Sosein as Subject Matter

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Abstract

(Neo)Meinongians in general, and Routley in particular, subscribe to the principle of the independence of Sosein from Sein (Routley 1980). In this paper, I put forward an interpretation of the independence principle that philosophers working outside the Meinongian tradition can accept. Drawing on recent work by Stephen Yablo and others on the notion of subject matter, I offer a new account of the notion of *Sosein* (Being-so) as a subject matter (or topic) and argue that in some cases *Sosein* might be independent from *Sein* (Being, Existence). The question whether numbers exist, for instance, is not part of the question of how numbers are, which is the topic mathematicians are interested in.

0 Introduction

(Neo)Meinongians in general, and Routley in particular, subscribe to the principle of the independence of *Sosein* from *Sein* (Routley 1980, Chapter I, sections II.2 and II.3). They believe that an object might have a certain nature, be a certain way, even though such object does not exist.

In this paper, I put forward a non-Meinongian interpretation of the principle of the independence of *Sosein* from *Sein* (henceforth, the *independence principle*), i.e., an interpretation of the principle that should be acceptable to philosophers working outside the Meinongian tradition. Such an interpretation of the independence principle is inspired by remarks from Stephen Yablo, a philosopher usually not classified as a Meinongian.

It will be clear that my interpretation of the principle of independence applies only to certain cases, and is not as general as Routley's favorite interpretation. I do not want to claim that the interpretation of the independence principle I discuss in this paper is the best available, or that it is compatible with the one endorsed by Routley, or that it is incompatible with Routley's interpretation. I just want to point out that philosophers working outside the Meinongian tradition can make sense of the notion of *Sosein* and the principle of the independence of *Sosein* from *Sein*. I think that clarifying how and why a non-Meinongian philosopher like Yablo appeals to the principle of independence is already an interesting point to make.

Here is how I propose to interpret the notion of *Sosein*: *Sosein* is a subject matter, i.e., a topic, a question. In particular, the *Sosein* of numbers can be identified with the question of how numbers are. Numbers' *Sein*, however, is the question whether numbers exist. The question whether numbers exist is not part of the question whether numbers are, so one can discuss how numbers are without discussing whether numbers exist.

Why should philosophers working outside the Meinongian tradition be interested in the notion of Sosein and the independence principle? Because the notion of *Sosein* and the principle of independence help to solve a neglected problem in the philosophy of mathematics, which I call the communication problem: How can mathematicians happily communicate despite having opposing views about the existence of abstract mathematical objects like numbers and sets? The answer, I submit, is that mathematicians are interested in how numbers and sets are, and that is a topic that can be addressed without addressing a different topic: whether numbers exist. The metaphysical question whether numbers exist, i.e., numbers' *Sein*, is not part of the mathematical question of how numbers are like, i.e., numbers' *Sosein*.

Section 1 introduces the communication problem and the solution I defend. Section 2 uses the work of Lewis (1988) and Yablo (2014a) to clarify the notion of subject matter or topic and to explain what it means for a topic to be part of another topic. Section 3 introduces my main argument for the conclusion that the question whether numbers exist is not part of the question of how numbers are. The argument is based on two premises, which are defended in sections 4, 5, 6 and 7. Section 8 concludes.

1 Motivation: the communication problem

The aim of this paper is to use the notion of *Sosein* to develop a solution to an interesting but neglected problem in the philosophy of mathematics:

> How is it that mathematicians can happily communicate despite having different views of [...] the existence [...] of mathematical objects? How can the ontological questions that philosophers sweat over be so irrelevant to actual practice? (Yablo 2001, p. 84)

I call this problem "the communication problem (in the philosophy of mathematics)". I want to explore the tenability of the following answer to the communication problem. When mathematicians do mathematics, they are interested in the question of how numbers are.[1] The ontological question whether numbers exist is not relevant for the question mathematicians want to discuss, thus, they tend to ignore it. The topic mathematicians

[1]I use the natural numbers as a paradigmatic example of mathematical objects.

are interested in is the *Sosein* of numbers; the *Sein* of numbers is a different topic, which mathematicians are happy to leave to metaphysicians.[2]

Note that I am saying only that disagreement *about the existence of numbers* has no impact on the mathematical practice and on communication in mathematics. Other kinds of philosophical disagreement, like disagreement about which methods of proof or which kinds of definitions are acceptable, can have a big impact on the mathematical practice. Constructivist and classical mathematicians do not accept the same theorems, so their philosophical disagreement has repercussions for their work as mathematicians. Similarly for the case in which one mathematician accepts impredicative definitions and another does not.

