Christine Ladd-Franklin and the Progress of Formal Logic

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Abstract

I want to highlight Christine Ladd-Franklin's contribution to logic, by placing her technical contribution to the algebra of logic in the broader context of her philosophical contribution to logic in general, concerning her view of the nature of logic and its role in philosophy. First, I will present the sense in which her algebra of logic means a progress within formal logic towards a higher level of formality. Second, I will focus on the contrast between the new symbolic form of logic and the traditional non-symbolic form of logic. Third, I will focus on the contrast between the symbolic form of logic and the "mathematical" form of symbolic logic that Russell and his ilk were trying to impose.

Christine Ladd-Franklin (1847-1930) is usually known among logicians for her technical contribution to the algebra of logic (Castrillo 1997) (Grattan-Guinness 2000, 151, 154-155). In contrast, less attention is usually paid to her general conception of logic and its place in philosophy. My goal in this paper is to balance this situation, by placing her technical proposal in the broader context of the contribution she also intended; namely, to make a certain conception of logic prevail (Pietarinen 2013, 153-156). First, I will present the sense in which her algebra of logic means a progress within formal logic towards a higher level of formality. Second, I will describe this progress in terms of a contrast between the new symbolic form of logic and the traditional non-symbolic form of logic, where the ability to deal with complexity is the main advantage of the new logic. Third, I will contrast logic in its application to mathematical thought and logic in its application to philosophical thought, where the abstract nature of Ladd-Franklin's symbolic logic will mark the difference with respect to the "mathematical" form of symbolic logic that Russell and his ilk were trying to impose.

1 A more formal Formal Logic

Christine Ladd-Franklin carried out a doctoral research in mathematics at Johns Hopkins University, under the supervision of Charles S. Peirce. The result was the paper "On the Algebra of Logic", which was published in a compilation of logical studies edited by Peirce (Ladd-Franklin 1883). In fact, this "mathematical" piece of work was the starting point of Ladd-Franklin's entrance into the field of philosophy: while her first published papers belonged in the field of mathematics (mainly, mathematical problems in the *London Educational Times*, and also scientific articles in *The Analyst* and the *American Journal of Mathematics*), from 1883 onwards her interest in abstract mathematics gave way to an interest in algebra as a method for thinking. It is true that from 1887 the bulk of her publications shifted to the field of psycho-physiology, but her concern with logic and philosophy remained (indeed, she lectured on logic at different universities), and it took shape in several sets of important texts (Green & LaDuke 2008).[1]

Concerning Ladd-Franklin's idea of logic, one of her most explicit accounts can be found in her review of *Elements of Logic* by the British logician Constance Jones, where she rejected Jones's definition of logic as a science of propositions. Ladd-Franklin admits that logic is in fact interested in propositions, but this is because of a prior interest in reasoning. According to her, logic should not be defined as a science of propositions, but as a science of reasoning. If the praxis of logic consists in the *a priori* generation of new knowledge from already known propositions (more specifically, "by piecing them together, to produce new propositions without fresh reference to the outside world" (Ladd-Franklin 1890b, 560)), the science of logic must consist in the systematic study and control of such reasoning processes: that is, the theory of logic will study what "drawing conclusions" amounts to, and how to guarantee that this process is correctly carried out.

To draw a conclusion from a set of premises is for Ladd-Franklin to extract the information that their combination allows:

> Reasoning may be defined as putting This and That together and extracting something Other, —something which has been asserted by the two premises together, but which contains, in the case of the syllogism, only half of what they assert.[2] It may be regarded (in its simpler forms) as the elimination of a common term (or terms) from simple propositions in *is* [...] (Ladd-Franklin 1912b, 643).

