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Robert Meyer's Publications on Relevant Arithmetic

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Abstract

The bibliography appearing below collects the publications in which Meyer's investigations into relevant arithmetic saw print. Each bibliographic item is accompanied by a short description of the text or other remarks. We include papers on relevant arithmetic coauthored by Meyer, but omit both Meyer's work on relevant *logic* and the work published independently by his collaborators.

• R. K. Meyer. Relevant arithmetic. *Bulletin of the Section of Logic*, 5:133–137, 1976.

◦ *This abstract offers a précis of the results and motivations outlined in Meyer's monographs* Arithmetic Formulated Relevantly *and* The Consistency of Arithmetic. $\mathsf{R}^\sharp$ *is formally defined and the relationships—both philosophical and formal—it bears to classical arithmetic are sketched out. Importantly, several notable results—*e.g.*, the Post and arithmetical consistencies of* $\mathsf{R}^\sharp$ *and the admissibility of the* γ *rule in* $\mathsf{R}^{\sharp\sharp}$*—are first announced—although not proven—in this abstract.*

◦ *This work is reprinted in this issue.*

• R. K. Meyer. *Arithmetic Formulated Relevantly*. Unpublished monograph, 1976.

◦ *Along with* The Consistency of Arithmetic*, one of two extended treatments on relevant arithmetic written by Meyer in 1976. Despite thematic overlap*

with its companion, this manuscript is gentler and more introductory in its tone, its case for $\mathsf{R}^\sharp$ *made through illustrations of how relevant arithmetic captures mathematical intuitions more closely than classical* PA*. The rest of the text serves as a guidebook acquainting the reader with interesting results about* $\mathsf{R}^\sharp$*, largely avoiding its companion's bellicose tone.*

◦ *This work is reprinted in this issue.*

• R. K. Meyer. *The Consistency of Arithmetic.* Unpublished monograph, 1976.

◦ *Like its companion* Arithmetic Formulated Relevantly*, this 1976 monograph is a rigorous philosophical investigation of the relevant arithmetic* $\mathsf{R}^\sharp$*. If the former work is a gentle invitation to* $\mathsf{R}^\sharp$*,* this *work enlists the* Post consistency *of* $\mathsf{R}^\sharp$*—along with the apparently* finitary *nature of its proof—to drive a sustained and vigorous series of arguments targeted at a purportedly overzealous reading of Gödel.*

◦ *This work is reprinted in this issue.*

• R. K. Meyer and C. Mortensen. Inconsistent models for relevant arithmetics. *Journal of Symbolic Logic*, 49(3):917–929, 1984.

◦ *This paper is notable for including the first appearance* in print *of a proof of a cornerstone of Meyer's program: the Post consistency of* $\mathsf{R}^\sharp$*. Notably, the paper demonstrates the utility of finite, inconsistent models of arithmetic as powerful tools for establishing results about relevant arithmetics.*

◦ *This work is reprinted in this issue.*

• R. K. Meyer and I. Urbas. Conservative extensions in relevant arithmetic. *Zeitschrift für mathematische Logic und Grundlagen der Mathematik*, 32:45–50, 1986.

◦ *This paper's main result is that all* positive *theorems of* $\mathsf{R}^\sharp$ *have a proof in which no instances of axioms including negation (*e.g.*, excluded middle) occur. Defining positive subsystems of a range of other relevant arithmetic, the authors extend this conservativity result to arithmetics like* $\mathsf{E}^\sharp$ *(arithmetic under the relevant logic* E*).*

• R. K. Meyer and C. Mortensen. *Alien Intruders in Relevant Arithmetic.* Technical Report TR-AR-9/87, Australian National University, 1987.

∘ *Running with Dedekind's description of nonstandard naturals as "alien intruders," this joint technical report examines the model theory of nonstandard elements in relevant arithmetic. The primary "Alien Intruder Theorem" is that there exist models of* $\mathsf{R}^\sharp$ *in which every* rational number *plays the role of a positive integer.*

∘ *This work is reprinted in this issue.*

• H. FRIEDMAN AND R. K. MEYER. WHITHER RELEVANT ARITHMETIC? *JOURNAL OF SYMBOLIC LOGIC*, 57(3):824–831, 1992.

