REVIEW: Vagueness and Degrees of Truth

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Nicholas J. J. Smith *Vagueness and Degrees of Truth*,
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Vagueness is one of the most persistent and challenging topics in the intersection of philosophy and logic. At least five other noteworthy books on vagueness have been written by philosophers since 1991 [2, 6, 11, 12, 15]. A (necessarily incomplete) bibliography that has been compiled for the Arché project *Vagueness: its Nature and Logic* (2004-2006) of the University of St Andrews lists more than 350 articles and books on vagueness until 2005.[1] Many new and interesting contributions have appeared since. The book under review is much more than yet another addition to this prolific discourse. Nicholas Smith manages to tackle two different tasks that are potentially in tension. On the one hand, he provides a comprehensive, systematic and well written account of various approaches to vagueness that have been debated so far. On the other hand, Smith carefully explains and defends his own theory of vagueness, called *fuzzy plurivaluationism*. Given the complex and almost unsurmountably large amount of relevant literature and the fact that theories of vagueness based on fuzzy logic have almost universally been rejected by philosophers so far this is no simple feat.

In the comments below, I will largely follow the structure of book. If along the way I cannot resist to make side remarks or even take issue with some of

[1] http://www.st-andrews.ac.uk/~arche/projects/vagueness/bibliography.shtml.

the presented theses, then this should just indicate that Smith does not shy away from boldly stating and defending partly quite controversial positions.

Vagueness and Degrees of Truth opens with a brief problem description and an overview on the six main sections that are divided into three parts. Section 1 of Part I, *Foundations*, provides a toolkit that covers basic notions of formal semantics not just for classical logics, but also for the more general algebraic setting of fuzzy logic. Section 2 is called *The Space of Possible Theories of Vagueness*. It presents competing approaches to vagueness not in form of a historic overview or by simply listing different theories, but rather takes four central features of the semantic picture offered by classical logic as focal point, and describes seven alternative concepts of reasoning under vagueness as different types of revisions of the classical picture. The four classical features are:

(*1a*) bivalence,

(*1b*) totality of the interpretation function,

(*1c*) truth functionality,

(*2*) each discourse has a unique intended interpretation.

Epistemicism is presented as an account of vagueness that does not deny any of those principles, but rather seeks to explain vagueness as a particular type of ignorance. We do not know, where to draw the line between bold and non-bold men in a 'sorites series' of man starting with Yul Brynner and ending with Andy Garcia (to borrow an example from Stewart Shapiro), where any two adjacent men are hardly distinguishable with respect to their hair. More generally: we do not know where to draw bounds for the extensions of vague predicates. Of course, all accounts of vagueness face this problem, called the *location problem* by Smith. Epistemicists maintain that there *is* in fact a last bold man in our sorites series, even if we have no means to identify him definitively. Smith sides with the semantic realism implied by this view, but denies that vague predicates have to refer to a single bivalent model, as will get clearer below.

Denying (1a) alone leads to additional truth values. Smith discusses the options for three-valued logics systematically and identifies the so-called problem of higher-order vagueness as one of two major objections to three-valued approaches to vagueness. (The other objection focuses on truth-functionality and is discussed in a later section.) It is essential for Smith's view that higher-order vagueness actually refers to two separate problems: the already mentioned location problem, and what he calls the *jolt problem*, that arises if one does not allow for gradual transitions between clear cases where the vague predicate applies and clear counter-cases. Smith suggests that three-valued accounts fare better with respect to the location problem than epistemicism. The former theories respect, at least broadly, a link between meaning and use that the epistemicist is forced to deny: rejecting bivalence allows to hold up the princi-

