Gödel's Property Abstraction Operator and Possibilism

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1 Introduction

Gödel's Ontological Argument, the most sophisticated and formal of ontological arguments, relies heavily on the notion of *positive property*, which according to Gödel, is a property "independent of the accidental structure of the world"; "pure attribution," as opposed to privation; "positive in the moral aesthetic sense."[1] Pure attribution seems likely to be related to the Leibnizian concept of perfection[2].

Gödel's Ontological Argument is even more distinctive because it employs a third-order modal logic with a property abstraction operator and property quantification into modal contexts. Gödel presents his argument without ever presenting the details of the formal system of logic being employed. The omission is serious because without it the philosophical presuppositions are hard to assess. Furthermore, Gödel never discussed an applied semantics to both explicate the modal operators and relate the formal representations in the argument to the intended meanings. In *Gödel's Ontological Argument* (2000)[3], the formal syntax and semantics of third-order modal logic with property abstraction were constructed, and a completeness theorem for third order modal Logic with property abstraction for faithful models was proved. I argued that it was not possible to develop a sufficient applied third-order modal semantics for Gödel's ontological argument nor was it possible for

[1] Sobel (1987:241-242)

[2] Feferman (1995:389)

[3] Randolph Rubens Goldman, *Gödel's Ontological Argument, Dissertation*, University of California at Berkeley (2000)

Gödel to distinguish between properties that clearly are distinct with regard to intensionality. Due to S5 modal redundancy some properties with multiple modal operators that can differ in meaning can be the same function from worlds to their extensions, and each can have the same extension in each world; consequently, their meaning cannot be distinguished in this system. In *Types, Tableaus, and Gödel's God*[4], Melvin Fitting reiterated this position with the problem for intensionality and reiterated the approach to examining the argument through the formal semantics of higher order modal logic.

However, there are far deeper problems with Gödel's ontological argument than the inability to distinguish meaning. Some problems are indicative of a ubiquitous problem within modal logic in general, namely the problem of creating an adequate applied semantics from formal modal logics with the use of possible worlds. A formal higher order modal logic argument may be an inappropriate forum for any author to formulate some natural language arguments involving enormous ontological cost. Other problems are unique to Gödel's use of third order modal logic and the property abstraction operator that can create properties from any third order modal logic formula with a free variable. I contend that Gödel's use of the property abstraction operator in the context of formal third order modal logic creates implicit philosophical assumptions that commit him to both possibilism (the belief in merely possible objects) and modal realism (the belief in possible worlds). In the first part of this paper I will present the formal syntax and semantics of third order modal logic with property abstraction, and in the second part I will discuss problems with the reasoning in Gödel's argument and the problematic implicit philosophical assumptions.

Jordan Howard Sobel has shown that Gödel's system is seriously defective and suffers from a kind of modal collapse, in that it can be derived that every existent has necessary existence, and that if any proposition is true, it is necessarily true [5]. I will therefore be addressing C. Anthony Andersen's revision of Gödel's Ontological Argument that at least is immune to the modal collapse discovered in Gödel's original argument [6]. As this was later revised

[4]Fitting, *Types, Tableaus, and Gödel's God*, Kluwer (2002)

[5]Sobel (1987: 250 - 253)

[6]Anderson I does not suffer the fate of the original argument, and Sobel's objection is blocked. The reason is that Sobel's objection relied on a consequence of the original version, which was that if x is God-like and has a property then that property is entailed by the property of being God-like. Thus, Sobel was able to create arguments which basically ran as follows: Let x possess the God-like property and imagine a world that has a y distinct from x having a certain essence, φ. Then the necessarily existing God-like being would have the property in that world of being such that there is something distinct from it having essence φ. Therefore this property would be positive and hence necessarily positive, and hence entailed by the necessarily exemplified property of God-likeness, and

I will refer to this as Anderson I.[7] I will refer to Anderson I as a Gödelian Argument in that it uses a formal third order modal logic argument to establish the existence of God and follows closely Gödel's original argument.

ANDERSON I:

Definition 1.1. $G^*(x) = \text{df } \forall\varphi[(\Box\varphi(x)) \leftrightarrow P(\varphi)]$ (The God-like property is changed to having necessary properties [i.e., a necessary property φ of x is a property of x such that $\Box\varphi(x)$].)

Definition 1.2. $\varphi\text{Ess}^*\ x = \text{df } \forall\gamma[(\Box\gamma(x)) \leftrightarrow (\Box\forall x[\varphi(x) \rightarrow \gamma(x)])]$ (An essence of x is a property φ that is such that for every property γ, x has γ necessarily if and only if γ is entailed by φ)

Definition 1.3. $\text{NE}^*(x) = \text{df } \forall\varphi[\varphi\text{Ess}^* x \rightarrow \Box\exists x\varphi(x)]$

Axiom 1.4. $P(\varphi) \rightarrow\sim P(\sim\varphi)$

$\sim\varphi = \text{df } x[\neg\varphi(x)]$

Axiom 1.5. $P(\varphi) \rightarrow [(\Box\forall x(\varphi(x) \rightarrow \gamma(x))) \rightarrow P(\gamma)]$ *(Unchanged)*

Axiom 1.6. $P(G^*)$ *(Analogous to Gödel's Axiom 3 but with the new definition of God-like)*

Axiom 1.7. $P(\varphi) \rightarrow \Box P(\varphi)$ *(Unchanged)*

Axiom 1.8. $P(NE^*)$ *(Analogous to Gödel's Axiom 5, but accommodating new definition of essence)*

Theorem 1.9. $P(\varphi) \rightarrow \Diamond\exists x\varphi(x)$

Proof. Deny, i.e., suppose $P(\varphi)$ and $\sim\Diamond\exists x\varphi(x)$. Then we have $\Box\forall x \sim\varphi(x)$, so $\Box\forall\text{x}[\varphi(x) \rightarrow x \neq x]$, and hence by Axiom 1.5, $P(x[x \neq x])$. However, $\Box\forall x[\varphi(x) \rightarrow x = x]$, and by Axiom 1.5 we have $P(x[x = x])$. $(\rightarrow\leftarrow)$ (to Axiom 1.4). $\Box$

Theorem 1.10. $G^*(x) \rightarrow G^*\mathit{Ess}^*x$

hence it is necessarily exemplified, and thereby it is necessary that there is a y with such an essence. The new version blocks this consequence by allowing neutral properties.

[7]C. Anthony Anderson (1990:295-296) Anderson's revision, Anderson II also contains the same problematic philosophical assumptions.

Proof. Part 1: Suppose $G^*(x)$ and $\Box\gamma(x)$. Then $P(\gamma)$ by definition of G^*, and hence by Axiom 1.7, $\Box P(\gamma)$. Also, by definition of G^*,

$$\Box[P(\gamma) \to [\forall z(G^*(z) \to \Box\gamma(z))]],$$

and so

$$\Box[P(\gamma) \to [\forall z(G^*(z) \to \gamma(z))]],$$

but by Modal distribution, we get $[\Box P(\gamma)] \to [\Box\forall z(G^*(z) \to \gamma(z))]$, and since $\Box P(\gamma)$, we have $\Box\forall z(G^*(z) \to \gamma(z))$. So we have shown that if something has the property of being God-like* and if it has a property necessarily, then that property is entailed by being God-like*.

Part 2: Suppose $G^*(x)$ and $\Box\forall z(G^*(z) \to \gamma(z))$, then by Axioms 1.5 and 1.6, $P(\gamma)$ and so by definition of G^*, $\Box\gamma(x)$. (Thus we have shown that if something is God-like* then it has a property necessarily if and only if that property is entailed by being G* (i.e. G* is an essence* of it). □

Theorem 1.11. $\Box\exists x G^*(x)$.

Proof.

$$G^*(x) \to [\mathrm{NE}^*(x) \wedge G^*\ \mathrm{Ess}^* x]$$
$$[\mathrm{NE}^*(x) \wedge G^*\mathrm{Ess}^* x] \to \Box\exists x G^*(x),$$

so $G^*(x) \to \Box\exists x G^*(x)$. But by theorem 1.9, $\Diamond\exists x G^*(x)$, and so we have $\Diamond\Box\exists x G^*(x)$, but this just gives in $S5$, by model redundancy, $\Box\exists x G^*(x)$. □

2 Syntax and Semantics of Third Order Modal Logic with Property Abstraction

The language will have in addition to individual variables and constants and function variables and constants both predicate variables and constants and second-order predicate variables and constants.The logical constants are: $\neg, \wedge, \vee, \to, \leftrightarrow, \forall, \forall^e, \Box, \Diamond, (,), \lambda, I^2$ "I^2" is a special constant, an exception to the classification of predicates as non-logical constants, and will stand for equality with regard to terms.$\forall^e$ is an Actualist quantifier. The logical constants $\exists$ and $\exists^e$ are defined in the usual way, i.e., $\exists\mu\varphi$ is defined as $\neg\forall\mu\neg\varphi$.

Given the language L of third order modal logic with property abstraction operator [8] , the formulas of L are defined recursively as follows:

[8]For a comprehensive treatment of the language see Randolph Rubens Goldman, Gödel's Ontological Argument, Dissertation, University of California at Berkeley (2000: 51-60) Presently I will use λ for property abstraction. Essentially, If φ is a formula with its free variables the individual variables $\mu_1, \ldots, \mu_k$ then $\lambda\mu_1, \ldots, \mu_k\varphi$ is a predicate constant.