In other words, the essence of reasoning is for her "the *fusing together* of two propositions, and the *emergence* of a third" (Ladd-Franklin 1890b, 560-561). This idea resembles the Aristotelian notion of *syllogismos* (that is, deductive reasoning) in the *Prior Analytics*:

> A "syllogismos" is an argument in which, certain things being posited, something other than what was laid down results by necessity because these things are so. By "because these things are so" I mean that it results through these, and by "resulting through these" I mean that no term is required from outside for the necessity to come about (Aristotle and Striker 2009, 24b18-22).

[1] A bibliography that tries to be as complete as possible can be found at the website: https://www.unav.es/gep/BibliografiaLaddFranklin.html.

[2] Theodore de Laguna refers to "[Ladd-Franklin's] half-humorous thesis, that the conclusion of the syllogism in Barbara omits precisely one half of what is contained in the premises", and explains that it is the symmetry of Ladd-Franklin's copula which allows to prove this result (de Laguna 1912, 398).

Both Ladd-Franklin and Aristotle put a strong emphasis on the idea of something resulting through a plurality of premises that have been previously considered as interrelated. In contrast, concerning the process of drawing conclusions from one single premise (immediate inference), Ladd-Franklin sometimes judges it to be a "relatively unimportant part" of logic as a theory (Ladd-Franklin 1912b, 643), although she is very happy on the other hand to collect examples of this kind of inference when she tries to make logic more concerned about real reasoning. In particular, as we will see soon, she gives examples of equivalent propositions that can be easily dealt with in algebraic terms but have been overlooked by traditional logic, such as "All ripe grapes are sweet" and "All grapes are either sweet or else unripe" (Ladd-Franklin 1890a, 88).

Now, the systematic study of the process of drawing conclusions, if approached from the philosophical (not empirical) standpoint, must be carried out from a formal (that is, abstract) point of view: the aim will be to understand what kind of thing this fusing toghether is, and what special subkinds of it need to be distinguished, by making the common elements salient, while disregarding the rest.[3] The crucial question is whether the new symbolic logic which was being born at the turn of the century (and which Ladd-Franklin was helping to shape) can be seen as implying a progress in the formality of logic. This certainly seems to be the appreciation of those who worked in its construction:

> Symbolic Logic is then that treatment of Formal Logic which employs, instead of many of the words of ordinary language, a system of special symbols which secure not only greater precision and compactness, but also *greater generality* in its discussions. (Huntington and Ladd-Franklin 1905) [my emphasis].

In sum, as Ladd-Franklin will put it later, the algebraic logic she is working on differs from the non-symbolic treatment of logic in being a "more formal Formal Logic" (Ladd-Franklin 1912b, 641), that is, in adopting a more *abstract perspective*. Namely, the new logic ignores differences, concerning not only "significant terms" but also concerning the two kinds of structural elements that play a role in reasoning, i.e.: "combinations of terms, or functions of terms, and statements of relations between terms" (Peirce, et al. 1960, 640). The extension of the use of symbols (beyond the mere use of letters as variables for "non-logical" content) has facilitated this abstract consideration of both operations and relations, which in turn has aided a more abstract consideration of the non-logical content itself.

First, concerning the generalisation of logical operations, it is well known that one of the aspects that can be ignored, when the appropriate symbols are used, is the distinction

[3] I tend to give little importance to discussions about "what it means to say that logic is formal", which follow the trend started by John MacFarlane (MacFarlane 2000). I take "form" to be a metaphorical expression with several potential meanings, and I follow Susanne Langer in simply connecting logical form with the human capacity of abstracting, that is, of considering what several things have in common, while neglecting the rest (Langer 1953). Whether this rest is taken to be matter, or content, or whatever, is irrelevant to my interests.

between the algebra of classes and the algebra of propositions: the terms $a, b, c \ldots$, can indifferently be interpreted as standing for classes or for propositions, so long as the operations "sum" and "product" are also interpreted in the corresponding abstract way (Ladd-Franklin 1892a, 127). Interestingly, Ladd-Franklin suggests that, in order to maintain the strict indifference between the class-interpretation and the proposition-interpretation, bivalence must be abandoned. In her opinion, limiting to 1 and 0 the values for abstract terms in the logical formulas is "a most unfortunate restriction" (ibid. 129). She makes an analogy between the relation between a general term and its instances (that is, the individuals belonging to the class, of which only a part may have the relevant properties) on the one hand, and the relation between a proposition and its "instances" across time on the other, of which only a part may have the property of being true: that is, she admits a proposition to be *sometimes true* and *sometimes false*. Reducing to 1 or 0 the range of values for propositions will be, for her, to assume that every proposition is *always true* or *always false*.