ple that a vague sentence is true if most competent speakers would confidently classify it as true and is false if most competent speakers would confidently classify it as false. Once we accept 'truth values' other than (simply) true and false it seems natural to go beyond three. Smith is sympathetic to fuzzy logic—or rather: fuzzy logic*s*—where usually the whole real unit interval is taken as truth value set. Indeed, he is the only philosopher I know of who cites, among other relevant sources, Petr Hájek's monograph [5], that has become a central base for research in deductive (mathematical) fuzzy logic. This is of some significance: contemporary fuzzy logic is still often motivated by 'reasoning under vagueness'. However the outlook is very different from traditional philosophical accounts of vagueness, that mostly reject a linearly ordered continuum of truth values. The criticism directed to the classic fuzzy logic papers by Zadeh [16, 17] and Goguen [4] only partly matches the agenda of Hájek and his colleagues, who investigate a broad family of logics, where, e.g., different truth functions for conjunctions are seen as equally valid candidate models for different natural language phenomena. The emphasis is on the characterization of general principles of inference with a graded notion of truth. In Part II, discussed below, Smith defends the existence of degrees of truth much more explicitly than is usually done in fuzzy logic. In Part I, he just points out that the fuzzy semantic picture does not locate vagueness in the relationship between language and world, but rather in the world itself. He correspondingly speaks of *worldly vagueness*.

Allowing for partial models, i.e., denying (1b), leads to different version of truth gap theories that are analogous to three-valued logics. More interestingly *supervaluationism*, one of the most prominent theories of vagueness, entails the view that the truth values of compound sentences are not recursively determined by the truth values of simpler sentences; i.e. it denies (1c). Quite different views on vagueness have been presented in reference to supervaluation. Smith suggests that the term supervaluationism should be confined to the theory that takes a single partial interpretation as base point and identifies (simple) truth with (formal) truth at all permissible classical extensions at this base point. This indeed matches the view proposed in Kit Fine's classic paper [3]. Smith recognizes that there are variants of this proposal, where it is denied that a single partial base interpretation can be identified as the intended one. However, he insists on an essential difference—that 'has not been appreciated in the literature'—between a view where the reference is to a single intended partial model together with constraints on admissibility of potential (formal) precisifications, and a concept where vague language is viewed as inherently indeterminate and thus sentences have to be evaluated with respect to a whole variety of interpretations that are compatible with linguistic practice. The latter view is called *plurivaluationism* by Smith and, with respect to the above mentioned features of the classical semantic picture, is identified as denial of (2). As a consequence, Achille Varzi, one of the most outspoken and prolific defenders of 'supervaluationism', (see, e.g., [13, 14]) has to be re-

classified as plurivaluationist according to Smith.

There is a further important approach to vagueness, called *contextualism* that maintains that the intended interpretation(s) change over time or with context. Such shifts of contexts may occur quickly and instantaneously during a conversation, which is the key to a contextualist solution of the sorites paradox. This view, most prominently elaborated in Stewart Shapiro's [11], doesn't fit easily into Smith schema of theories arising vis-á-vis the classical features 1a, 1b, 1c, and 2. Smith argues that contextualism should not be seen as a theory of vagueness in its own right, but rather is compatible with all other mentioned theories. The diachronic semantic indeterminacy implied by contextualism is not seen as a feature particular to explaining vagueness, but rather as a more general phenomenon of language use. I am not sure that contextualists like Shapiro will or should accept this view. But this is not the place to defend contextualism as proper theory of vagueness. In any case, Smith seems happy to also accommodate context shifts into fuzzy plurivaluationism, his own preferred theory.

Smith only very briefly considers the suggestion that intuitionistic logic provides an appropriate formalization of reasoning under vagueness. This view is labelled 'asserting nothing' and characterized as a 'no semantics' approach that therefore is simply out of the range of theories discussed in the book. While this is a fair remark in reference to Hilary Putnam's [9] take on vagueness from the intuitionistic point of view, I think that more could be said here. While there are hardly any hard-core followers of Brouwer's intuitionism still around, who would indeed deny the role of formal semantics as taken for granted here, most contemporary logicians recognize the adequateness of intuitionistic logic for a wide range of application scenarios, also from a semantic point of view. In such contexts so-called Kripke models, in particular in the explicitly epistemic version introduced by Beth, are employed in a fashion that is perfectly analogous to applications of modal logics or fuzzy logics, for that matter. The idea that such models provide a key to formal reasoning with vague concepts has very recently been taken up by Matthias Baaz, who refers to earlier work of Dana Scott.