I Atomic formulas$_1$

i If ρ is a predicate variable or predicate constant of degree n and $\eta_1, \ldots, \eta_n$ are terms,[9] then $\rho\eta_1, \ldots, \eta_n$ is an atomic formula$_1$.

ii If Γ is a second-order predicate variable or predicate constant of degree n and $\rho_1, \ldots, \rho_n$ are predicate variables or predicate constants, then $\Gamma\rho_1, \ldots, \rho_n$is a second-order atomic formula$_1$.

II Formulas$_1$

i All atomic and second-order atomic formulas$_1$ are formulas$_1$.

ii If φ and ψ are formulas$_1$ and μ is a variable then the following are also formulas$_1$:

$$\neg\varphi, (\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \rightarrow \psi), (\varphi \leftrightarrow \psi), \Box\varphi, \Diamond\varphi, \forall\mu\varphi, \forall^e\mu\varphi, \exists\mu\varphi, \exists^e\mu\varphi$$

iii Other than the above, nothing else is a formula$_1$.

Predicate constants$_1$ obtained via the abstraction operator:
If φ is a formula$_1$ with its free [10]
variables the individual variables $\mu_1, \ldots, \mu_k$ then $\lambda\mu_1, \ldots, \mu_k\varphi$ is a predicate constant$_1$ of degree k.

I Formulas$_{N+1}$:

i If ρ is a predicate variable or predicate constant of degree k or a predicate constant$_N$ of degree k obtained by the abstraction operator and $\eta_1, \ldots, \eta_k$ are terms, then $\rho\eta_1, \ldots, \eta_k$ is an atomic formula$_{N+1}$ of degree k.

ii If Γ is a second-order predicate variable or predicate constant of degree k and $\rho_1, \ldots, \rho_k$ are predicate variables or predicate constants or predicate constants$_N$ obtained by the abstraction operator, then $\Gamma\rho_1, \ldots, \rho_k$ is a second-order atomic formula$_{N+1}$.

[9]Terms are recursively defined as follows:

1. If ϖ is any individual variable or individual constant then ϖ is a term
2. If δ is an n-ary function constant or n-ary function variable and $\eta_1, \ldots, \eta_n$ are terms then $\delta\eta_1, \ldots, \eta_n$ is a term.

[10]see Goldman (2000: 56-59)

II Formulas$_{N+1}$:

1. All formulas$_N$ are formulas$_{N+1}$.
2. All atomic formulas$_{N+1}$ and second-order atomic formulas$_{N+1}$ are formulas$_{N+1}$.
3. If φ and ψ are formulas$_{N+1}$ and μ is a variable then the following are also formulas$_{N+1}$:

   $\neg\varphi, (\varphi \wedge \psi), (\varphi \vee \psi), (\varphi \rightarrow \psi), (\varphi \leftrightarrow \psi), \Box\varphi, \Diamond\varphi, \forall\mu\varphi, \forall^e\mu\varphi, \exists\mu\varphi, \exists^e\mu\varphi$
4. Other than the above, nothing else is a formula$_{N+1}$.

PREDICATE CONSTANTS$_{N+1}$ OBTAINED VIA THE ABSTRACTION OPERATOR:

1. 1) If φ is a formula$_{N+1}$ with its free variables the individual variables $\mu_1, \ldots, \mu_k$ and which is such that the highest level of predicate constant obtained by the abstraction operator contained in it is N then $\lambda\mu_1, \ldots, \mu_k\varphi$ is a predicate constant$_{N+1}$ of degree k.

FORMULAS: φ is a formula if and only if there is a k such that φ is a formula$_k$. By an atomic formula, I mean an atomic formula$_1$ or second-order atomic formula$_1$.

Abbreviations:

1. '$\eta_1 = \eta_2$' abbreviates $I^2\eta_1\eta_2$ where η_1, η_2 are terms.
2. If ρ and σ are n-ary predicate expressions, '$\rho = \sigma$' abbreviates

   $$\Box\forall\alpha_1, \ldots, \forall\alpha_n[\rho\alpha_1, \ldots, \alpha_n \leftrightarrow \sigma\alpha_1, \ldots, \alpha_n]$$

   where $\alpha_1, \ldots, \alpha_n$ are individual variables.
3. If Λ_1 and Λ_2 are n-ary second-order predicate expressions, '$\Lambda_1 = \Lambda_2$' abbreviates $\Box\forall\pi_1, \ldots, \forall\pi_n[\Lambda_1\alpha_1, \ldots, \alpha_n \leftrightarrow \Lambda_2\alpha_1, \ldots, \alpha_n]$ where $\pi_1, \ldots, \pi_n$ are predicate variables.

It will be obvious by context as to what abbreviation is being employed.

The formal semantics of third order modal logic necessary to interpret such a Gödelian Argument [11] are as follows:

Ω is an L-model if there are A, B and R such that

SEMANTICS: $\Omega = \langle A, B, R, \rangle$ where A is a set, possibly the null set, $A \subset B$, B is non-empty, and R is a function with the constant symbols of L as domain, and such that for all natural numbers n:

[11] In Goldman (2000), Cocchiarella's (1969) system for second order modal logic was extended to third order modal logic with some modifications.

1. for all individual constants υ in $L, R(\upsilon) \in B$.
2. for all n-place predicate constants σ in $L, R(\sigma) \subseteq$ nth Cartesian product of B.
3. for all n-place function symbols (i.e., either function constants or variables) δ in $L, R(\delta)$ belongs to the set of all functions from the nth Cartesian product of B to B.

Where $\Omega = \langle A, B, R \rangle$, we set $\Lambda(\Omega) = A$ and $\Pi(\Omega) = B$, and refer to $\Pi(\Omega)$ as the possibilia of the world Ω. Intuitively, we may view $\Lambda(\Omega)$ as the set of objects existing in the world Ω, while $\Pi(\Omega)$ is the set of possibilia of Ω. Remark on Notation: Given an L-model $\Omega = \langle A, B, R, \rangle, \upsilon$ an individual constant, δ an n-place function constant, and σ an m-place predicate constant, I will sometimes use υ_Ω in place of $R(\upsilon), \delta_\Omega$ in place of $R(\delta)$, and σ_Ω in place of $R(\sigma)$.

Definition 2.1. An I-indexed family $\langle \Omega i \rangle i \in I$ of L-models is a *world system* for L if for all j and $k \in I$:

1. $\Pi(\Omega_j) = \Pi(\Omega_k)$ (The possibilia of worlds in a world system are the same)
2. $\cup\Lambda(\Omega_k)(k \in I) = \Pi(\Omega_j)($ The union of the domains of existing objects over the worlds in a world system is equal to the set of the possibilia of any world in the world system) [12]

We speak of the members of I as the reference points of the world system. Intuitively, all worlds have the same possibilia but may differ with regards to the objects existing in them.

Definition 2.2. Give a world system $\langle \Omega_j \rangle\ j \in I$, we say that X is an *n-ary attribute* in $\langle \Omega_j \rangle j \in I$, if X is a function with domain I such that for all $j \in I, X_j \subseteq$ nth Cartesian product of $\Pi(\Omega_j)$.

Definition 2.3. Given a world system $\langle \Omega_j \rangle j \in I$, we say Y is an *n-ary e-attribute* if Y is an n-ary attribute in $\langle \Omega_j \rangle j \in I$ such that for all $j \in I, Y_j \subseteq$ nth Cartesian product of $\Lambda(\Omega_j)$.

Definition 2.4. $\langle \langle \Omega_j \rangle j \in I, \langle F_n \rangle n \in w, \langle E_n \rangle n \in w \rangle$ is a *Secondary World System* for L if $\langle \Omega_j \rangle j \in I$is a world system for L and if for all $m \in w$, every member of F_m is an m-ary attribute in $\langle \Omega_j \rangle j \in I$, and every member of E_m is an m-ary e-attribute in $\langle \Omega_j \rangle j \in I$, and $E_m \subseteq F_m$.

[12]This represents a change from Cocchiarella's system where it needed only be that $\cup\Lambda(\Omega_k)(k \in I) \subseteq \Pi(\Omega_j)$. The change is in order to establish the soundness of the new axiom $\Diamond\exists^e \alpha\alpha = \beta$.

Intuitively, a secondary world system is a collection of worlds together with some properties, (properties being functions that map worlds to sets which function as the extension of the property in that world).

MODELS FOR THE THIRD-ORDER MODAL LOGIC SYSTEM:

Let us call predicate symbols that range over predicates (of individuals) second-order symbols. Then a β- model (or third order world system) is $\langle\langle\Omega_j\rangle j \in I, \langle F_n\rangle n \in w, \langle E_n\rangle n \in w, \langle \cup F_n, R^*_j\rangle j \in I, \langle Q_n\rangle n \in w\rangle$where, for all j, for each n-ary second-order predicate constant $\Lambda, R^*_j(\Lambda) \subseteq$ nth Cartesian product of $\cup F_n$ and Q_n is a set of second-order n-ary-attributes, i.e. every member G of Q_n is a function defined on domain I s.t. for all $j \in I, G(j) \subseteq$ nth Cartesian product of $\cup F_n$. We will also require that for all second-order predicate constants Λ, there is a G in Q s.t. for all $j, G(j) = R^*_j(\Lambda)$. This will be denoted by G_Λ.

Intuitively, a third order world system extends a secondary world system by including properties of properties. This is required for the Gödel Ontological Argument.

By an assignment of values to variables in a Third-Order World System, we mean a function s on the Set of Predicate and Individual Variables s.t.

1. for each individual variable $\alpha, s[\alpha] \in \Pi(\Omega_j)$ for some j; each individual variable is assigned some possible object.

2. for each $n \in w$, for each n-place predicate variable $\pi, s[\pi] \in F_n$.; each n-place predicate variable is assigned an n-ary attribute.

