c(c(A)) \supseteq c(C))$. For the second, we can substitute $c(B)$ for $c(A)$ into $c(c(C) \supseteq c(A))$, which would then contain $c(c(C) \supseteq c(B))$. These results follow the $\to$-substitutions of A for B into antecedent positions and of B for A into consequent positions, as mentioned in §3. Note that the transitivity of containment is used in obtaining these results.

For Permutation (or Commutation, as Plumwood calls it in the latter part of section III), $(A \to .B \to C) \to .B \to .A \to C$, we can consider its rule-form, $B, A \to .B \to C \Rightarrow A \to C$. Let $c(B) \in T$ and $c(A \to .B \to C) \in T$, that is $c(A) \supseteq c(c(B) \supseteq c(C))$, where a containment statement is contained in that of A. However, $c(A \to C) \in T$ need not be the case since this means that the simple content of C would be contained in that of A, regardless of the truth of B.

For Disjunctive Syllogism, $A \mathbin{\&} (A \supset B) \to B$, as it occurs in section IV of Plumwood's paper, or its deductive equivalent $\sim A \mathbin{\&} (A \vee B) \to B$, as we would prefer because the disjunction is made explicit. We consider its rule-form $\sim A, A \vee B \Rightarrow B$, putting $c(\sim A) \in T$ and $c(A) \cap c(B) \in T$. Thus, $c(A)^* \in T$ and, by p9 of the above content semantics, either $c(A) \in T$ or $c(B) \in T$. Unlike the three laws considered above, appeal is not made to the differences in containment complexity between antecedent and consequent. Here, we appeal to a property of negation represented by the operator $*$ on contents. In particular, there is nothing in the content semantics or indeed in the logic MC to prevent A from being a simple inconsistency, which allows

$c(A)$ and $c(A)^*$ to both be true.[4] The truth of $c(A)$ would then mean that $c(A) \in T$ or $c(B) \in T$ can be satisfied without $c(B)$ being true. This refutes the rule-form of Disjunctive Syllogism and hence its $\rightarrow$-form.

We next present a syntactic criterion that is obtained by examining the different levels of content containment used to reject Plumwood's first three laws. Syntactically, the difference in levels of content containment is reflected in the depths of entailment occurring in the whole formula. This can be seen by comparing Conjunctive Syllogism, $(A \rightarrow B) \,\&\, (B \rightarrow C) \rightarrow .A \rightarrow C$, and the Modus Ponens Axiom, $A \,\&\, (A \rightarrow B) \rightarrow B$. Their respective representations in the content semantics are as follows.

$$\text{CS: } c(c(c(A) \supseteq c(B)) \cup c(c(B) \supseteq c(C))) \supseteq c(c(A) \supseteq c(C)).$$

$$\text{MPA: } c(c(A) \cup c(c(A) \supseteq c(B))) \supseteq c(B).$$

In considering CS, we see that the containment statements $c(A) \supseteq c(B)$ and $c(B) \supseteq c(C)$ will contain the containment statement $c(A) \supseteq c(C)$, due to the transitivity of $\supseteq$. Thus, these containment statements are all of the same application level, due to their corresponding $\rightarrow$-formulae being of the same depth in CS. (We define depth recursively below.)

In contrast, considering MPA, the content of A, whatever A is about, does not interact with the content of the containment statement $c(A) \supseteq c(B)$, and similarly the content of B does not interact with $c(A) \supseteq c(B)$ either, because of the differences in their respective meanings. Even if A is a containment statement itself, it would not interact with the containment of itself in B due to the different levels, which is reflected in their meanings. So, the union of $c(A)$ and $c(c(A) \supseteq c(B))$ would be a disjoint union of two separate components and so cannot yield $c(B)$.

Also, we compare MPA with its deductive equivalent, the rule-form of contraction (CR), $A \rightarrow .A \rightarrow B \Rightarrow A \rightarrow B$, where a similar argument applies. Its representation in the content semantics is as follows.

$$\text{CR: if } c(c(A) \supseteq c(c(A) \supseteq c(B))) \in T \text{ then } c(c(A) \supseteq c(B)) \in T.$$

We let $c(c(A) \supseteq c(c(A) \supseteq c(B))) \in T$, that is $c(A) \supseteq c(c(A) \supseteq c(B))$. We need to prove $c(A) \supseteq c(B)$. Again, the two $c(A)$'s of the premise do not

[4]The fact that MC allows contradictions to occur can be seen from the non-triviality of an inconsistent set theory, set up using MC, as proved in Brady [1989] and in [2006].

interact as they are of different levels of containment. Indeed, as for MPA, the two contents $c(c(A) \supseteq c(B))$ and $c(A)$ do not overlap, preventing the conclusion from being drawn.