Part II of the book is laconically titled *Vagueness*. Some might wonder why a definition of vagueness is only attempted after the detailed and systematic presentation of alternative theories of vagueness. But in fact this order of presentation appropiately acknowledges that any definition that goes beyond introductory hints on borderline cases, blurred boundaries, and sorites paradoxes cannot be neutral with respect to alternative theories of vagueness. Smith explicitly links 'two tasks: the task of finding the correct theory of vagueness and the task of finding an adequate definition of vagueness.' The suggested definition invokes a Closeness principle: a predicate $\mathtt{F}$ is vague just in case for any two objects $\mathtt{a}$ and $\mathtt{b}$, if $\mathtt{a}$ and $\mathtt{b}$ that are very similar with respect to $\mathtt{F}$, then the sentences $\mathtt{Fa}$ and $\mathtt{Fb}$ are very similar in respect of truth. This actually has to be refined a bit by additionally insisting on the existence of some $\mathtt{F}$-connected,

F-diverse set S of objects. F-connectedness means that all objects of S can be connected by a chain of objects, where adjacent members of the chain are very close in F-relevant aspects. F-diversity means that there are objects a and b in S that are not very similar in respect of truth. The relation of Closeness to the widely discussed principle of Tolerance is carefully explained. The latter states that if a and b are very similar with respect to F, then Fa and Fb are identical in respect to truth. Smith claims that accepting Closeness implies that in many circumstances 'we act as if we believe Tolerance.' This explains why the sorites paradox is compelling. But unlike Tolerance, Closeness allows us to reject the inductive premise in a sorites argument. (This is only a brief hint on how Smith explains and defends his definition in view of competing accounts.)

Obviously the definition of vagueness as Closeness presupposes that sentences can indeed be more or less similar to each other in respect of truth. It is only a small step from this definition to the implicit central claim of the book: to explain vagueness, talk of degrees of truth is not only useful, but essential. All theories that do not allow for intermediary truth values fail to accommodate Closeness and *a forteriori* the essence of vagueness. To someone disinclined to accept degrees of truth in the first place, Smith's definition of vagueness as Closeness may sound question begging. For example, an epistemicist will deny that Fa and Fb can be very close, but not identical in respect of truth. She will insist that Fa and Fb can only be either both true or both false or else one is true and the other is false. The latter case can occur even if we have no means to distinguish a from b is respect of F. But I think that such a criticism of Smith's approach were misdirected. His definition of vagueness is not intended to be compatible with all (prima facie) reasonable accounts of vagueness. It is rather the core of his own theory and has to be judged in its intended context. It is defended not in isolation but in explicit competition with alternative theories of vagueness.

Nevertheless, I have a worry of my own here—one that actually applies to almost all philosophical accounts of vagueness that I have seen so far. Smith, like his colleagues, is convinced that there must be *the* one and only correct theory of vagueness, and in the same vein only one correct definition of vagueness can exist. While it is obvious that contradicting accounts cannot all be simultaneously correct, I think that vagueness is such a complex and multifaceted phenomenon that quite different approaches can still be illuminating and adequate in various ways. Moreover, it should be clear that formal logic and corresponding semantic concepts necessarily abstract away considerably from actual linguistic practice and that we cannot settle questions about the 'correct logic of vagueness' solely by reference to empirical data or by appeal to intuitions about correct inference and use of language.

Let me try to illustrate these remarks by a brief glimpse at contemporary research on vagueness in linguistics. Like in philosophy, various competing semantic accounts of vague language are discussed by linguists. (See, e.g., [1, 7, 8, 10] and references therein for some important contributions.) But, as