I am also open to the possibility that a disagreement about the nature of numbers might have an impact on the mathematical practice. Nominalists believe that there are no numbers and platonists believe that numbers exist, but nominalists and platonists usually agree that numbers are abstract, in the sense that numbers, if they exist, lack causal powers and spatiotemporal location. Disagreement about the abstractness of numbers might in principle have an impact on mathematical practice. For instance, Linnebo (2018, section 2.5) suggests that:

> [if mathematical] objects had spatiotemporal locations, then actual mathematical practice would be misguided and inadequate, since pure mathematicians ought then to take an interest in the locations of their objects, just as take an interest in the locations of animals. (Linnebo 2018, section 2.5)

In any case, let me stress that here I am interested only in explaining how mathematicians are able to communicate despite having opposing views *about the existence of numbers*. I am open to the possibility that other kinds of philosophical disagreement might lead to disagreement about which theorems should be accepted or which methods of proof should be adopted.[3]

My answer to the communication problem is that (i) in normal circumstances mathematicians talk about how numbers are, and (ii) whether numbers exist is not part of the

[2]The answer to the communication problem I am going to defend is inspired by Yablo (2014a, pp. 90-1) and Yablo (2001, p.84). It is not by accident that I confine my attention to the communication problem in the philosophy of mathematics: although “communication problems” similar to the one discussed here might arise also in other areas of discourse, the solution I defend is meant to apply only to numbers and “preconceived objects” (see section 5).

[3]One anonymous referee proposed the following variation on the communication problem: “[h]ow is it that mathematicians can happily communicate despite having different views of the metaphysical nature of mathematical objects?” For the reasons mentioned above, I think we should distinguish the way in which disagreement about the existence of numbers and disagreement about the nature of numbers might affect communication in mathematics. My formulation of the communication problem uses only elements of the quotation from Yablo 2001, p. 84, precisely because I think that disagreement about the nature of mathematical objects might affect mathematical practice (again, think about case of classical vs. intuitionist mathematicians).

question of how numbers are. This answer to the communication problem makes use of the notion of subject matter or topic. In the following section I will draw on work by Lewis and Yablo to make these notions precise. But let me first give you an informal justification of my appeal to the notion of subject matter as the key to solve the communication problem.

Prime numbers are plentiful sounds like an uncontroversial claim. *Numbers exist* sounds like a much bolder claim. This is strange, given that on standard assumptions prime numbers are plentiful entails numbers exist and we recognize such an implication (Shaffer 2009). How can an uncontroversial claim un-controversially entail a controversial claim (Yablo 2017)?

It is important to reflect on a feature of the argument, *prime numbers are plentiful* ∴ *numbers exist*. "Something changes – some cognitive switch is pulled – en route from [the premise] to [the conclusion of the argument] in a way that bears on our judgments of clear vs controversial" (Yablo 2014b).

Prime numbers are plentiful seems to be a number-theoretic claim, the kind of claim that working mathematicians feel comfortable making. *Numbers exist* seems to be a metaphysical claim, the kind of claim working mathematicians tend not to make. One reason experts might refrain from making this sort of claim is that they concern an issue that the experts recognize as outside their field of expertise.

The hypothesis I want to explore is that mathematicians refrain from asserting numbers exist because such a sentence introduces a new topic, one that is different from the topic they feel entitled to talk about: **Whether numbers exist** is a topic that pertains to metaphysics, not to mathematics.[4] Number theory is about **numbers**, not about **the existence of numbers**. The change that takes place en route from prime numbers are plentiful to numbers exist is a change in subject matter.

Summing up: The subject matter of (pure) number theory is **how numbers are**, i.e., numbers' *Sosein*. The question **whether numbers exist** is not a part of the question of **how numbers are**. This explains why mathematicians do not bother about metaphysical questions like the existence of numbers.

Borrowing Carnap's (1950) jargon, one might call **how numbers are** the internal question of number theory and the question **whether numbers exist** a question external to number theory. That's ok, but note that the solution to the communication problem defended here is not committed to the idea, endorsed by Carnap, that external questions, taken at face value, are meaningless. Nor is the solution to the communication problem defended here committed to the claim that external questions are meaningful: No position concerning the meaningfulness of external questions (taken at face value) is defended here. Note also that the present solution to the communication problem is not based on Carnap's theory of frameworks; rather, the present solution to the communication problem is based on a theory of subject matter and a connection between communication and subject matter. These two elements are not present in Carnap's proposal. In addition, in the following

[4]Notation: phrases in **bold face** refer to subject matter.

sections I discuss a metaphysical view (the idea that numbers are preconceived objects) that is not part of Carnap's original proposal.