∘ *The central result of this joint paper—that* γ *is* not *admissible in* $\mathsf{R}^\sharp$*—crucially affected the trajectory of Meyer's program. The authors observe that the complex ring* $\mathbb{C}$ *serves as a model of* $\mathsf{R}^\sharp$*. But as a countermodel to a* PA *theorem QRF,* $\mathbb{C}$ *witnesses that some theorems of* PA *are unprovable in* $\mathsf{R}^\sharp$*, which entails that* γ *fails for* $\supset$*.*

• J. K. SLANEY, R. K. MEYER, AND G. RESTALL. LINEAR ARITHMETIC DESECSED. *LOGIQUE ET ANALYSE*, 39:379–388, 1996.

∘ *Meyer defined a* secondary formula *to be a* φ *for which both* $\varphi \to 0 = 0$ *and* $0 \neq 0 \to \varphi$ *are provable and showed that in* $\mathsf{R}^\sharp$ *every zeroth-degree formula is a secondary formula,* i.e.*,* $\mathsf{R}^\sharp$ *is* secsed*. This paper identifies a connection between an arithmetic's being secsed and validity of* contraction *in its underlying logic, which the authors examine in the context of the linear arithmetic* $\mathsf{LL}^\sharp$*.*

∘ *This work also appears as ANU Technical Report TR-ARP-2/96.*

• R. K. MEYER. KURT GÖDEL AND THE CONSISTENCY OF $\mathsf{R}^{\sharp\sharp}$. IN P. HAJEK, EDITOR, *GÖDEL '96: LOGICAL FOUNDATIONS OF MATHEMATICS, COMPUTER SCIENCE AND PHYSICS*, PAGES 247–256. CAMBRIDGE UNIVERSITY PRESS, CAMBRIDGE, 1996.

∘ *This paper includes reworked selections from* The Consistency of Arithmetic *critical of Hilbert and Gödel. A description of a finite model for* $\mathsf{R}^{\sharp\sharp}$ *follows, leading to a discussion of the sense in which this model "finitarily proves the consistency of* $\mathsf{R}^{\sharp\sharp}$*." Special attention is paid to a tension between this "finitary" proof and the nonconstructivity of Meyer's proof of* γ *admissibility in* $\mathsf{R}^{\sharp\sharp}$*.*

• R. K. MEYER AND G. RESTALL. '*STRENGE*' ARITHMETICS. *LOGIQUE ET ANALYSE*, 42:205–220, 1999.

◦ *This paper tackles a host of topics related to modal notions in relevant arithmetic. Observing that there exist* restricted *senses in which* $\mathsf{R}^\sharp$ *avoids the traditional* fallacies of modality, *a more traditional presentation of modality is applied to* $\mathsf{E}^\sharp$, *on which the authors prove a number of interesting features,* e.g., *that* $\mathsf{E}^\sharp$ authentically *commits no fallacies of modality. Finally, applying metavaluations to* $\mathsf{E}^{\sharp\sharp}$ *is shown to yield a* complete *theory of arithmetic* $\mathsf{TE}^\sharp$ *which, the authors suggest, may have a stronger claim to being "true arithmetic" than* $\mathsf{R}^{\sharp\sharp}$.

• R. K. Meyer. ⊃E is admissible in "true" relevant arithmetic. *Journal of Philosophical Logic*, 27(4):327–351, 2004.

◦ *After some interesting discussion defending the thesis that* $\mathsf{R}^{\sharp\sharp}$ *characterizes* true arithmetic, *the paper provides a proof that* γ *is admissible in* $\mathsf{R}^{\sharp\sharp}$ *(in contrast to its inadmissibility in* $\mathsf{R}^\sharp$*). The methods developed to prove the result—although infinitary and highly nonconstructive—are independently interesting.*

• R. K. Meyer. Fallacies of division. In M.-H. Lee, editor, *Proceedings of the XXII World Congress of Philosophy*, volume 13, pages 71–80. 2012.

◦ *Meyer recalls his stock suggestion that* $0 = 2$ *does not* relevantly *entail* $0 = 1$ *and considers an objection involving division. This leads to an argument that some classical theorems characterizing division are* au fond *elaborate cases of* fallacies of relevance.