Second, concerning the abstract treatment of the connecting link between aggregates of terms, the great achievement of the new form of logic is the generalisation of the copula, which, in De Morgan's words, has been "made as abstract as the terms" (Ladd-Franklin 1960, 368). Again, it is the use of symbols that has favoured the abstract consideration of the copula, in which only what is common to all instances of drawing conclusions is made salient: for example, in Peirce's system, only transitivity needs to be taken into account, irrespective of whether you read the symbol as "implies", "is included", "entails", "is always followed by", or even "is an ancestor of". This means again that the reading of the formulas in terms of classes or in terms of propositions does not affect the nature of the connection, which has been captured in purely abstract terms (Ladd-Franklin 1890a, 562). Still further, the abstract consideration allows Ladd-Franklin to achieve a greater independence from linguistic forms of expression: she abandons transitive relations (which are asymetrical) in favor of her preferred symmetrical relations for the canonical expression of propositions, which are one of her most significant contribution to the algebra of logic in general and to the analysis of syllogisms in particular (Uckelman 2021, 537) (Pérez-Ilzarbe 2022).

In the logic of classes, the generalisation of the copula as a "counter" in an abstract game (which is no more identified with the "is" or traditional logic) had a well-known price that most algebraic logicians were happy to pay: the much discussed abandonment of the existential import of universal affirmative propositions (Wu 1969). "No consistent Logic of universal propositions is possible except with the convention that they do not imply the existence of their terms" (Ladd-Franklin 1890a, 88). This is indeed a strong departure from the Aristotelian tradition, even though the specific character of the novelty tends to be misunderstood. Ladd-Franklin in fact seems to be making the mistake of interpreting the traditional view as if a general assumption of existence was attached to any proposition, as if any proposition involved a commitment with non-empty classes (Ladd-Franklin 1892b, 527, note 1). But the actual point in the traditional interpretation concerns no rejection of empty classes, but the commitment with being that is implied by the *affirmative* copula "is", so that the implication of existence is related to *affirmative propositions* (both universal and particular), and is absent in negative ones (Parsons 2021). In contrast, the new approach relates the implication of existence to *particular* propositions, since "some" is read in terms of "there are", and hence as implying existence (both in the case of the affirmative and in the case of the negative particular proposition).

Therefore, concerning universal affirmative propositions, which lack an explicit commitment to "some" terms, the algebraic logician is free to opt for a neutral reading, which is seen as a mere convention adopted for the sake of logical manipulation. The interesting point is that this convention yields again a higher level of abstraction, since the new way of drawing conclusions includes the old one as a specific case: if you take (in general) "Every a is b" as meaning "Every *a that there is* is b", when you also know (as a particular case) that there is in fact at least one a, you can just separately state this fact (Ladd-Franklin 1890b, 563).

Particular propositions, then, acquire an important logical role, since there will be occasions when it will be necessary to make the affirmation of existence explicit (Ladd-Franklin 1889, 561). Moreover, the existential commitment (or the lack of it) can be more abstractly included in the symbolism by the use of the Special Terms of logic ("everything" and "nothing", which contain the expression of the universe of discourse). This leads Ladd-Franklin to defend the view that every proposition is concerned with existence (either as an affirmation of existence or as a denial of existence), as we will see in the next section.