In the following sections, I use recent work on the notion of subject matter by Stephen Yablo and others to explain what a subject matter is and what it means for subject matter **m** to be a part of subject matter **n**. Using these definitions, I then identify the two subject matters how numbers are and whether numbers exist and defend the thesis that the first does not include the second.

2 Subject matters and their parts

I take seriously the idea that the communication problem should be solved by connecting the notions of communication and subject matter. This is natural: Conversations are about topics, and if a topic is not part of what is being discussed in a given conversation, then it is natural for those engaging in that conversation to ignore it. I want to find a subject matter on which mathematicians can agree despite disagreeing over the existence of numbers, and I claim **how numbers are**, i.e., numbers' *Sosein*, is the subject matter I am looking for: **how numbers are** that does not include the topic **whether numbers exist**.

In order to make the notion of subject matter and the relation of a subject matter being part of another precise, I need a theory of subject matter. This section provides the elements of a theory of subject matter needed for the rest of the paper. I should point out that I use one of the simplest theories of subject matter available, i.e., Lewis' theory. I choose this theory not because I believe that is the best one but to keep things as simple as possible. More sophisticated theories of subject matter, like Fine's (2017) or Yablo's (2014a), are not needed here.

A subject matter, or topic, is what our books, papers, conversations etc., are about. It is the issue we address, the question we try to answer in these conversations, books, papers, etc.. A question can be identified with a set of propositions, $\{p_1, p_2, ..., p_n, ...\}$: These propositions can be thought of as the answers to the interrogative sentence that we use to express the question. The topic **the number of stars** can be identified with the set of propositions {there are no stars, there is one star, there are two stars, ...}, which are the answers to the interrogative sentence *how many stars are there*? (cfr. Lewis 1988). Two worlds agree on subject matter **m** if the same **m**-answer is true in both worlds.

The set of propositions {there are no stars, there is one star, there are two stars, ...} conceived as a family of sets of possible worlds is a partition of the set of possible worlds (every possible world belongs to one and only one of the propositions contained in the set), and partitions induce equivalence relations: Given the subject matter **m**, two sets are **m**-equivalent if they belong to the same **m**-cell; therefore, subject matters like the numbers of stars can be also defined as equivalence relations on the set of possible worlds.

There might be cases where taking subject matters to be equivalence relations might not be the best approach. The different answers to *how many stars are there?* are incompatible with each other, but there seem to be questions that admit more than one correct answer: *Where can I get an Italian newspaper?* is one of Yablo's examples. The simplest option might be just to take a subject matter to be a set of propositions. This will do for our purposes.

A broader subject matter covers more issues, so it should be easier for two worlds to disagree about it, which means that the subject matter should have smaller cells. The broadest possible subject matter should have cells containing only one possible world. The trivial subject matter, which all worlds agree on, is simply the set of all worlds. If subject matters are partitions (equivalence relations), then **m** includes **n** iff **m** is a refinement of **n**; i.e., every **m**-cell is included in an **n**-cell. A more general definition is that **m** includes **n** if and only if every answer to **m** entails an answer to **n** and every answer to **n**, is entailed by an answer to **m**. So the topic **your last weekend** includes the topic **your last Sunday** because every answer to the question *what did you do last weekend?* entails an answer to the question *what did you do last Sunday?*, and every answer to the latter question is included in an answer to the former.

3 *Sosein* and *Sein* as questions

Numbers' *Sein* as a question is the pair of answers {there are numbers, there are no numbers}.[5] It is commonly assumed that numbers are modally extreme: the existence of numbers is either a metaphysical impossibility or a metaphysical necessity.[6] If this were the case, one answer to the question whether numbers exist would be a necessary truth and the other a metaphysical impossibility. If we accept the idea that numbers are modally extreme, then the question of **Numbers' *Sein*** becomes part of the question of **Numbers' *Sosein***. Here is why: if numbers are modally extreme, then **Numbers' *Sein*** is the pair $\{\top, \bot\}$, where $\top$ is a proposition true in all worlds and $\bot$ is a proposition that is true in none. The answers to the question of **Numbers' *Sosein*** divide into two groups: those, like *numbers are such to include infinitely many primes*, that are conceptual necessities, and those, like *numbers are such to include only finitely many primes*, that are conceptual impossibilities. This means that **Numbers' *Sosein***, conceived as the set of the propositions expressed by its answers, is the pair $\{\top, \bot\}$. So, on the hypothesis that numbers are modally extreme, **Numbers' *Sosein*** and **Numbers' *Sein*** turn out to be the same question.[7]

[5] Given that my purpose here is to show that the principle of the independence of *Sosein* from *Sein* can be accepted also by non-Meinongian philosophers, I treat *Sein* as existence conceived in the most orthodox way, which I take to be Quine's way. My point is not that the orthodox account of existence is correct, but that even in an orthodox account of existence, there is room to argue that how numbers are is in independent (in the sense explained in the paper) from the question whether numbers exist.