3. for each $n \in w$, for each n-place second-order predicate variable $\Delta, s(\Delta) \in Q_n$; each n-place second-order predicate variable is assigned a secondary n-ary attribute.

The extensions of term or predicate expressions are relativized not only to third-order world systems but also to the indices and assignments (of values to variables that may occur free in the term or predicate expression). In addition to the extension at an index of a predicate constant σ or secondary predicate constant Λ in L, we will also speak of the intension of σ, respectively Λ, in a third-order world system.

Definition 2.5. Extension of a term or predicate expression (with respect to an assignment) Let $\langle\langle\Omega_j\rangle j \in I, \langle F_n\rangle n \in w, \langle E_n\rangle n \in w, \langle \cup F_n, R^*_j\rangle j \in I, \langle Q_n\rangle n \in w\rangle = \beta$ be a third-order world system for L, ϑ an assignment in β, and let $j \in I$. Then

1. if α is an individual variable, ext$[\alpha, \beta, j, \vartheta] = \vartheta[\alpha]$.

2. if υ is an individual constant, $\text{ext}[\upsilon, \beta, j, \vartheta] = \upsilon_{\Omega j}$. [13]

3. if π is a predicate variable, $\text{ext}[\pi, \beta, j, \vartheta] = \vartheta[\pi]_j$.

4. if σ is a predicate constant in L, then $\text{ext}[\sigma, \beta, j, \vartheta] = \sigma_{\Omega j}$.

5. if δ is an n-place function constant in L, and $\eta_1, \ldots, \eta_n$ are terms of L, then $\text{ext}[\delta(\eta_1, \ldots, \eta_n), \beta, j, \vartheta] = \delta_{\Omega j}$ ($\text{ext}[\eta_1, \beta, j, \vartheta], , ext[\eta_n, \beta, j, \vartheta]$)[Uniqueness follows from the Recursion Theorem and the fact that terms are freely generated. For a discussion of this, see Enderton] [14]

6. if Δ is an n-place secondary predicate variable, $\text{ext}[\Delta, \beta, j, \vartheta] = \vartheta[\Delta]_j$.

7. if Λ is an n-place secondary predicate constant, $\text{ext}[\Lambda, \beta, j, \vartheta] = R_j^*(\Lambda)$.

Definition 2.6. Intention of predicate constant σ of L, or secondary predicate constant Λ of L : Let $\langle\langle\Omega_j\rangle j \in I, \langle F_n\rangle n \in w, \langle E_n\rangle n \in w, \langle \cup F_n, R_j^*\rangle j \in I, \langle Q_n\rangle n \in w\rangle = \beta$ be a third-order world system for L, $\text{int}[\sigma, \beta]$ = the function f with domain I and such that for some assignment ϑ in β, and for each $j \in I$, $f(j) = \text{ext}[\sigma, \beta, j, \vartheta] = \sigma_{\Omega j}$.

$\text{int}[\Lambda, \beta]$ = the function f with domain I and such that for some assignment ϑ in β, and for each $j \in I$, $f(j) = \text{ext}[\Lambda, \beta, j, \vartheta]$.

Definition 2.7. Intention of a predicate variable π or a second-order predicate variable Δ (with respect to an assignment): Let $\langle\langle\Omega j\rangle j \in I, \langle F_n\rangle n \in w, \langle E_n\rangle n \in w, \langle \cup F_n, R_j^*\rangle j \in I, \langle Q_n\rangle n \in w\rangle = \beta$ be a third-order world system for L, ϑ an assignment in β, and let $j \in I$. Then $\text{int}[\pi, \beta, \vartheta] = \vartheta[\pi]$ and $\text{int}[\Delta, \beta, \vartheta] = \vartheta[\Delta]$.

Definition 2.8. SATISFACTION AND TRUTH: Let β- model $\langle\langle\Omega j\rangle j \in I, \langle F_n\rangle n \in w, \langle E_n\rangle n \in w, \langle \cup F_n, R_j^*\rangle j \in I, \langle Q_n\rangle n \in w\rangle$ be a third order world system for L, ϑ an assignment in β, and let $j \in I$. Then

1. for all $n \in w$, for all n-place predicate variables or predicate constants ρ of L, and for all terms $\eta_1, \ldots, \eta_n$ in L, the assignment ϑ satisfies $\rho\eta_1, \ldots, \eta_n$ in β at j iff $\langle \text{ext}[\eta_1, \beta, j, \vartheta], \ldots, \text{ext}[\eta_n, \beta, j, \vartheta]\rangle \in \text{ext}[\rho, \beta, j, \vartheta]$; for all $n \in w$, for all n-place second-order predicate variables or second order predicate constants Λ and for all $\sigma_1, \ldots, \sigma_n$ where for each $j \leq n$, σ_j is either a predicate constant or predicate variable, assignment ϑ satisfies $\Lambda\sigma_1, \ldots, \sigma_n$ in β at j iff $\langle \text{int}[\sigma_1, \beta, \vartheta], \ldots, \text{int}[\sigma_n, \beta, \vartheta] \in \text{ext}[\Lambda, \beta, j, \vartheta]$; ϑ satisfies $I^2\eta_1\eta_2$ in β at j iff $\text{ext}[\eta_1, \beta, j, \vartheta] = \text{ext}[\eta_2, \beta, j, \vartheta]$.

[13]if $i \neq j$, then there is no requirement in Cocchiarella's system that for an individual constant $\upsilon, \upsilon_{\Omega j} = \upsilon_{\Omega i}$

[14]Enderton (1972: p 27,99)

2. If φ and ψ are formulas of L, then ϑ satisfies $\varphi \rightarrow \psi$ in β at j iff either ϑ does not satisfy φ in β at j or ϑ does satisfy ψ in β at j ; ϑ satisfies $\neg\varphi$ in β at j iff ϑ does not satisfy ψ in β at j.

3. Satisfiability of formulas with modal operators: if φ is a formula of L, then ϑ satisfies $\Box\varphi$ in β at j iff for all $k \in I$ ϑ satisfies φ in β at k; if φ is a formula of L, then ϑ satisfies $\Diamond\varphi$ in β at j iff for some $k \in I$ ϑ satisfies φ in β at k.

4. if φ is an L-formula, and α is an individual variable, then ϑ satisfies $\forall\alpha\varphi$, (where $\forall$ is the possibilist quantifier), in β at j iff for all $x \in \Pi(\Omega_j)$, (the possible objects in Ω_j), $\vartheta[\alpha \backslash x]$ (by convention this means the assignment which is exactly like ϑ except for assigning x to α) satisfies φ in β at j.

   i ϑ satisfies $\forall^e\alpha\varphi$, (where $\forall^e$ is the actualist quantifier), in β at j iff for all $y \in \Lambda(\Omega_j)$, (the objects existing in Ω_j), $\vartheta[\alpha \backslash y]$ satisfies φ in β at j.

5. if φ is an L-formula and π is a n-place predicate variable, then ϑ satisfies $\forall\pi\varphi$ in β at j iff for all $X \in F_n, \vartheta[\pi/X]$ satisfies φ in β at j, and

   ii ϑ satisfies $\forall^e\pi\varphi$ in β at j iff for all $Y \in E_n, \vartheta[\pi/Y]$ satisfies φ in β at j.

6. ϑ satisfies $\Delta(\pi)$ at j iff $\vartheta(\pi) \in \vartheta(\Delta)_j$.

7. ϑ satisfies $\forall\Delta\varphi$ at j in β (Δ is an n-ary second-order predicate variable) iff for all X in $Q_n, \vartheta[\Delta/X]$ satisfies φ at j in β.

8. ϑ satisfies $\lambda\alpha_1, \ldots, \alpha_n\varphi\eta_1, \ldots, \eta_n$ at j in β iff ϑ satisfies $\varphi[\alpha_1/\eta_1, \ldots \alpha_n/\eta_n]$ at j in β.

9. If Λ is an n-place second-order predicate variable or n-place second-order predicate variable and $\sigma_1, \ldots, \sigma_n$ are predicate expressions and for any m, if for some formula φ with free variables $\alpha_1, \ldots, \alpha_k, \sigma_m$ is a predicate constant obtained via the abstraction operator from φ, i.e. σ_m is $\lambda\alpha_1, \ldots, \alpha_k\varphi$, then ϑ satisfies $\Lambda\sigma_1, \ldots, \sigma_n$ in β at j iff ϑ satisfies $\exists\pi^k[\pi^k = \lambda\alpha_1, \ldots, \alpha_k\varphi \wedge \Lambda\sigma_1, \ldots, \sigma_{m-1}, \pi^k, \sigma_{m+1}, \ldots, \sigma_n]$ in β at j.

Definition 2.9. A formula φ is said to be true in a third-order world system β for L at index j if φ is satisfied in β at j by every assignment in β.

Definition 2.10. A formula φ is valid in β if it is true in β at every index.

Definition 2.11. Normal Third-Order World System Let β- model $\langle\langle\Omega j\rangle j \in I, \langle F_n\rangle n \in w, \langle E_n\rangle n \in w, \langle\cup F_n, R_j^*\rangle j \in I, \langle Q_n\rangle n \in w\rangle$ be a third-order world system for L. Then β is normal if we have that:

$$\exists\pi\Box\forall\alpha_1, \ldots \forall\alpha_n[\pi\alpha_1, \ldots, \alpha_n \leftrightarrow \varphi(\alpha_1, \ldots, \alpha_n)]$$

and

$$\exists^e\pi\Box\forall\alpha_1, \ldots \forall\alpha_n[\pi\alpha_1, \ldots, \alpha_n \leftrightarrow [\varphi(\alpha_1, \ldots, \alpha_n) \wedge \exists^e\sigma\sigma\alpha_1, \ldots, \alpha_n]]$$

are valid in β where φ is any formula with free variables $\alpha_1, \ldots, \alpha_n$, π is a n-place predicate variable not occurring in φ, σ is an n-place predicate variable, and we require that $\exists\Delta\Box\forall\pi_1, \ldots, \forall\pi_n[\Delta\pi_1, \ldots, \pi_n \leftrightarrow \varphi(\pi_1, \ldots, \pi_n)]$ is also valid in β where φ is any formula with $\pi_1, \ldots, \pi_n$ free and Δ is an n-place secondary predicate variable not occurring in φ.