In sum, the new symbolic logic has favoured some "simplified ways of looking at things" (that is, an abstract look), which on the other hand must not be judged to be exclusive of the new logic, but they belong to the very essence of *logic* in general (Ladd-Franklin 1890a, 75). Ladd-Franklin finds in the work of Peirce the most successful development of this point of view, with the effect of "at once simplifying and extending the whole body of logical doctrine (not merely its symbolic exposition)" (Ladd-Franklin 1892a, 127). Now, the extension of the logical doctrine is one of Ladd-Franklin's main concerns. She thinks that "ordinary logic" has become too limited in scope, and has thus fallen far removed from "actual thinking" and "everyday life". These shortcomings (and the extension needed in view of them) are mainly related to the analysis of categorical propositions.

2 Symbol logic and the traditional analysis of propositions

"Ordinary logic" was the kind of logic mostly practiced in Ladd-Franklin's time, when the new symbolic logic was in fact a minority enterprise, and had to face the reluctance of "longlife" logicians (Moktefi 2019). This way of doing logic was reflected in the usual textbooks (Van Evra 2008) and in the "mechanical logical exercises of the schools" that were familiar to Ladd-Franklin before she discovered the new way, thanks to Peirce's teaching (Ladd-Franklin 1916, 717). This state of the discipline was an outworn heritage of the revival of Aristotelian logic from the late sixteenth to the early nineteenth century: a tradition which continued seeing the syllogism as the central concern of logical doctrines, but which had lost the subtleties and technical developments which were the great achievements of late-medieval and post-medieval logic (Dutilh Novaes & Read 2016).

The main characteristics of this way of doing logic can be summarized in the following points:

a) Simple standard categorical propositions are the focus of logical analyses. This simple standard form basically consists of two terms (subject and predicate) united by a copula, the result of a two-termed simplifying analysis which usually leaves out negative terms and any other kind of complex or non-standard terms. Examples such as “no non-human is literate”, “every man’s donkey runs”, “every man or donkey runs”, and “every man who is white runs”, which were extensively discussed in medieval and post-medieval logic, were later deemed too complex to deserve attention. This neglect gave rise to the false idea that Aristotelian syllogism was a rigid and limited structure, instead of considering it to be (as it was in its origin) a general abstract structure, that can be filled with terms of any kind of complexity (d’Ors 1981, 1106-1279).

b) Propositions are divided according to a twofold quantity (universal/particular) and a twofold quality (affirmative/negative), thus obtaining four standard forms of categorical propositions, named by the letters A, E, I, O, and captured in the structures “Every A is B” (A form), “No A is B” (E form), “Some A is B” (I form), and “Some A is not B” (O form). Ladd-Franklin, in contrast, will follow De Morgan in dividing the logical space into eight basic propositional forms (Pérez-Ilzarbe 2022, 86).

c) Propositional structures are understood in terms of predication, that is, in terms of saying something about something. Accordingly, the validity of syllogistic structures is exhibited, in the first place, by means of the principles “dici de omni” and “dici de nullo”, which are based on the transitivity of the predicative relation. The tradition soon formulated some rules of thumb from these principles, the famous “rules of the syllogism” (concerning the distribution of the middle term and the quality of the propositions involved) that became usual in logical textbooks (Keynes 1906, n. 199).

The main problem with this worn-out form of traditional logic, Ladd-Franklin thinks, lies in its inability to deal with complex structures. She complains, for example, that this received form of logic does not provide the tools for recognising the equivalence between “All students of chemistry are also students of either biology or physics” and “Students of chemistry who do not study physics all study biology” (Ladd-Franklin 1889, 544-545). In contrast, the abstract point of view of the new symbolic logic easily allows to understand this equivalence in terms of a general rule which applies to any expression containing a transitive copula: “any term which enters the subject as a factor is-the-same-thing-as its negative in the predicate as an alternative” (Ladd-Franklin 1890a, 87-88).