[6] The phrase "modally extreme" comes from Yablo (2012).

[7] If you identify a question with the set of its possible answers (those that are true in at least one world),

Fortunately, the idea that the hypothesis that numbers exist must be either necessary or impossible has recently been challenged (Field 1989, Rosen 2006, Yablo 2014a): There are ways to make sense of the idea that the question whether numbers exist is contingent, in the sense that neither the hypothesis that numbers exist nor the hypothesis that numbers do not exist is *incoherent* or *absurd*.

Perhaps when we are dealing with standard metaphysical modalities the existence of numbers is either necessary or impossible, but there might be other modalities beyond the metaphysical ones. Say that X is impossible relative to a set of constraints C iff C logically entails the negation of X. Metaphysical impossibility is what you obtain when C is the set of all the constraints about how the world might be. When C is a smaller set, we obtain a different sort of modality (Yablo 2014a, Appendix to Chapter 5).

One way to put this point is to think whether a certain possibility is excluded by the nature/essence of a certain object (Fine 1984) or the nature of a plurality of objects (Correia 2006). As argued by Fine (1984), given the existence of Socrates, the existence of {Socrates} is a metaphysical necessity, but the existence of {Socrates} is not part of Socrates' essence. Let 's-necessary ψ' express that it lies in the nature of Socrates that ψ. The idea is that (modulo the existence of Socrates) it is necessary that {Socrates} exists but not s-necessary that {Socrates} exists.

If we admit stronger modalities than the standard metaphysical ones, both the hypothesis that there are numbers and the hypothesis that there are no numbers might turn out to be possible relative to this non-standard modality. Following Rosen (2006), I take the relevant modality here to be necessity in virtue of the essence of numbers.

I will say more about the notion of the essence of numbers in the following section, but the main point here is that the notion of essence I am using is such that it is necessary in virtue of the essence of numbers that if numbers exist, each number has a successor, but it is not necessary in virtue of the essence of numbers that numbers exist. In Rosen's words:

> It may be that no relation deserves the name '$\in$' unless it satisfies the pairing axiom, just as nothing deserves the name 'bachelor' unless it is male. But it is not in the nature of bachelorhood to be instantiated; and likewise, it is not in the nature of the epsilon relation that something should bear it to something else. You can know full well what set membership is supposed to be – what it is to be a set, what the word 'set' means-without knowing whether any sets exist, and hence without knowing whether Pairing is true. (Rosen 2006, p. 18)

Numbers' *Sosein*, conceived as a question, is the set of answers {numbers are such that ..., numbers are such that __, numbers are such that _ _, etc.}. I argue that the correct answers to the question about how numbers are (numbers are such to include a smallest element, numbers are such to include infinitely many primes, etc.) entailed by the essence of numbers and count as necessary when considering the modality 'necessary relative to

then **Numbers' *Sosein*** = **Numbers' *Sein*** = $\{\top\}$.

the essence of numbers'. Incorrect answers to the question of how numbers are will be taken to be impossibilities relative to the same modality.

My argument for the claim that **Numbers' *Sein*** is not contained in **Numbers' *Sosein*** is that the correct answers to the question of how numbers are do not entail an answer to the question whether numbers exist and, therefore, applying the definitions from the previous section, the subject matter **Numbers' *Sein*** is not part of the subject matter **Numbers' *Sosein***. Here is the argument in schematic form:

(P1) Answers to the question of how numbers are like are determined by numbers' essence.

(P2) Numbers' essence does not determine whether numbers exist.

(C) So the answers to the question of how numbers are do not entail an answer to the question whether numbers exist.

At least three questions need to be answered: (i) What does the phrase "the essence of numbers" mean? (ii) Why should all the correct answers to the question of how numbers are be entailed by the essence of numbers? (iii) Why does the existence/non existence of numbers fail to be entailed by the essence of numbers?

The following sections are dedicated to answering these questions.

4 The essence of numbers

The notion of essence at play here is the one made popular in contemporary work in metaphysics by Fine (1984). The essence of an object o is specified by an answer to the question "What is o?" Aristotelians, for instance, maintain that it lies in the essence of Socrates to be human, so their answer the question "What is Socrates?" is "a human being". The notion of essence can be also applied to pluralities of things (Correia 2006): Humans are essentially rational beings, and bachelors are essentially unmarried men.[8]

Fine and Correia consider essence a primitive notion, in particular, they argue that essence should not be analyzed in modal terms: That o is essentially F does not mean that in all possible worlds in which o exists, Fo. This might lead one to think that the notion of essence is irremediably obscure. But I think something informative can be said about the essence of natural numbers.