Definition 2.12. Let φ be any formula whose free variables are individual variables. Then β is normal for φ if we have that:

$$\exists\pi\Box\forall\alpha_1, \ldots, \forall\alpha_n[\pi\alpha_1, \ldots, \alpha_n \leftrightarrow \varphi(\alpha_1, \ldots, \alpha_n)]$$

and

$$\exists^e\pi\Box\forall\alpha_1, \ldots, \forall\alpha_n[\pi\alpha_1, \ldots, \alpha_n \leftrightarrow [\varphi(\alpha_1, \ldots, \alpha_n) \wedge \exists^e\sigma\sigma\alpha_1, \ldots, \alpha_n]]$$

are valid in β where φ has as its free variables $\alpha_1, \ldots, \alpha_n$; π is a n-place predicate variable not occurring in φ, and σ is an n-place predicate variable.

Lemma 2.13. *Semantical Lemma: If φ is an L-formula, β a third order world system for L, j a reference point of β, and s an assignment, and β is normal for all $\psi \in C(\varphi)$*[15] *then:*

1. *if ξ is an individual constant of L and s satisfies $\exists\alpha\Box\alpha = \xi$ in β at j, then s satisfies $\varphi[\alpha/\xi]$ in β at j iff $s[\alpha/ext(\xi, \beta, j, s)]$ satisfies φ in β at j.*

2. *If ρ is a predicate constant of L of the same arity as σ, then s satisfies $\varphi[\sigma/\rho]$ in β at j iff $s[\sigma/int(\rho, \beta)]$ satisfies φ in β at j. If ρ is a predicate variable of L of the same arity as σ, then s satisfies $\varphi[\sigma/\rho]$ in β at j iff $s[\sigma/int(\rho, \beta, s)]$ satisfies φ in β at j.*

[15] $C(\varphi)$ is a recursively defined set containing instances of predicate constants defined via property abstraction within φ. For details see Goldman (2000: 67-68)

3. *If* Λ *is a secondary predicate constant of* L *of the same arity as* Δ *, then* s *satisfies* $\varphi[\Delta/\Lambda]$ *in* β *at* j *iff* $s[\Delta/int(\Lambda,\beta)]$ *satisfies* φ *in* β *at* j*. If* Λ *is a secondary predicate variable of* L *of the same arity as* Δ *, then* s *satisfies* $\varphi[\Delta/\Lambda]$ *in* β *at* j *iff* $s[\Delta/int(\Lambda,\beta,s)]$ *satisfies* φ *in* β *at* j*.*

(The proof is a straightforward induction on the complexity of formulas.)

AXIOMS: The axioms that provide the Completeness result consist of all the generalizations of the following forms.

Sentential Axioms

1. $\varphi \to (\psi \to \varphi)$
2. $[\varphi \to (\psi \to \chi)] \to [(\varphi \to \psi) \to (\varphi \to \chi)]$
3. $(\sim\varphi \to \sim\psi) \to (\psi \to \varphi)$

Modal Axioms

4. $\Box\varphi \to \varphi$
5. $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$
6. $\Diamond\varphi \to \Box\Diamond\varphi$

Quantificational Axioms for possibilia and attributes

7. $\forall\alpha(\varphi \to \psi) \to (\forall\alpha\varphi \to \forall\alpha\psi)$ where α is an individual variable and $\forall\pi(\varphi \to \psi) \to (\forall\pi\varphi \to \forall\pi\psi)$ where π is a predicate variable.
8. $\varphi \to \forall\alpha\varphi, \varphi \to \forall\pi\varphi$, where α and π respectively are individual and predicate variables not occurring free in φ.
9. $\exists\pi\pi = \lambda\alpha_1, \alpha_2, \ldots, \alpha_n\varphi$ for n-place predicate variable π not occurring in φ, where the free variables in φ are $\alpha_1, \alpha_2, \ldots, \alpha_n$
10. $\exists\alpha I^2\alpha\xi$ where ξ is a term and α does not occur in ξ
11. $I^2\xi\eta \to (\varphi \to \psi)$, where φ is an atomic formula$_1$ and η is a term in it, and ψ is obtained from φ by replacing one or more occurrences of η by an occurrence of ξ.
12. $\Diamond I^2\alpha\beta \to \Box I^2\alpha\beta$

Existence Axioms

13. $\forall^e\pi(\varphi \to \psi) \to (\forall^e\pi\varphi \to \forall^e\pi\psi)$

14. $\forall \pi \varphi \rightarrow \forall^e \pi \varphi$

15. $\forall^e \pi \Box \exists^e \sigma \Box \forall \alpha_1, \ldots, \forall \alpha_n [\sigma \alpha_1, \ldots, \alpha_n \leftrightarrow (\pi \beta_1, \ldots, \beta_k \wedge \varphi(\beta_1, \ldots, \beta_k))]$ where $\{\alpha_1, \ldots, \alpha_n\} \subseteq \{\beta_1, \ldots, \beta_k\}$ and π and σ are distinct predicate variables not occurring in φ.

16. $\exists^e \pi \pi = \lambda \alpha_1, \ldots, \alpha_n \exists^e \sigma \sigma \alpha_1, \ldots, \alpha_n$

17. $\forall^e \alpha_1, \ldots, \forall^e \alpha_n \varphi \leftrightarrow \forall \alpha_1, \ldots, \forall \alpha_n [\exists^e \pi \pi \alpha_1, \ldots, \alpha_n \rightarrow \varphi]$

Additional Equality Axiom

18. $\Box I^2 \xi \xi$ where ξ is a term

Third Order Axioms

19. $\forall \Delta(\varphi \rightarrow \psi) \rightarrow (\forall \Delta \varphi \rightarrow \forall \Delta \psi)$ (needed for Generalization Theorem).

20. $\varphi \rightarrow \forall \Delta \varphi$, where Δ is a second-order predicate variable not occurring free in φ (also needed for Generalization theorem).

21. $\exists \Delta \Box \forall \pi_1, \ldots, \forall \pi_n [\Delta \pi_1, \ldots, \pi_n \leftrightarrow \varphi(\pi_1, \ldots, \pi_n)]$ and the axiom is an axiom schema (needed for the overall Henkin construction in the desired completeness result).

    We can not use an analogue of (12) and rightly so, for given that in some worlds two predicates are co-extensional, it does not mean they are necessarily so. We do, however, need the following axioms.

22. $\rho = \sigma \rightarrow (\varphi \rightarrow \psi)$, where φ is a second-order atomic formula and ψ is obtained from φ by replacing one or more occurrences of ρ with σ.

23. $\forall \Delta \varphi(\Delta) \rightarrow \varphi(\Lambda)$, where Λ is an secondary predicate expression (either a second order predicate variable or second-order predicate constant) free for Δ in φ.

Axiom for the Abstraction Operator

24. $\Box(\varphi(\eta_1, \ldots, \eta_n) \leftrightarrow \lambda \alpha_1, \ldots, \alpha_n \varphi[\alpha_1/\eta_1, \ldots, \alpha_n/\eta_n])$

Additional S5 Axiom

25. $\Diamond\exists^e\alpha I^2\alpha\beta$[16]

Note: Axioms like 23, which essentially defines the universal quantifier, cannot be given when quantifying over properties or individuals, but restricted versions of it in these cases can be derived. The problem comes in with terms, i.e. with function symbols of individuals. Something like $\sigma(\alpha)$ is not a rigid designator and can vary from world to world, so we cannot necessarily get $\forall\alpha\varphi(\alpha) \to \varphi(\xi)$ when ξ is a complex term and φ is not modal free. For instance, $\forall\alpha\forall\beta[\alpha = \beta \to \Box\alpha = \beta]$, but it is not necessarily the case that $[\delta(\alpha) = \delta(\beta) \to \Box\delta(\alpha) = \delta(\beta)]$. This is not a problem at the second-order predicate level.

The Inference Rules are Modus ponens, Possibility Interchange (an occurrence of $\Diamond\varphi$ can be replaced by $\neg\Box\neg\varphi$ and vice versa), Necessitation, and that an occurrence of $\varphi \wedge \psi$ can be replaced by $\neg(\varphi \to \neg\psi); \varphi \vee \psi$ by $\neg\varphi \to \psi; \varphi \leftrightarrow \psi$ by $(\varphi \to \psi) \wedge (\psi \to \varphi)$; and vice versa.

Derivations: A derivation of φ from Γ is a sequence $\psi_1, \ldots, \psi_n$ such that $\psi_n = \varphi$ and for each $j \leq n$ either

i ψ_j is in $\Gamma \cup E$ (where E is the set of axioms)

ii for some k and m less than j, ψ_j is obtained from ψ_k and ψ_m by modus ponens

iii for some k less than j, ψ_j is obtained from ψ_k by necessitation

iv for some k less than j, ψ_j is obtained from ψ_k by possibility interchange

v for some k less than j, ψ_j is obtained from ψ_k by an above truth functional rule

We write this by $\Gamma \vdash \varphi$. If Γ is the null set, and φ is derivable from the set of axioms alone, we write $\vdash \varphi$.