The crucial advance that the new logic has brought about is the understanding that subject and predicate are not the final units of analyis, and that dividing them into parts allows the possibility that these parts are moved back and forth in the proposition:

> The secret of the great command which Symbolic Logic has over complicated trains of reasoning is wholly contained in the fruitful idea that subject and predicate are not necessarily indivisible wholes, but that they can be broken up and their separate elements shifted at pleasure from one side of the copula to the other (Ladd-Franklin 1889, 545).

This is essential to extend the scope of logic towards forms of argument that are not usually taken into account, and this is how the symbols of algebra have helped in overcoming the limitations of traditional logical analysis, too commited in Ladd-Franklin's opinion to the subject/predicate analysis. Curiously enough, the fact that the new logic was developed by mathematicians (interested in posing and solving artificial problems of a higher and higher degree of complexity) has been beneficial to logic as a branch of philosophy (interested in real problems of actual reasoning). Thanks to the work of these mathematicians, logic was developed towards the analysis of propositions with any number of terms, thus containing the classical analysis of two-termed proposition as a particular case.

When terms are taken as the final unit of logical analysis, the question as to what is said of what is no more approached from the point of view of predication, but from the point of view of the mere combination of terms. The possibility (allowed in abstract calculi) of shifting terms from one side of the copula to the other implies that "subject" ceases to be the thing you are talking about, to become just the combination of terms placed to the left of the copula, and "predicate" ceases to be what you say about that thing, to become just the combination of terms placed to the right of the copula. This allows for a treatment of complexity in an abstract (and thus very simple) way, which in turn implies a new look at the copula, which can be understood as a connecting term which not only contains the "is" that is needed for predication, but also brings with it "all the quantity and quality of the proposition incorporated within it" (Ladd-Franklin 1912b, 641, note 3). This is what Ladd-Franklin calls the "figured copula" (ibidem), which allows a systematic treatment of the set of all possible copulas (which is the same as propositional forms), for whose disctinction only three aspects are relevant: quality, quantity, and symmetry.

The role of the copula is crucial from a meta-logical perspective, since it allows to differentiate each of the several logical systems from each other, and to assess them from a purely formal point of view. At this point, Ladd-Franklin's contribution in the search for more adequate forms of expression is her proposal to adopt symmetrical copulas as canonical forms of expression, due to their advantages from the point of view of symbol manipulation. The apparent naturalness of the universal affirmative non-symmetrical copula (the one used in the expression "a is b") looses its value when more complex propositions are involved: in these cases, it becomes more natural to use a form of expression in which all the terms play the same role (that is, it is more natural to choose a symmetrical copula) (Ladd-Franklin 1889, 556-557). If all terms play the same role, their place in the proposition becomes completely irrelevant: this means an advance from the point of view of having a simpler set of rules, but in addition, the great practical benefit of symmetrical copulas is that they make the fallacy of a "wrong conversion" utterly impossible to commit (Ladd-Franklin 1912b, 646).

The possibility of shifting terms from one side of the copula to the other leads naturally to the consideration of the limit-cases of this transposition: cases in which all the terms are placed on the same side of the copula. This has led Ladd-Franklin to defend the use of the Special Terms of Logic ("everything" and "nothing"), which are special indeed, since from the syntactic point of view they behave as terms, but from the semantic point of view their meaning is not variable, but fixed.

In normal circumstances, these terms tend to remain "unexpressed" (that is, they play an implicit role in the proposition), but there might be some occasions when it is convenient to make them explicit. For example, the complete expression of a subject-predicate proposition will reveal its four-termed nature: "*Everything* which is a is either b or else *non-existent*" (Ladd-Franklin 1990a, 88). This expression reveals the lack of existential commitment of the universal affirmative proposition.