Every account of the natural numbers must characterize them somehow. A Neo-Fregean account of natural numbers (Hale and Wright 2003) takes numbers to be the values of the

[8]When I talk about numbers' essence, the notion of essence I have in mind is what Correia (2006) calls "generic essence" (what it is to be a number) as opposed to objectual essence (the essence of an individual number). When I say that it is part of the numbers' essence to include infinitely many primes, I am not claiming that it lies in the essence of each individual number to include infinitely many primes, but that "[n]umbers are of a type to include infinitely many primes" (Yablo 2014a, 91). See Correia (2006) for more on the distinction between generic and objectual essence.

number function, a function governed by Hume's Principle (the number of Fs = the number of Gs if and only if there is a $1-1$ correspondence between Fs and Gs).

A structuralist account of natural numbers takes natural numbers to be the elements of an omega sequence (Benacerraf 1965, Shapiro 1997), where omega sequences are defined as any system of objects satisfying the second-order Dedekin-Peano axioms.[9] According to other accounts, the key principle governing number talk is that the number of Fs $= n$ if and only if there are n Fs (Rayo 2013, see also Rosen and Yablo MS).

Whichever way numbers are characterized, this characterization is taken to reflect an essential feature of numbers. It is a natural gloss to the Fregean account that numbers are essentially the value of the number-of function (Rosen 2017, Rosen and Yablo (MS), Rosen 2010). Similarly, it seems not merely true that 2 is the number of the Fs if and only if there are exactly two Fs: The property of being the number of all the pluralities composed of two elements strikes us as an essential feature of the number 2.

Some philosophers argue in favor of a certain account of natural numbers on the grounds that this account correctly ascribes to natural numbers their essential features. Gómez-Torrente's (2015) case for the view that natural numbers are cardinality properties is based on the claim that such a view delivers the correct results about the essential features of numbers. And Crispin Wright's invocation of 'Frege's constraint' in support of a Neo-Fregean account of natural numbers is based on the idea that such an account makes the possibility of using natural numbers to count things an essential feature of natural numbers.[10]

In section 6 I focus on a specific account of natural numbers (and mathematical objects in general) offered by Kit Fine (2005). But the notion of the nature of numbers should be intelligible independently of the choice of a specific theory of the natural numbers. The essence or nature of natural numbers is simply the way natural numbers are characterized in your favorite account of natural numbers. In the next section, I defend the idea that the way numbers are (their *Sosein*) is fixed by their essence by arguing that mathematical objects are preconceived objects.

5 Preconceived objects

In this section, I defend the idea that numbers are preconceived objects. The doctrine that numbers are preconceived objects plays an important dialectical role in this paper: It serves to motivate premise (P1) of the argument from section 3.

[9]Perhaps one might add the requirement that the interpretation of '$<$' be computable (Benacerraf 1996, Halbach and Horsten 2005, Horsten 2011).

[10]"Frege's Constraint explicitly incorporates the additional thought that [the] essence [of mathematical objects like the natural or real numbers] is to be located in the applications; and so much was tacitly built into my characterizations above of the basic metaphysical questions which a satisfactory foundation for a particular pure mathematical theory should address, in particular in the central role accorded to the question what kinds of thing the numbers in question are numbers of" (Wright 2000, 325).

When we are dealing with a concrete object, the object usually has very few essential features: My essential features include my kind, my origin and few other things (Yablo 2002). If one wants to find out whether I am blond, there is no way to answer the question by considering only my essence. You need to look at me in order to answer the question whether I am blond, which means that in possible worlds where I am absent the question whether I am blond or not does not have an answer. Given an object o and a feature F that o possesses only accidentally (if at all), the answer to question whether o is F is not entailed by the essence of o. Concrete objects have plenty of accidental features, hence the question of how they are cannot be answered just by considering their essences. The *Sosein* of concrete objects outstrips their essence.

With abstract objects like numbers, the situation seems different. Of course, numbers have accidental features, if we consider their relations with concrete objects: Zero is the numbers of dragons in some worlds (those with no dragons) but not in others (those with dragons). But if we focus on the relations that a number has to other numbers and the intrinsic properties of numbers (if there are any), these features seem to be essential features: 17 is essentially prime and essentially larger than 15.

Abstract objects like numbers and sets have been called "preconceived objects" by Yablo. This means that "[e]ither they should have feature F, given their job description, or they don't have feature F" (Yablo 2010, p. 7). We can take the job description of numbers to be their nature or essence, i.e., the way they are characterized in your favorite account of such objects.