Soundness[17]: : Let Γ be a set of L-formulas and Φ be a single L-formula. Then if Φ is derivable from Γ, Φ is satisfied by every normal β - model with assignment ϑ that satisfies every member of Γ [18]. A fortiori, if Φ is a logical

[16] This axiom represents a shift from Cocchiarella's version and is more in keeping with the philosophical intuition of S5 that every possible object must be in at least one possible world.

[17] See Goldman (2000: p 77-79)

[18] Cocchiarella's original system does not have equality as a primitive and rather defines $\eta = \xi$ as $\forall\rho(\rho(\eta) \to \rho(\xi))$. Cocchiarella's version can be made sound without equality as a primitive by defining identity in terms of indiscernibility ($\eta = \xi$as $\forall\rho(\rho(\eta) \leftrightarrow \rho(\xi))$ and by employing identity-standard models. This is important in particular for his Axiom 11, $\forall\pi(\pi\eta \to \pi\zeta) \to (\varphi \to \psi)$, where π is a unary predicate variable, and ψ is an atomic

theorem (i.e., Φ is derivable from the axioms alone) then Φ is valid with respect to normal β - models. The proof of this is straightforward. One checks that each axiom holds in any normal β - model, which can be seen from the Soundness in third-order logic with regard to faithful models, and in S5 modal logic and that the definitional axiom of the property abstraction operator hold from the way satisfaction is defined, and then one checks the inference rules, and modus ponens, possibility interchange, and necessitation clearly preserve soundness (for a nice discussion of soundness with respect to faithful models, see Shapiro[19]).

COMPLETENESS THEOREM FOR THIRD-ORDER MODAL LOGIC

If Γ is a consistent set of formulas of the countable language L, then there exists a normal third order world system Ξ (with countably many possibilia and countably many attributes and second-order attributes) and assignment ϑ s.t. for some reference point j of Ξ, every member of Γ is satisfiable by ϑ in Ξ at j.

This constitutes a weak completeness result for uncountable languages in itself. A strong result also follows from it and the fact that given , a consistent set of formulas of an uncountable language, we could take the set of all finite subsets of Γ (call it J) and for j in J, given U_j, a model for j, we know there is an ultrafilter D over J such that the ultraproduct is a model of Γ (for each φ in Γ, look at the set of all j, s.t. φ belongs to j. Call it φ^*. Let $E = \{\varphi^* : \varphi \text{ in } \Gamma\}$ then E has the finite intersection property and can be extended to an ultrafilter D over J in the standard way.

Lemma 2.14. *Let E be any subset of $S(J)$ and let D be the filter generated by E then D is a proper filter iff E has the finite intersection property*

Lemma 2.15. *D is an ultrafilter over J iff D is a maximal proper filter*

Theorem 2.16. *If E is any subset of $S(J)$ and E has the finite intersection property then there exists an ultrafilter D over J which contains E (follows from Lemma 2.14 and 2.15 and Zorn's Lemma)*

The fact that the ultraproduct will be a model of Γ follows from an easy application of Los theorem. $\{j \in J : U_j \models \varphi\}$ extends φ^* and hence belongs to D so $\Pi_D U_j \models \varphi$ for each φ in Γ.

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formula and η is a term in it, and φ is the formula obtained from ψ by replacing an occurrence of η with ζ , i.e., $\psi[\eta/\zeta]$) and his Axiom 12 ($\Diamond\forall\pi(\pi\alpha \rightarrow \pi\beta) \rightarrow \Box\forall\pi(\pi\alpha \rightarrow \pi\beta)$ where π is a unary predicate variable)

[19]Shapiro (1991: p 71-89)

The proof of Completeness was originally presented in Gödel's Ontological Argument (2000)[20] along the basic ideas of the Henkin proof of Completeness in First-Order Logic and an edited version of the Cocchiarella proof for Completeness in Second Order Modal Logic with adaptations to a Third-Order Modal Logic system, the revised faithful models, and the inclusion of a property abstraction operator. A somewhat similar account is later found in Fitting's Types, Tableaus, and Gödel's God.[21]

3 Philosophical Assumptions

In the previous section I presented a pure semantics for the third order modal logic employed by Gödel. In this context, the ontological argument is a mathematical one, and the only commitment is to set theory or a fragment of set theory. However, the argument is not meant to be merely mathematical, but rather an actual argument for the existence of God. A positive property is not merely some mathematical aspect of a β-model $\langle\langle U_j\rangle j \in I, \langle F_n\rangle n \in w, \langle E_n\rangle n \in w, \langle \cup F_n, R^*_j\rangle j \in I, \langle Q_n\rangle n \in w\rangle$ (i.e., an element of Q_n for some n), but something with the intended meaning of "moral aesthetic". The conception of God also brings with it other intended meanings for properties, such as "omniscience", "moral rectitude", "being the cause of the world", or "transcendent to the world". The pure semantics does not tell us what the modal operators are supposed to mean. Thus, an applied semantics is essential. Yet, the applied semantics must do more than explain the modal operators. It must also relate the formal representations in the argument to the meanings of properties that Gödel and others ascribe to God such that the ontological argument becomes an argument that can be construed as an argument for the existence of God. Additionally, given the use of the property abstraction operator employed by Gödel, a host of properties involving de re assertions[22] are allowed. Thus, the question arises: What are the philosophical assumptions implicit in Gödel's use of third order modal logic?

I see six major philosophical assumptions that are highly problematic for Gödel's argument.

[20] Randolph Rubens Goldman, Gödel's Ontological Argument (2000), Dissertation, University of California at Berkeley

[21] Melvin Fitting, *Types, Tableaus, and Gödel's God*, Kluwer (2002)

[22] A modal formula φ is de re if there is a formula in φ that consists of a modal operator followed by a subformula that contains a free instance of an individual variable or an individual constant. For example, $\exists x \Box Rx$ is de re because within Rx x is free and x is only bound by the existential quantifier within φ outside the scope of the modal operator, $\Box$.

I The existence of *possible worlds*

II The existence of *merely possible objects* (Actualism v. Possiblism)

III Bivalence holds with respect to *possible objects*, i.e. assuming $[\varphi(z) \vee \neg\varphi(z)]$ for all possible entities z for non-modal φ

IV De re assertions make sense for *possible objects*

V *Possible objects* have necessary properties or not, i.e., $\Box\varphi(x) \vee \sim \Box\varphi(x)$ for all possible objects.

VI Intrinsic and Extrinsic Properties of possible objects, including those defined with respect to merely possible objects, have second order properties or not and have necessary second order properties or not. Internal, external, and extrinsic relations between possible objects have second order properties or not and have necessary second order properties or not.

V is an instance of IV so I will discuss them together. VI is probably the most problematic for any attempt to obtain to satisfactory applied semantics for Gödel's Ontological Argument.

3.1 Possible Worlds

A proponent of modal realism (the doctrine that there are possible worlds) contends that the interpretations of the set-theoretic structures as possible worlds characterize the real truth conditions for modality and that the possible worlds themselves are genuine features of modal reality. The modal operators can then be conceived of in terms of quantifications over these possible worlds. $\Box\varphi$ is true when φ is true in all possible worlds, and $\Diamond\varphi$ is true when φ is true in some possible world. In order to formulate an applied semantics for modal logic, various philosophers have put forth theories of what a possible world is.

Alvin Plantinga has argued that there is an intended applied semantics that he refers to as the Canonical Conception.[23] Under the Canonical conception, the $\langle U_j \rangle j \in I$ in the β-model would represent possible worlds. Given that $\Omega = \langle A, B, R, \rangle$ represents one of those possible worlds, B would represent the set of possible objects, which is the same set in all possible worlds

It is important to stress the difference between the pure and applied semantics for modal logic, and the difference between "a possible world" and

[23] Plantinga (1979:254)

the set-theoretic structure which is intended to represent a possible world. In the pure semantics, there are only the set-theoretic structures themselves. The pure semantics can investigate logical validity in the modal language, but it does not supply the truth conditions for the modal concepts we employ in our reasoning about modality. The possible worlds of modal realists that supply such truth conditions range from the concrete worlds of David Lewis to the more abstract entities postulated by Alvin Plantinga

In *Counterfactuals*, David Lewis writes:

> I believe that there are possible worlds other than the one we happen to inhabit. If an argument is wanted, it is this. It is uncontroversially true that things might have been otherwise than they are. I believe and so do you, that things might have different in countless ways. Ordinary language permits the paraphrase: there are many ways things could have been besides the way they actually are. On the face of it, this sentence is an existential quantifier. It says that there exist many entities of a certain description, to wit ' ways things could have been'. I believe that things could have different in countless ways; I believe permissible paraphrases of what I believe; taking the paraphrase at its face value, I therefore believe in the existence of entities that might be called 'ways things could have been': I prefer to call them 'possible worlds'. [24]

David Lewis' paraphrase does not seem so innocent. While it might be correct to exchange, 'Things could have been different in countless ways', with 'There are countless ways in which things could have been different', it seems that the paraphrase 'There are many ways things could have been besides the way they actually are' is a subtle distortion. In other words, 'ways things could have been' is different from 'ways in which things could have been different'. What allows David Lewis to draw the conclusion that there really are entities called 'ways things could have been'? If this seems suspicious, it pales in comparison to the leap taken when he assumes that 'ways things could have been' are really spatio-temporally distinct concrete universes, as he goes on to state in *On the Plurality of Worlds*:

> There are countless other worlds, other very inclusive things. Our world consists of us and all our surroundings, however remote in time and space; just as it is one big thing having lesser things as parts, so likewise do other worlds have lesser other worldly

[24] Lewis (1973)

> things as parts. The worlds are something like remote planets, and they are not remote. Neither are they nearby. They are not at any spatial distance whatever from here. They are not far in the past or future, nor for that matter near; they are not at any temporal distance whatever from now. They are isolated: There are no spatiotemporal relations at all between things that belong to different worlds. [25]

What justifies this leap? Lewis answers this himself:

> Why believe in a plurality of worlds? - Because the hypothesis is serviceable, and that is a reason to think it is true. [26]

How can mere serviceability support the enormous ontological cost of such worlds? I do not find any of David Lewis' arguments satisfactory.[27] However, it is interesting that modern String theorists in physics have postulated the existence of additional dimensions and a multiverse[28] of which our universe is just one of many. Nonetheless even such universes would not explicate modality because we would still have modal beliefs that the multiverse itself could have been different.