In her mature theory of the Special Terms, Ladd-Franklin defends the thesis that "an existence-term is always involved in every possible statement, and it is entirely at our discretion whether we make it explicit or not" (Ladd-Franklin 1912b, 651). In other words, every proposition is an "existence proposition" in the sense of being "concerned with existence": a) some of them are explicit and precise affirmations of existence (in the form "a exists"); b) the rest contain an implicit concern with existence, be it an affirmation of existence (in the form "a is b", which implicitly asserts the existence of the combination ab) or a denial of existence (in the form "no a is b", which implicitly denies the existence of the combination ab).

When Ladd-Franklin discusses the import of propositions, she is very careful not no privilege any of the two aspects of terms (extension/intension). She explains that the copula includes a fourfold implication, resulting from the fact that terms have a double meaning (class-meaning/attribute-meaning): "Every term is a double-edged machine—it effects the separating out of a certain group of objects and it epitomises a certain complex of marks" (Ladd-Franklin 1890b, 561). Since each term (with its two-fold meaning) can occupy two different places in the proposition (subject position/predicate position), four implications result: that certain objects are the same as certain objects, that certain objects are in possession of certain qualities, that a certain quality-complex entails another quality-complex, and that a certain quality-complex is indicative of the presence of certain objects. From the logical point of view, when you assert a proposition you are *equally* affirming all the four. In my opinion, this is the point behind Ladd-Franklin's insistence that the copula has "a double force". Even if we interpret this in terms of the force/content distinction (Boyd 2022), what she is trying to stress about the content is not the mere existence of a connection between subject and predicate, but the *fourfold nature* of the asserted connection.

Now, the Special Terms behave in the same way as any term, so that in their class-meaning they tell us about "existent *things*", and in their attribute-meaning they tell us about "things *which exist*". For example, when you exclude ab from the universe of possible combinations, your denial includes both "no things are *ab*" and "whatever is *ab* is-not existent". Analogous considerations can be made if the system is interpreted as a calculus of propositions. The point is, in any case, that the copula carries with it all four implications.

The discussion concerning existence, and the adequate way of expressing it in propositions, was a hot topic of the moment, specially in America, since the "Six Realists" (Professors Perry, Montague, Holt, Pitkin, Spaulding, and Marvin) presented their program for a scientific philosophy in 1910 (Holt, et al. 1910), as a united reaction against Josiah Royce's "refutation of realism" (MacKinnon 1985, 328-331). Ladd-Franklin's solution leaves open, in her logical papers, what must be the exact "special character" attached to "existence" or "reality" (Ladd-Franklin 1912b, 653), which logic can "throw no light upon." In the logic of classes, "existence" just means "occurrence within a given domain of thought" (be it the space-time physical world or any imaginable domain). In the logic of propositions, it just means "possible state of affairs" (be it actual or merely entertained in thought).

All these subtleties are examples of the kind of progress that symbolic logic is bringing to the field, in contrast with the traditional analysis. Ladd-Franklin highly values collaborative work and expects achievements in the long run: she sees every symbolic advancement as a small step in the right direction, paving the way for "more carefully thought and more detailed" developments (Ladd-Franklin 1912, 641, note 3). It remains to be examined the debate concerning the alleged mathematical character of the new logic.

3 Symbol logic versus mathematical logic

The awkward position of symbolic logic between philosophy and mathematics is still a problem today, and it was perceived as a major problem in these days, both in America and in Europe (Peckhaus 1999) (Grattan-Guinness 2004) (Marion & Moktefi 2014). Taking a clear stance in this controversy, Ladd-Franklin prefers to call the new logic simply "symbol logic" rather than "mathematical logic" (Ladd-Franklin 1916).