My claim is that the essence of numbers contains the answer to the question whether numbers possess feature F. Numbers' *Sosein* does not outstrip numbers' essence. Numbers are such that they include infinitely many primes because it is in the nature of omega sequences to include infinitely many elements occupying prime-number positions. Or because it is a consequence of Hume's principle that there are infinitely many prime numbers, or because some other account of the nature of numbers entails that numbers must include infinitely many primes. So the truth value of *prime numbers are plentiful* is determined by the nature of numbers. Saying that numbers are preconceived objects means that what holds for *prime numbers are plentiful* holds for every sentence of pure arithmetic: The truth-values of these sentences are fixed by the numbers' job description.[11] Of course, this does not mean that it is always possible *for us* to figure out whether the numbers' job description does entail that numbers are such that __: If we take second-order Peano arithmetic as numbers' job description, there will be no mechanical way to check whether a certain number sentence is or is not a consequence of the numbers' job description (Yablo 2010, introduction).

My defense of the idea that numbers are preconceived objects is based on the idea, shared by many platonists, that numbers have their properties essentially and that the

[11] It might be tempting to compare Yablo's idea that numbers are preconceived objects and that the truth value of sentences of pure number theory is determined by numbers' job description with some of Routley's ideas (see Routley 1980, 22-24). I won't attempt such a comparison here.

essence of numbers is given by the way they are characterized by what we take to be the best account of numbers.[12] Both assumptions can in principle be rejected, but I think they are likely to be part of many attractive accounts of what numbers are.

There is a general reason why platonists, who believe that mathematical objects exist, should consider mathematical objects preconceived objects. The reason is that taking mathematical objects to be preconceived offers arguably the best line of defense against one prominent challenge against platonist positions, the so-called epistemological challenge against platonism (Liggins 2010). The challenge for the platonist is to explain how we can form reliable beliefs about a domain of mind-independent and causally inert abstract mathematical objects without invoking mysterious relations connecting concrete mathematicians with abstract mathematical objects. In the words of Hartry Field:

> special 'reliability relations' between the mathematical realm and the belief states of mathematicians seem altogether too much to swallow. It is rather as if someone claimed that his or her belief states about the daily happenings in a remote village in Nepal were nearly all disquotationally true, despite the absence of any mechanism to explain the correlation between those belief states and the happenings in the village (Field 1989, 26-7)

Mark Balaguer (1995) has argued that the best reply to this challenge consists in making a sharp distinction between our knowledge of Nepalese villages and our knowledge of mathematical objects. In the case of Nepalese villages, it is not enough to conceptualize them in order to obtain knowledge about them, whereas in the case of mathematical objects, "all we have to do in order to attain [...] knowledge is to conceptualize [...] a mathematical object" (Balaguer 1995). Why is having a (coherent) conception of a mathematical object enough to know it, whereas having a coherent conception of a Nepalese village is not enough to attain knowledge of it? The most natural answer is that mathematical objects are preconceived objects, whereas Nepalese villages are not.

Balaguer (1995, 1998) is one example of a philosopher who comes very close to accepting the view that mathematical objects are preconceived; Linnebo, whose view that mathematical objects are "thin" (Linnebo 2008), is another one.

In the following section, I consider *procedural postulationism*, an account of (our knowledge of) mathematical entities defended by Kit Fine (2005) and argue that this account vindicates the hypothesis that numbers are preconceived objects. This will provide further evidence in favor of my claim that the most attractive accounts of our knowledge of mathematical objects vindicate the idea that mathematical objects are preconceived objects. Nonetheless, it is important to note that in this section I have given a general motivation to endorse the doctrine that mathematical objects are preconceived objects, a doctrine that, in turn, support P1 of the argument from section 3. So, whether you like procedural postulationism or not, the reasons presented in this section to endorse P1 still apply.[13]

[12] Subject to the qualifications mentioned above.

[13] I have argued that platonists should be attracted by the doctrine that mathematical objects are pre-

6 Procedural postulationism

According to procedural postulationism (Fine 2005), mathematical objects are introduced by an act of postulation. This act of postulation does not consist in laying down a set of axioms, as in the classical version of postulationism. Rather, to postulate something is to give a command, which is expressed by an imperative, rather than an indicative sentence.

Commands might be simple or complex, depending on whether their formulation contains other commands. Given commands A and B, the complex command $A; B$ can be formed: it prescribes to execute A and then B. Similarly, given command A, the complex command A^* can be formed: A^* demands to keep iterating A any finite number of times.

The postulation that introduces natural numbers is obtained from two commands (expressed here by sentences in the imperative mode):

(Zero) Introduce a number!
(Successor) For every number introduced, introduce its successor!