Examples of abstract possible worlds include Alvin Plantinga's maximal states of possible affairs and *lingusitic ersatzism*. *Lingusitic ersatzism* is the view that possible worlds are maximally consistent sets of sentences from some natural language, such as English. Linguistic ersatzism faces two significant challenges. It lacks the power of expressibility necessary to represent all the possibilities. [29] David Lewis has argued against linguistic ersatzism by showing that, given a Euclidean space-time with Democritean physics, the world can be completely characterized by specifying which space-time points have matter and which do not. The cardinality of space-time points in Euclidean space is the cardinality of the continuum. The cardinality of possible arrangements of matter is equal to the cardinality of subsets of the set of points, which is the size of the power set of the continuum, but sentences are finite strings in a countable language and hence countable. Thus the cardinality of all sets of sentences is that of the continuum. Hence, the cardinality of possible worlds is at most the continuum, and so there are not enough linguistic possible worlds. Another problem confronting linguistic

[25] Lewis (1986: p2)
[26] Lewis (1986: p3)
[27] for an excellent critique of David Lewis' modal realism, see Chapter 3 in Chihara (1998: p. 76- 141)
[28] Randall (2005: 334-351)
[29] Lewis: (1986: p 143)

ersatzism is that possible worlds described in terms of maximally consistent sets of sentences suffer from a kind of circularity. These possible worlds are defined in terms of consistency, but, because the sentences of the language employed already have meanings (and are not a pure formal language for which syntactic consistency can easily be defined), consistency in this case could only be understood in terms of the possible worlds themselves.

Robert Adams believes possible worlds are maximal consistent sets of propositions. However, such a view leads to a violation of Cantor's theorem. If such a maximal consistent set of propositions existed, then the union of it and the set of all the negations of members of it would be the set of all propositions. Given a set of all propositions, its power set exists. However, for each element x of the power set there is associated in a one to one manner the proposition that *I believe some member of x to be true*. Hence, a violation of Cantor's theorem occurs.[30] Plantinga's version also leads to a violation of Cantor's theorem. Another difficulty is that Plantinga's maximal states of affairs are supposed to represent ways in which the world might have been, and how such a representation would take place is controversial. [31]

All versions of possible worlds seem to suffer from various deficiencies. For an excellent and more complete discussion see Chihara. [32]

3.2 Possibilism

Possibilism is the doctrine that there are merely possible objects. There are several philosophical issues associated with possibilism. Is an ontological commitment to possibilia justifiable? Can possibilia have properties? Another involves the notion of transworld identity. How can a merely possible object be identified from one world to another? Meinong believed that there were merely possible objects and even impossible objects, such as a round square.[33]Meinong only refers to these merely possibly objects through definite descriptions like, for example, 'the golden mountain.'[34] Belief in the existence of such objects incurs ontological cost. Furthermore, if there are such objects, then we should expect there to be non-qualitative properties possessed by one merely possible object as opposed to another (i.e., essences) but if such merely possible objects are identified with definite descriptions, then it seems unclear how non-qualitative properties could be distinguished. There could have been two golden mountains that were qualitatively indis-

[30]Jubien (1988: p 306-307)
[31]for elaboration of the difficulties Plantinga faces see Chihara (1998: p 101 - 141)
[32]Chihara (1998: 76-141)
[33]Chisholm (1960: 82-84)
[34]Chisholm (1960:9)

cernible, 'the golden mountain that is qualitatively indistinguishable from another', and 'the other qualitatively indistinguishable golden mountain'. What would it mean for a possible object to have the property of being identical with the first and not the second? Moreover, what is there to identify the same possibilia over different worlds? For example, if we are to believe that there is a merely possible object associated with 'the golden mountain' and that this possible object might have been higher (in another world), then how can the possible object 'the golden mountain' in the world in which it is higher be identified with the possible object 'the golden mountain' in the world in which it is not?

Anderson accepts Possiblism in the context of the Ontological Argument, because he endorses the Cocchiarella system for his applied semantics. [35] Plantinga does not. Plantinga, who is both an Actualist (he does not believe in merely possible objects) and Serious Actualist, (he does not believe that something can have a property in a world in which it does not exist), has tried to formulate a definition of essence consistent with his philosophical views. According to Plantinga,[36] an essence is a property F such that there is a world w and an object x such that:

1. x has F in every world in which it exists
2. there is no $w\sharp$ and no y $y \neq x$ such that y has F in $w\sharp$

Plantinga believes essences are necessary entities and contends that in defining essences he has avoided a commitment to merely possible objects. Jager's applied semantics [37] was constructed in conformance with Plantiga's views on Actualism and Serious Actualism in that the extension of n-ary predicates would consist only of those subsets of the nth Cartesian products of elements that existed at the world. If Anderson tried to adopt such a Serious Actualist semantics it would necessitate that all n-ary attributes in a world system be e-attributes. In Jager's system the elements that exist at a world w are the set of all essences exemplified in w. Thus the system is purported to also be Actualistic according to Plantinga because he believes essences actually exist.[38] However, there is a problem for Plantinga. Chihara pointed out the variable x occurs within the scope of a world variable $w\sharp$ which itself occurs within the scope of w.[39] Plantinga must demonstrate how there can be a possible individual x of world w that has some property

[35] Anderson (1990: 300)
[36] Plantinga (1974: 72)
[37] Jager (1982)
[38] Chihara (1998: 116)
[39] Chihara (1998:118)

F in all worlds in which it exists and which is such that in any other world $w\sharp$ has another property, namely the property of it being the case that if there is any object y distinct from it in $w\sharp$, then y lacks property F, and still avoid a commitment to x being a merely possible individual. Anderson, on the other hand, makes no such attempt, and the ontological argument he gives is straightforwardly committed to possibilia.

3.3 Bivalence with respect to possible objects

Even though we can make sense of a statement such as

There could have been a unicorn named Kate which might have been standing outside or not standing outside,

how can we understand a statement about the merely possible object, Kate, such as

Kate is standing outside or Kate is not standing outside

to be true in a world where no unicorn exists? What does truth at a world mean for a sentence referring to an object that does not exist at the world? A claim of this sort could neither be consistent with Actualism nor with Serious Actualism. Hillary Putnam declares that there is a conceptual relativity which must be examined with regard to the abstract notion of an object [40], let alone a merely possible object. Is there really a logical or metaphysical notion of 'object' independent of the language? Putnam contends that the answer to a question such as 'How many objects are there in the room?' is not independent of the framework of the language. If the notion of 'object' may be subject to skepticism, how much more vulnerable to skepticism is the notion of 'merely possible object'?

3.4 *De re* assertions make sense for *possible objects* including

3.5 *Possible objects* have necessary properties or not, i.e., $\Box\varphi(x) \vee \neg\Box\varphi(x)$ for all possible objects.

What can be made of *de re* assertions predicating a modal property with respect to a merely possible object? For example

1. *Kate is necessarily hungry.*

[40] Putnam (1988):112-113

2. *Kate possibly injures her leg.*

3. *Kate is necessarily hungry or Kate is not necessarily hungry.*

Can possible objects even have necessary properties or not, i.e., $\Box\varphi(x) \vee \neg\Box\varphi(x)$ for all possible objects? Presupposition V is a feature of the pure semantics, as it is satisfiable in any third order structure, but it is controversial under a natural language interpretation, especially in the context of merely possible objects. Presupposition IV for merely possible objects is an even greater assumption than II, and is certainly inconsistent with both Actualism and Serious Actualism.

Anderson's definition of essence: $\varphi \text{Ess} x = \text{df}\forall\gamma[(\Box\gamma(x)) \leftrightarrow (\Box\forall x[\varphi(x) \rightarrow \gamma(x)])]$

(An essence of x is a property φ that is such that for every property γ, x has γ necessarily if and only if γ is entailed by φ.)The definition becomes controversial when we see that it is not only essential to be able to make sense of the quantification over possible objects within $\Box\forall x[\varphi(x) \rightarrow \gamma(x)]$, but to also make sense of quantification over all properties including those properties defined through property abstraction with respect to merely possible objects outside the scope of the modal operators in $\forall\gamma[(\Box\gamma(x)) \leftrightarrow (\Box\forall x[\varphi(x) \rightarrow \gamma(x)])]$. Theorem 1.10, $G(x) \rightarrow G$ Ess x, inherits the same concerns.