So, the depths of $\rightarrow$-formulae create a hierarchy of contents. (For depths, see the formal definition below.) We can use this to create a syntactic criterion to reject false laws of logic such as MPA which, though syntactic, is extracted from the content semantics. Such a syntactic criterion is the DRC, which we set out as follows, as in Brady [1984a] where it was introduced and proved for a range of weak relevant logics including MC.

Indeed, MC satisfies the Depth Relevance Condition (DRC): if $A \rightarrow B$ is a theorem then A and B have a common variable at the same depth in both A and B.

This is a tighter version of the Relevance Condition, taking into account the depths of variables in A and in B.

We formally define the depth of subformula C in A, $d(C, A)$ recursively as follows:

$d(A, A) = 0$.

If $d({\sim}C, A) = n$ then $d(C, A) = n$.

If $d(C \mathbin{\&} D, A) = n$ then $d(C, A) = n$ and $d(D, A) = n$.

If $d(C \vee D, A) = n$ then $d(C, A) = n$ and $d(D, A) = n$.

If $d(C \rightarrow D, A) = n$ then $d(C, A) = n + 1$ and $d(D, A) = n + 1$.

Thus, the depth of C in A is the number of iterated $\rightarrow$'s, starting from A, working one's way down to the subformula C within the formula tree of A.

What the DRC indicates is that the meanings of subformulae (including variables) vary as to their depth in the overall formula. Indeed, Logan [2022] shows that different substitutions can be made on the same variable occurring at different depths within a formula with impunity, which implies that such a variable has a different meaning at each depth at which it occurs.

The DRC can do much useful work in weeding out false laws, including Plumwood's first three, as we see below. Further, the DRC is quite definitive in its application in contrast with the content semantics itself.

Exp. $A \mathbin{\&} B \rightarrow C \rightarrow .A \rightarrow .B \rightarrow C$.

Since the A's are of the same depth, we consider its derived instance, $A \to .B \to A \,\&\, B$, where the A's are of different depths, failing the DRC.

Hyp. Syll. $A \to B \to .B \to C \to .A \to C$ and
$A \to B \to .C \to A \to .C \to B$.

Clearly, the A's and B's in the antecedents $A \to B$ are of one lower depth than those in the two consequents, and so the DRC fails these formulae as they stand.

Perm. $(A \to .B \to C) \to .B \to .A \to C$.

Since the C's are of the same depth, we consider the derived instance, $B \to .B \to C \to C$, where the B's are of different depths.

5 Conclusion.

Plumwood's paper is a very welcome work that should have been published around 1968 when it was written. It strives to determine the pros and cons of particular logics, rather than just exploring technical results related to a particular system or to systems in general. Such pros and cons are essential in determining a good entailment logic. We have avoided going into much of Plumwood's detail, as other authors will probably do some analysis of their own on many of her points. We have kept within Brady's general approach to logic, as we feel that we can differentiate ourselves from other authors by doing so. This can be especially seen in the relationship between entailment and deducibility in §2 and the use of content semantics to invalidate Plumwood's "false laws" and some other unprovable formulae of the logic MC of meaning containment in §4.

When Plumwood was writing the paper, she was amid a melting pot of ideas. There was uncertainty about what logic was appropriate and there was a need for such a paper to sift through the various arguments. Entailment was proposed as the converse of deducibility with a view to help characterize entailment and sufficiency also played a role in its conceptual thinking. We have succeeded in sharpening these concepts but, as we have argued in §2, neither are capable of delineating a suitable logic of entailment. This paper also provides an opportunity to use content semantics to invalidate some "false laws" of MC, generally extending such work initiated in Brady [2006].

Plumwood used suppression and illicit replacement to reject the majority of her "false laws" but she unfortunately overstepped in her rejection of the Affixing replacement rules, Richard Routley having introduced these laws much later in Routley, Meyer, Plumwood and Brady [1982], following on from discussion of the weaker relevant logics in the late 70s. Further, her conditional support for Factor is not appropriate given the argument presented in section V, showing that its conjunctive and disjunctive forms together lead to irrelevance. Apart from that, Plumwood's argumentation was generally sound in rejecting her "false laws". So, we offer overwhelming support for her courageous program.

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