The complex instruction that generates the natural number system is:

(NUMBERS) Zero;Successor*

Procedural postulationism captures the intuition that the natural number sequence is generated by starting from zero and iterating the successor function (which obviously must satisfy the usual constraints).[14] Fine (2005) presents procedural postulationism in the context of a logic, procedural logic, that allows to figure out what is the result of performing a certain task starting from a certain state. Using this logic, it is possible to prove that the execution of NUMBERS yields a state in which second-order Dedekind-Peano axioms are true. In this sense, second-order axioms for arithmetic can be derived from the postulate NUMBERS.

To make it vivid how the introduction of numbers into the domain of discourse proceeds according to procedural postulationism, Fine asks us to imagine a genie that automatically executes every order we give to her. My proposal is to understand the job description of numbers in terms of the task the genie is to perform in order to introduce numbers into our domain of discourse. So the job description of numbers = NUMBERS = the description of the genie's job.

conceived. What about anti-platonists? Even anti-platonists should acknowledge a distinction between correct mathematical claims and incorrect ones. For anti-platonists, the only factor that could make a mathematical sentence correct/incorrect is whether the sentence is in accord with our conception of some relevant kind of mathematical objects. In other words, according to anti-platonists, whether it is correct or not to attribute to a mathematical object a certain feature F depends on whether our characterization of that object entails that it has feature F, which means that the object is preconceived.

[14] The constraints on the successor function mentioned by Fine (2005, p. 107) are that it is a function from natural numbers to natural numbers, that every number has a unique successor, and that if a number is the predecessor of a given number or if a number has no predecessors, this holds of necessity (which means that you can introduce successors by postulation, but not predecessors).

According to procedural postulationism, not everything can be postulated. The first constraint on postulation is that the instructions given to the genie must be consistent. This means you cannot postulate into existence contradictory objects.

Consistency is not the only constraint on postulation. You cannot postulate into existence God or a universal lover (an individual loving all the elements of the domain), even though such postulations are consistent. You can postulate only the existence of a certain kind of objects. Which ones? The answer has to do with the kind of predicates that are used in the postulations:

> It is legitimate to postulate by means of predicates such as '$\in$' or '$<$' or 'successor', but not by means of predicates such as 'person' or 'loves' or 'divine'. But what then is the relevant difference in the predicates? I should like to suggest that it lies in how the predicates are to be understood. Predicates of the first kind are in a certain sense formal; they are simply to be understood in terms of how postulation with respect to them is to be constrained. [...] Say that a predicate is postulational if its meaning is entirely given by a set of postulational constraints. Our view is that we are entitled to postulate by means of postulational predicates, as long as we stay within the constraints by which they are defined. However, we are not entitled to postulate by means of any other predicates, since there is then an independent question, not to be settled by postulation alone, of what their application should be. Thus it is only when the predicates have no content beyond their role in postulation that they may legitimately be used as vehicles of postulation. (Fine 2005, p. 107)

Fine's notion of objects defined by postulational predicates seems very close to Yablo's notion of preconceived objects. The procedural postulationist accepts that predicates such as 'number' or 'successor' "have no content beyond their role in postulation" (Fine 2005: 107). If we take numbers' job description = NUMBERS = the postulate that introduces numbers into our domain of discourse, we can paraphrase the idea that numeric predicates "have no content beyond their role in postulation" as the claim that the features of numbers are fixed by NUMBERS= numbers' job description, which means that numbers are preconceived objects.[15]

In the previous section, I offered general reasons in favor of the thesis that mathematical objects are preconceived. In this section, I used procedural postulationism as one example of an account of our knowledge of mathematical objects that supports the thesis that mathematical objects are preconceived objects. This completes my case in favor of premise P1 of the argument from section 3.

[15] The numbers' job description should be identified with NUMBERS + the constraints mentioned in footnote 12.

7 Essence and existence

Premise 1 from the argument of section 3 can be paraphrased by saying that when we are dealing with preconceived objects like numbers, the answer to the question of how these objects are is determined by the way these objects are *supposed to be* (Yablo 2010, Introduction).

I now turn to premise 2 of the argument from section 3. The premise says that the nature of numbers does not entail that numbers exist. Given the assumption that numbers are preconceived objects, this can be reformulated in this way: The fact that numbers are supposed to be a certain way does not entail that they exist. I hope this sounds plausible. In any case, this section provides a defense of the second premise of the argument from section 3.

Brakeless trains are dangerous or *objects with no forces acting on them do not accelerate* strike us as true claims, even though there are no brakeless trains (hopefully) and no objects without forces acting on them (see Yablo 2014, pp. 90-1). The laws of physics determine how objects of a certain kind (brakeless trains and objects not subject to any force) should behave, but the laws of physics do not determine whether objects of that kind exist.