The commitment of the Ontological Argument to possibilism begets the vexing philosophical issues discussed earlier. Does it make sense with regard to Axiom 1.5 to talk about entailment when the quantification over merely possible objects is permitted? Will not the truth values for some possible objects be genuinely non-existent? If Axiom 1.5 is blocked at this point, the Ontological Argument collapses. Recall the cornerstone of the Gödel argument: $P(\varphi) \rightarrow [(\Box\forall x(\varphi(x) \rightarrow \gamma(x))) \rightarrow P(\gamma)]$ is responsible for deriving

Theorem1.9 $P(\varphi) \rightarrow \Diamond\exists x\varphi(x)$, and property abstraction is utilized in the proof of Theorem 1.9 and is an essential feature of Gödel's argument.

In *Worlds of Possibility*, Charles Chihara offers an anti-Realist account of the semantics of modal logic compatible with Actualism, and he provides a connecting theorem between the relativized notion of truth in a mathematical model and the absolute notion of truth under a natural language interpretation.[41]

Adhering to Chihara's definition of *NL-interpretation* as conforming to a model of modal logic[42] , for any NL-interpretation ϑ representing a third-order world system β- model, $\langle\langle Uj\rangle j \in I, \langle Fn\rangle n \in w, \langle En\rangle n \in w, \langle\cup Fn, Rj^*\rangle j \in I, \langle Qn\rangle n \in w\rangle$, it is likely, at least, to have bijections f_j from $\Lambda_j(\Omega_j)$ to the

[41]Chihara (1998: 190-259)
[42]Chihara (1998:229-239)

extension of $\vartheta(\Lambda)$ (the English predicate asigned by ϑ to delimit the domain of the quantifiers with respect to individuals) and (given second-order property Δ), g_j from $\cup E_n$ to $\vartheta(E)$ (the delimitation under ϑ of the domain of the quantifiers with respect to attributes) s.t. for every n-ary predicate σ and for every n-tuple in $\Lambda(\Omega_j)$, $\langle \lambda_1, \ldots, \lambda_n \rangle \in R_j(\sigma)$ iff $\langle f_j(\lambda_1), \ldots, f_j(\lambda_n) \rangle \in \text{ext}[\sigma/\vartheta]$, (the extension of σ under ϑ), and for every n-tuple in $\cup E_n$, $\langle \rho_1, \ldots, \rho_n \rangle \in R^*_j(\Delta)$ iff $\langle g_j(\rho_1), \ldots, g_j(\rho_n) \rangle \in \text{ext}[\Delta/\vartheta]$, and conformance of the NL-interpretation ϑ to a β- model

$$\langle \langle Uj \rangle j \in I, \langle Fn \rangle n \in w, \langle En \rangle n \in w, \langle \cup Fn, Rj^* \rangle j \in I, \langle Qn \rangle n \in w \rangle$$

would include that for every Ω_j in β, the world could have been such that there were f_j and g_j via which there was a representation of the world, and that the world could not have been such that for no Ω_j in β were there functions f_j and g_j via which there was a representation of the world.

Now then, one might be able to extrapolate Gödel's Ontological argument as an argument for God if the use of NL-interpretations included a bijection from objects in the actual world to the domain of some Ω_j in the β- model which succeeded in making "omniscient" and "good" well-defined over this domain and "positive" well-defined over the domain of properties. The problem is that in a β- model, it will be the case that either $\langle \lambda_1, \ldots, \lambda_n \rangle \in R_j(\sigma)$ or $\langle \lambda_1, \ldots, \lambda_n \rangle \notin R_j(\sigma)$, and that either $\langle \rho_1, \ldots, \rho_n \rangle \in R^*_j(\Delta)$ or $\langle \rho_1, \ldots, \rho_n \rangle \notin R^*_j(\Delta)$. For such properties as "goodness" and "omniscience", however, there will inevitably be statements that are not simply either true or false, but meaningless.

The goal of the anti-realist endeavor is to provide a connecting theorem between the satisfaction of a formula in a mathematical structure and the truth of a formula under an NL-interpretation which conforms to the structure. A connecting theorem would provide a link between two kinds of interpretations, and would, in effect, provide an answer to Davidson's problem, as it is related to third-order modal logic. In general, a *connecting theorem* states that a formula is satisfied by a β-model if and only if the NL-interpreted formula was in fact true under the NL-interpretation that conforms to the structure. Conformance itself would have to be defined in terms of a modal primitive, since such an account is not meant to be a reductive account of what modality is. Conformance would also need to defined in order to make clear how the β-model would represent the objects and qualities alluded to by the NL-interpretation. Furthermore, conformance would have to be defined within the constraints of the anti-realist endeavor. Apparently, an anti-realist faces many encumbrances against the development of an adequate notion of conformance with respect to the structures of third-order modal logic and against the construction of a connecting theorem.

Through the use of the property abstraction operator in Gödel's system, many new properties can be created. These properties must be given natural language interpretations. In Gödel's system, for every formula with free individual variables, there must be a property associated with that formula. For instance,

1. $[\neg\exists^e zz = x \wedge \exists y[\neg\exists^e zz = y \wedge \Box R^2 yx]]$
2. $[\neg\exists^e zz = x \wedge \Diamond R^1 x]$
3. $[\exists y[\neg\exists^e zz = y \wedge (\Diamond R^1 y \wedge y \neq x)] \wedge \Box R^1 x]$

are formulas with the free individual variable x. Hence, by Gödel's liberal employment of the property abstraction operator, we have the properties

1. $\lambda x[\neg\exists^e zz = x \wedge \exists y[\neg\exists^e zz = y \wedge \Box R^2 yx]]$
2. $\lambda x[\neg\exists^e zz = x \wedge \Diamond R^1 x]$.
3. $\lambda x[\exists y[\neg\exists^e zz = y \wedge (\Diamond R^1 y \wedge y \neq x)] \wedge \Box R^1 x]$.

In any given normal β-model, the sentences

1. $P\lambda x[\neg\exists^e zz = x \wedge \exists y[\neg\exists^e zz = y \wedge \Box R^2 yx]] \vee \neg P\lambda x[\neg\exists^e zz = x \wedge \exists y[\neg\exists^e zz = y \wedge \Box R^2 yx]]$
2. $P\lambda x[\neg\exists^e zz = x \wedge \Diamond R^1 x] \vee \neg P\lambda x[\neg\exists^e zz = x \wedge \Diamond R^1 x]$
3. $P\lambda x[\exists y[\neg\exists^e zz = y \wedge (\Diamond R^1 y \wedge y \neq x)] \wedge \Box R^1 x] \vee \neg P\lambda x[\exists y[\neg\exists^e zz = y \wedge (\Diamond R^1 y \wedge y \neq x)] \wedge \Box R^1 x]$

are valid. For example, let $R^2 yx$ be given the NL-interpretation 'y is transcendent to x', and let $R^1 x$ be given the NL interpretation 'x has moral rectitude'.

What are the actual properties that are represented by

1. $\lambda x[\neg\exists^e zz = x \wedge \exists y[\neg\exists^e z(z = y) \wedge \Box R^2 yx]]$
2. $\lambda x[\neg\exists^e zz = x \wedge \Diamond R^1 x]$
3. $\lambda x[\exists y[\neg\exists^z z = y \wedge (\Diamond R^1 y \wedge y \neq x)] \wedge \Box R^1 x]$?

In what way can an anti-realist and an Actualist coherently say that there is a property of being a merely possible object and having another merely possible object necessarily transcendent to it, or that there is a property of being a merely possible object which possibly has moral rectitude, or that there is a property of both necessarily having moral rectitude and being distinct from a merely possible object that possibly has moral rectitude?

3.6 Second Order Properties of Properties and Relations of Possible Objects

What does it mean for the properties in the paragraph above to be positive or not positive? Presupposition VI is the most challenging obstacle that would need to be addressed satisfactorily in order for anyone to show that Gödel's Ontological Argument is indeed an argument for the existence of God, and it is a problem that arises directly from the use of the property abstraction operator.

There could have been two other students of Socrates, one named Kate and the other, her twin, named Karen. These non-actual possible objects would have had properties if they had existed, and other objects would have possessed properties in relation to these objects. What are these properties and what distinguishes them? What would it mean for a property of Kate to have a certain second order property and another property of Karen to not have that second order property? What would it mean for an object to possess a property in relation to Kate and not to Karen? For instance, let $\varphi(\alpha)$ be

1. $\neg(\exists^e\beta\beta = \xi) \wedge (\exists\beta\Box\beta = \xi) \wedge \neg(\exists^e\beta\beta = \eta) \wedge (\exists\beta\Box\beta = \eta) \wedge \neg(\xi = \eta) \wedge (\Box R^2\alpha\xi) \wedge \neg(\Box R^2\alpha\eta)$, where ξ and η are individual constants.

   and let $\psi(\alpha)$ be

2. $\neg(\exists^e\beta\beta = \xi) \wedge (\exists\beta\Box\beta = \xi) \wedge \neg(\exists^e\beta\beta = \eta) \wedge (\exists\beta\Box\beta = \eta) \wedge \neg(\xi = \eta) \wedge \neg(\Box R^2\alpha\xi) \wedge (\Box R^2\alpha\eta)$.

By property abstraction, we would obtain $\lambda\alpha\psi(\alpha)$ and $\lambda\alpha\varphi(\alpha)$, and any β-model would satisfy $[\Box\Lambda\lambda\alpha\psi(\alpha)\vee\neg\Box\Lambda\lambda\alpha\psi(\alpha)]\wedge[\Box\Lambda\lambda\varphi(\alpha)\vee\neg\Box\Lambda\lambda\alpha\varphi(\alpha)]$ for any second order predicate constant Λ. This is saying that the property of being necessarily R^2- related to the merely possible object η and not necessarily R^2- related to the other merely possible object ξ is necessarily a Λ-property or not necessarily a Λ-property and that the property of being necessarily R^2- related to the merely possible object ξ and not necessarily R^2- related to the other merely possible object η is necessarily a Λ-property or not necessarily a Λ-property. But in what way is this meaningful in an applied semantics?