far as I can see, a particular kind of context dependency and semantic indeterminacy plays the central role in almost all linguistic models of vagueness. Reference to degrees of truth is rejected for methodological reasons. While hedging, and predicate modifiers like *very*, *clearly*, *definitely* receive attention, truth conditions are invariably formulated respecting bivalence. Indeed, I think that most linguists would argue that talk of degrees of truth, while not outright wrong, is unnecessary to explain linguistic phenomena pertaining to vagueness. Moreover, attention to graduation alone is insufficient for a full account of vagueness. Rather the following is often deemed essential for vagueness: speakers with complete information about an object a and a predicate F may accept or reject an utterance of form Fa without thereby calling in question their linguistic competence. We do not necessarily have to hedge or to admit degrees of truth to justifiably assert Fa even if a is a borderline case for F. However, we do have to take into account the conversational context, containing a record of relevant previous utterances, in a fully adequate semantic model of vague language. It should be pointed out that Smith is not inimical to such linguistic theories of vagueness. In fact, as already indicated, his own fuzzy plurivaluationism takes into account indeterminacy and is deemed fully compatible with contextualism. Still, Smith insists that semantic indeterminacy and context dependency are not part of the nature of vagueness. While this disagreement might largely be considered a matter of different emphasis rather than of principle, it does bear on the treatment of so-called higher-order vagueness. Smith writes that "[t]he demand for higher and higher orders of borderline is just the demand for a gradual transition from the cases where the predicate clearly applies to the cases where it clearly does not apply [...] and this idea of a gradual transition is captured in Closeness [...] Thus, in light of the Closeness definition, the phenomenon of 'higher-order vagueness' may be seen as really being part of plain old vagueness." However, if one thinks that shifting conversational contexts and hence a particular form of semantic indeterminacy and instability have to play a central role in any account of vagueness, then it is not clear at all that higher-order vagueness is already subsumed by 'plain vagueness'. Contextualists like Shapiro [11] rather see the need for refinements of their semantic model that allow to discriminate between, e.g., clear borderline cases and borderline cases of borderline cases. Like-minded linguists, like Chris Barker [1], even go further and strive for models that explain the difference of meaning between, e.g., 'definitively very tall' and 'very definitively tall.' I am not sure that fuzzy plurivaluationism alone already provides all resources needed to address such problems about meaning in presence of vagueness.

Part III of the book is called *Degrees of Truth*. Smith recognizes that it is not enough to insist that an adequate theory of vagueness should countenance degrees of truth. Nor is it enough to simply refer to research in fuzzy logic and point out—as some logicians to in this context—that sorites style arguments can be formalized in particular fuzzy logics in ways that let the in-

ductive premise of the argument appear as close to truth, while avoiding contradiction. Appropriately, Smith devotes a whole chapter (5 – *Who's Afraid of Degrees of Truth?*) to replies to various worries about degree theoretic accounts of vagueness in general and fuzzy logic in particular. Some of the complaints are shown to rest on misunderstandings or unjustified assumptions. Other objections are deflected by proposing modifications and additions to what Smith calls the 'standard fuzzy view.' I cannot discuss all of this rich, well presented material here, but rather want to pick out two items.

I am certainly not the only logician who is tempted to answer to the objection that fuzzy logic violates principles of classical logic with the reply that only questionable metaphysical assumptions can lead to the view that classical logic is the one and only correct way to model formal reasoning. Smith has a quite different reply in store. He shows that the classical consequence relation can in fact be derived from fuzzy semantics. However, while the corresponding concept is certainly quite interesting even from a purely technical point of view, one should point out that it comes at a rather high price: consequence cannot any longer be understood as preservation of designated truth values. Moreover Smith's notion ceases to coincide with classical consequence if we insist on the presence of Łukasiewicz implication (or, in fact, of any other connective that is the residuum of a t-norm modeling conjunction; as required in Hajek's approach to fuzzy logic, mentioned earlier). Still, I am very much in sympathy with pointing out that logical consequence can be defined in quite different ways in a fuzzy setting. In fact one could go even further here and hint, e.g., at various concepts of graded consequence that have been discussed in the literature. Similarly, one may point out that in mathematical fuzzy logic a broad class of truth functions are investigated as candidates for modeling conjunction and correspondingly also implication. The underlying idea is that in natural language (as well as in certain technical contexts) different meanings can be coherently associated with conjunction, and certainly also with if-then-relations.