I argued that in the case of numbers, their job description determines the laws of arithmetic, which, in turn. determine all the features of numbers. I want to argue now that just as the laws of physics do not determine whether brakeless trains exist, the laws of arithmetic do not determine whether numbers exist.

My argument relies on what Rosen (2006) calls the anti-Anselmian conception of essence. According to this conception, the essence of a certain kind of objects might exclude as incoherent the hypothesis that such objects exist but do not possess features $A, B, C, ...$ But the hypothesis that a kind of objects has no instances can never be ruled out as incoherent just on the basis of consideration related to the essence of that kind of objects. In the words of Kant (quoted also by Rosen):

> If, in an identical proposition, I reject the predicate while retaining the subject, contradiction results . . . But if we reject subject and predicate alike, there is no contradiction; for nothing is then left that can be contradicted. To posit a triangle and yet to reject its three angles is contradictory; but there is no contradiction in rejecting the triangle together with its three angles. (Kant, Critique of Pure Reason, A 595/B623)[16]

Our conception of the natural numbers, i.e., the way that natural numbers are supposed to be, is enough to exclude the hypothesis that the sequence of natural numbers might have a maximal element, but not enough to exclude the possibility of there being no numbers.

[16]The Anti-Anselmian conception of essence is not immediately incompatible with the doctrine the existence is a property (as Rosen himself notes). Let's define a kunicorn as an existing unicorn: Then it is part of the nature of kunicorns to exist, but this does not entail that the kind kunicorn is instantiated (the kunicorn example is due to Raymond Smullyan).

I think that the anti-Anselmian conception of essence is very plausible if we identify the essence of the numbers with their job description. Consider the procedural postulationist idea that our conception of the natural numbers is essentially a procedure to generate them by introducing a first element and then iterating the execution of the successor function. Just as we can specify the truth conditions of an indicative sentence without specifying whether that sentence is true or false, so we can specify the conditions under which a certain order would be executed without automatically executing such an order.[17]

NUMBERS specifies a certain task that the genie should execute, but this leaves it open whether the task has been executed or not. The content of an instruction is one thing; the execution of the instruction is another. Even admitting that in some cases it might be enough to give an instruction for such instruction to be executed, it is not merely the fact the instruction has a determinate content that guarantees that the instruction is executed.[18]

I think that an argument in favor of the idea that an instruction might have content although it has not been fully executed can be found if we turn our attention from the sequence of natural numbers to the set theoretical universe. At least according to some philosophers, an important feature of the set theoretical universe is that it is open-ended:

> the hierarchy of sets that exist [...] is regarded as potential in character: however many sets have been formed, it is mathematically possible to form even more. (Linnebo 2013, p. 208)

The task assigned to the genie in the case of sets is essentially one that cannot be completed: No matter how many sets the genie forms, (s)he will never form all the sets that can be formed. In this sense, the task assigned to the genie is an instruction to form more and more sets, going on forever (Yablo 2004, p. 153): It is an order that can never be fully executed.[19] This does not mean that it cannot be partially executed, but it shows that there will always be a gap between the instruction for set formation and its execution, which is the point I wanted to make.

[17] Call an order trivial when you need to do nothing in order to obey it. Fine seems to acknowledge that NUMBER is not trivial in this sense (see Fine 2005, p. 108).

[18] Fine seems to accept that the mere fact that NUMBERS has a certain content (i.e. that necessarily, if such an order gets executed, then the resulting domain satisfies certain conditions) by itself does not entail that such an order has been executed. See Fine (2005, p. 100), where he remarks that in order for an order to be executed more is required than the mere consistency of the order: at the very least, it is also required that the someone gives the order. Someone more pessimistic than Fine would probably believe that even giving the order might not be enough for the order to be executed. But anyway, the point remains that the determining the content of an order is one thing; establishing whether the order has been executed is another thing.

[19] The instruction for the genie in the set-theoretic case is to form the empty set and iterate the following command: whenever there are some things in the domain, form their set (Fine 2005, 93, see also Yablo 2004, p.153). The genie is supposed to form sets as fast as possible, so the procedure of forming sets from pluralities of existing objects should be iterated in the transfinite.

8 Conclusions

In this paper, I have provided a new interpretation of the principle of the independence of Sosein from Sein. My interpretation departs from traditional ones in many important ways, which I do not have the space to discuss here. Nonetheless, I think that my interpretation of the principle of independence has some value, because it explains why philosophers working outside the Meinongian tradition (like, for instance, Yablo) might find the principle attractive.

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