It seems that to even attempt a meaningful applied semantics, Gödel must be committed to merely possible objects, essential properties of possible objects and some form of modal realism, and thereby contracts the host of problems connected to such commitments. A distinguishing feature of the conformance worked out by Chihara (in the first order case) is that only a certain bijection from the domain of the structure to actual objects in the world need be constructed. However, the universes of β-models contain both attributes (functions from possible worlds to the extension of the property or relation in the world) and second order attributes (also functions from possible worlds to the extension of the second order property or second order relation in the world), and through the Gödelian property abstraction operator, a host of properties and relations can be formed which must be represented in any normal β-model. It is difficult to ascertain what these could correlate to in the actual world.

Another problem is that in order to construct a notion of conformance, the domains in which it makes sense to speak of the various natural language predicates anticipated in the Ontological argument must be restricted. It does not make sense to say of a particle in quantum mechanics that it is not morally good, although, perhaps, it might make sense to say of an electron that it is not transcendent to some particle. Furthermore, in order to obtain a connecting theorem, one must delimit the domain of the quantifiers to just those things that satisfy an English predicate assigned by the NL-interpretation. As the Fundamental Connecting Theorem by Charles Chihara [43] shows, satisfaction of a sentence in a model to which an NL-interpretation conforms is linked to the truth of the NL-interpreted sentence when the domain of the quantifiers has been so delimited. For instance, Chihara gives an example of what he refers to as an "NL proto-interpretation", where the domain of quantifiers is delimited by the predicate "is one of the pair, Bill Clinton or Hillary Clinton." $\neg\Diamond\exists x\exists y\exists z[x \neq y \ \wedge \ x \neq z \ \wedge \ y \neq z]$ would be satisfied by any S5 model that conforms to the NL- proto-interpretation (provided equality is included in the language), and so the connecting theorem would provide that it truly is the case that there could not be three distinct objects, if it is stipulated that the world is such that there are only Bill and Hillary Clinton, but it does not provide that it truly is the case that there could not be more than three objects. In the NL-interpretation for the Gödel Ontological Argument, the domain would certainly have to be, at the very least, restricted to sentient beings in order to make sense of such predicates as "is omniscient" and "is moral". Hence, the universe of properties would have to be restricted, and consequently the domains of second-order properties would have to be restricted. In the Gödel Ontological Argument, the claim is being made that God necessarily exists. However, when we examine the definitions

Def 1: $G^*(x) = \text{df}\forall\varphi[(\Box\varphi(x)) \leftrightarrow P(\varphi)]$

Def 2: $\varphi\text{Ess}^*x = \text{df}\forall\gamma[(\Box\gamma(x)) \leftrightarrow (\Box\forall x[\varphi(x) \rightarrow \gamma(x)])]$

[43] Chihara (1998: 252)

Def 3: $\text{NE}^*(x) = \text{df}\forall\varphi(\forall\text{Ess}^*x \rightarrow \Box\exists x\varphi(x))$ (Necessary Existence),

we discover that all three are defined in terms of universal quantifications over all properties. To satisfy the restrictions that would have to be placed on the universe of properties by the NL-interpretation, a lot of properties would have to be left out. Accordingly, even if it were possible to define an adequate notion of conformance in the case of third order modal logic, and even if it were possible that a β- model conforming to the NL interpretation for the Gödel Ontological Argument satisfied $\Box\exists xG^*x$ by way of $G^*(x) \rightarrow (G^*\text{Ess}x)$ and the use of the definition of NE* to get $[\ \text{NE}^*(x) \wedge G^*\text{Ess}^*x] \rightarrow \Box\exists xG^*(x)$, we could not know that it truly was the case that God necessarily existed. How could we know that we could truly capture the notion of essence in its intended sense, in which it is defined in terms of all properties and not just some properties?

Another problem is that the connecting theorem would only apply for certain suitable natural language predicates, because there might be certain NL-interpretations which conform to no models. Chihara cites the claim by Descartes in the *Meditations* that the soul is essentially indivisible as one such troublesome example of non-conformance with respect to first-order logic.

> In particular, think of how many souls there might have been? Is there any cardinal limit to what the totality of souls (in Descartes's sense of that term) might have been? At this point, one might begin to wonder whether any set theoretical structure could capture what we are trying to express. [44]

Chihara also points out that there may be no set theoretic structure that could conform to NL-interpretations which contains a degree of vagueness, unclarity, excessive size, or complexity. [45] The NL-interpretation required for the Gödel Ontological Argument may be one such candidate. Certainly such second order predicates as "being independent of the accidental structure of the world" or "being a pure attribution" or "positive in the moral aesthetic sense" contain a significant degree of vagueness. Furthermore, is there any limit to the size of what possible objects there might have been? We could argue that conformance would be something extremely difficult to obtain, given the complexity of the NL-interpretation required for the Gödel Ontological argument and the diverse and ambiguous predicates under consideration. Certainly the burden is on the proponent of the argument that to

[44] Chihara (1998: 271)
[45] Chihara (1998: 272)

show that conformance can achieved. Furthermore, the burden is on the proponent to establish that the formal third-order modal logic argument adapts itself to an ontological argument.

It is rather striking that Gödel worked on this argument in third order modal logic sporadically over a period of thirty years, and never once attempted to construct an applied semantics or even intimate how one can be achieved. Perhaps his ontological Platonism with regard to set theory influenced him to avoid this issue. Gödel believed that the objects of set theory exist independently of our constructions and that we have a perception of individual sets that can enable us to recognize the truth of the axioms of set theory with regard to these objects.[46] For Gödel the Continuum Hypothesis in set theory for example would have a definite truth value in the intended model of sets even though the statement has been proven to be independent of the axioms. However there is still is rather large chasm between a perception of sets and a perception of possible worlds and merely possible objects.

4 Conclusion

Thus, for all the reasons discussed, it appears improbable that Gödel could find a way to show that the abstract argument of pure third order modal logic that he presents actually translates into an argument for the existence of God, no matter which type of applied semantics (modal realism/anti-realism, possibilism/actualism) he wishes to adopt.

I will now address two main arguments that a proponent of the ontological argument might make. She could insist that the argument could be reformulated using ordinary English, in order to avoid all the problems connected to conformance in conjunction with a NL-interpretation of the formal argument or the pitfalls connected to both modal realism and possibilism. It would then be legitimate to ask what reformulation could capture the meaning of the Gödel Ontological argument if there is no ontologically innocent way to express it. Furthermore, how could such a reformulation lead to a proof of the conclusion in the third-order modal logic? Recall that the proof made use of both the problematic property abstraction and the preliminary result that $G^*(x) \to (G^*\text{Ess}x)$, and that Anderson's definition of essence carries with it the same problems the reformulation would be trying to avoid, namely a commitment to possible objects, their essential properties, and possible worlds. The reformulation into ordinary language would have to stray far enough away from the problematic assumptions involving

[46] Godel (1964: 262)

the third order modal logic that it would most likely lose the very appeal of Gödel's Ontological argument, which is its formal nature. An argument cast into such a reformulation approaches being more of a declaration that God exists based on that very assumption than what one might consider to be a genuine ontological argument.

A proponent of Gödel's Ontological Argument might also claim that she could both bypass any potentially troublesome commitments to possible worlds and possible objects and avoid the troubles in the construction of a connecting theorem by adopting the position that the use of third order modal logic and its formal semantics function merely as a heuristic device to illustrate a proof of God's existence from axioms that are, she claims, obviously true under the desired NL -interpretation. Such a proponent would have to explain why, if the system of third-order modal logic can be used so clearly and effectively to prove results that will be true under the desired NL-interpretation, it is not possible to provide a satisfactory notion of conformance and a connecting result. She would have to explain how she avoids the problem central to the property abstraction operator (as discussed above), which seems inevitable. By Axiom 1.5, the property associated with any valid formula with one individual free variable is positive, as it is entailed by any positive property (of which, by Axiom 1.8, necessary existence is one). For example, the property

$$\lambda x[\Box[\forall\Delta[(\Delta R^1 \wedge (\neg\exists^e zz = x \wedge \Diamond R^1 x)) \leftrightarrow (\Delta R^1 \wedge (\neg\exists^e zz = x \wedge \Diamond R^1 x))]]]$$

is positive. But what does it mean when R^1 is given the natural language interpretation of 'is good', for instance? Finally, the proponent would need to explain how she can construe Gödel's Ontological Argument as a proof of God's existence, given the vagueness of the notion of positive property itself. It seems, at least, that the burden should be on the proponent to provide some justification.

The ineffectiveness of Gödel's argument can in part be traced to Axiom 1.5. There is a logical aspect to positiveness distinct from the Leibnizian notion. Leibniz took the position that a positive property cannot be analyzed in terms of the negation of anything, and this strictly speaks to the internal nature of the property. Gödel says that if there is a logical implication involving a positive property and another property (the sort specified by Axiom 1.5) then that other property is also positive. Furthermore, while Leibniz restricted positive properties more or less to qualities which could be described in natural language without the use of a property abstraction operator, Gödel allows for a much broader range of properties, including those associated with propositional functions of a single variable. Since it is a much more formal argument, it must be evaluated through the formal semantics, and, for this to be done, it would be imperative that a bridge could be drawn from the formal semantics to the intended natural language meaning of the argument. To posit an assumption that God exists seems to impose less ontological cost than to assume the existence of merely possible objects, possible worlds, intrinsic properties of possible objects, and second order properties of intrinsic properties of possible objects.

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