She admits that the term "mathematical" can be understood in at least two different senses. In its specific sense, "mathematics" is a calculus with a particular content: according to her, it concerns quantity. But in a wide sense of "mathematics", she admits that "every form of deductive reasoning is mathematical, in the sense of being highly abstract and subject to formal rules of operation, which allow it to be carried out without knowledge of what things are" (Ladd-Franklin 1889, 546). Now, the highly abstract character of logic has been fostered by the use of symbols, but Ladd-Franklin insists that symbolism is a very useful though not essential tool (Ladd-Franklin 1889, 545) (Ladd-Franklin 1890a, 88). Even more, not every symbolism is adequate, and hence her search for the most adequate system of symbolic logic, and her negative appraisal of Russell's form of symbolic logic (Pietarinen 2013, 153-156) (Anellis 2004-2005).

The goodness of a symbolic system can be measured with respect to the three goals of a logical calculus: expression, rules for immediate inference, and rules for reasoning (Ladd-Franklin 1889 549). A bad choice in the expressive step might introduce irrelevant elements that the system will carry with it throughout its development. The main concern about Russell's form of logic is that it was oriented to the interest of mathematicians, forgetting the interest of philosophers, and thus introducing elements that can be very relevant from the mathematical perspective, but irrelevant from the perspective of reasoning in general (Ladd-Franklin 1918, 177). In this respect, Ladd-Franklin feared that symbolic logic would be identified with Peano's system, "in which everything is sacrificed to the mathematician's ways of thinking" (Ladd-Franklin 1912, 641).

Ladd-Franklin distinguishes two fields of application of the new symbolic logic: reasoning in general, and the foundation of knowledge in particular. This second strand is again connected with the preoccupations of the time, which include as special concerns the problem of the foundations of mathematics and the problem of constructing a "scientific philosophy" (Wilson 1990). These two particular projects are connected to each other, since philosophy took the foundational project in mathematics as a model to follow, and both are connected to the progress of logic, since it was the development of modern logic that showed the need to make explicit the primitive elements (undemonstrated premises and undefined terms) of any system of thought (Ladd-Franklin 1912, 641).

It might be true that logic has a mathematical side (the part of logic that applies to the foundation of mathematics), but logic must also have a non-mathematical, but strictly philosophical, side: the part of logic that applies to reasoning in general, and to the foundation of knowledge in general. For this project, Ladd-Franklin finds Bertand Russell's "mathematical logic" to be particularly inadequate: she harshly labels it as a "one-sided and amorphous form of logic" (Ladd-Franklin 1912b, 663). I want to mention just two of Ladd-Franklin's criticisms, which are related to each other: the use in logic of "that uncanny term, the variable" (ibid., 661), and the excessive attention paid to individuals (American Philosophical Association 1918, 177). In my opinion, her qualms about the introduction of individuals in logic (and about its correlate, the use of variables as standing for any individual) are due to the fact that she considers its introduction in logical calculi to be a desertion from the maximally abstract stance of formal logic. In abstract algebra, reference to individuals can be disregarded when the view is put on classes: hence Ladd-Franklin's criticism of the membership relation. She sees no point in distinguishing a new relation apart from class inclusion, these are irrelevant differences that can be ignored when reasoning is approached from the point of view of the elimination of a middle term. This and no other is for Ladd-Franklin "the problem of logic":

> To throw the multiform propositions of real life into a single standard form of expression, to condense the information that interests us by the elimination of certain terms which we do not care for, and to state the information which is left in the form of any terms which we happen to wish to see described (Ladd-Franklin 1889, 563).

4 Ladd-Franklin's legacy

Christine Ladd-Franklin saw herself as making an important contribution to the advancement of logic. Maybe the fact that she did not gather a school of disciples around her explains the fact that her work was not as recognised as it might have been. To my knowledge, her only disciple was Eugen Shen, who did a good work in disseminating Ladd-Franklin's main achievements (Shen 1927) (Shen 1929), but who did not manage to start a Laddian tradition.

Fortunately, Ladd-Franklin is being recognized as one of the founding grandmothers of modern logic (Janssen-Lauret forthcoming), and a growing interest in her work can be appreciated, day after day, in conferences and publications. I must thank the editor of this Special Issue for this oportunity to share some of the ideas of this great logician.

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