Of more importance, at least in my view, is Smith's reply to an often heard, major objection to the fuzzy logic view: namely that intermediate truth values taken from the real unit interval impose an artificial precision that clashes with our semantic intuitions. While we seem to have no troubles to process the vague information that Peter is a rather nice guy, it seems virtually impossible to defend the view that what is or should be meant by this statement is that Peter is nice to degree 0.873, or to any other such precise degree. In response to this objection, Smith proposes that the fuzzy logic picture has to be complemented by an account of semantic indeterminacy. While fuzzy (many-valued) interpretations take care of what Smith calls *worldly vagueness*, it is essential that in a given discourse we do not refer to a unique intended interpretation, but rather to a whole set of such interpretations. Thus *fuzzy plurivaluationism* suggests a semantic picture consisting of two components: fuzzy interpretations and a one-to-many relation between language and those interpretations,

that are admissible in a given context. To my mind at least, the second component is just as essential for this theory of vagueness as is the suggested definition of vagueness via Closeness. In any case, plurivaluation also nicely takes care of a related objection to truth degrees represented by numbers between 0 and 1: namely that it seems wrong to say that all sentences in a given discourse can be linearly ordered according to their individual degrees of truth. If, in addition to knowing that Peter is rather nice, we are informed that he is rather small, does it make sense to insist that fully grasping this information entails judging the degree of truth of 'Peter is nice' as being either strictly smaller, strictly larger, or identical to that of 'Peter is small'? Obviously such worries disappear if we accept that the corresponding degrees might be ordered in different ways in different, but equally admissible fuzzy interpretations.

In conclusion, *Vagueness and Degrees of Truth* is a densely argued, engagingly presented monograph on vagueness, that is neither just a survey of philosophical accounts of vagueness nor just a particular addition to the range of corresponding theories—although it is both of these things as well. There can be no doubt that it is a 'must read' for anyone engaged or just seriously interested in the debate on reasoning in face of vagueness. Moreover, I am convinced that the book is accessible and useful also to newcomers to the topic: to enter a lively and prolific field guided by a fine and comprehensive example of topical research is probably more appropriate and certainly more attractive than merely reading through introductory accounts and overview articles about vagueness.

References

[1] Chris Barker: The Dynamics of Vagueness. *Linguistics and Philosophy*, 25(1):1-36, 2002.

[2] Linda C. Burns: *Vagueness: an Investigation Into Natural Language and the Sorites Paradox*. Dordrecht, Kluwer Academic Publishers, 1991.

[3] Kit Fine: Vagueness, Truth and Logic. *Synthése* 30, 265-300, 1975.

[4] Joseph Goguen: The Logic of Inexact Concepts. *Synthése* 19, 325-373, 1968/69.

[5] Petr Hájek: *Metamathematics of Fuzzy Logic*. Kluwer, 1998.

[6] Rosanna Keefe: *Theories of Vagueness*. Cambridge University Press, 2000.

[7] Chris Kennedy: Vagueness and Grammar: The Semantics of Relative and Absolute Gradable Adjectives. *Linguistics and Philosophy*, 30(1):1-45, 2007.

[8] Manfred Pinkal: *Logic and Lexicon*. Kluwer Academic Publishers, Boston, 1995.

[9] Hilary Putnam: Vagueness and Alternative Logic. In Putnam: *Realism and Reason, Philosophical Papers, III*, 271-286.

[10] Robert van Rooij: Vagueness and linguistics. In: *Vagueness: A Guide*, G. Ronzitti (ed.), Springer, forthcoming.

[11] Stewart Shapiro: *Vagueness in Context*. Clarendon Press, Oxford, 2006.

[12] Roy Sorenson: *Vagueness and Contradiction*. Clarendon Press, Oxford, 2001.

[13] Achille Varzi: Vagueness, Logic, and Ontology. *The Dialogue, Yearbooks for Philosophical Hermeneutics* 1:135-154, 2001.

[14] Achille Varzi: Supervaluationism and its Logic. *Mind* 116: 633-675, 2007.

[15] Timothy Williamson: *Vagueness*. London, Routledge, 1994.

[16] Lotfi Zadeh: Fuzzy Sets. *Information and Control* 8:338-353, 1965.

[17] Lotfi Zadeh: Fuzzy Logic and Approximate Reasoning. *Synthése* 30:407-428, 1975.

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