Collapsing Arguments for Facts and Propositions

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Received by Greg Restall
Published November 27, 2008
http://www.philosophy.unimelb.edu.au/ajl/2008

Abstract: Kurt Gödel argues in "Russell's Mathematical Logic" that on the assumption that, contrary to Russell, definite descriptions are terms, it follows given only several "apparently obvious axioms" that "all true sentences have the same signification (as well as all false ones)." Stephen Neale has written that this argument, and others by Church, Davidson, and Quine to similar conclusions, are of considerable philosophical interest. Graham Oppy, responding to this opinion, says they are of minimal interest. Falling between these is my opinion that implications of these arguments for propositions and facts are of moderate philosophical interest, and that these arguments provide occasions for reflection of possible interest on fine lines of several theories of definite descriptions and class–abstractions.

0 Introduction

0.1 According to Stephen Neale

"A *collapsing argument* is an argument designed to show that there are fewer items of a given kind than might be supposed. Alonzo Church. . . , W. V. Quine. . . , and Donald Davidson. . . have used collapsing arguments to undermine several philosophical theses, most notable (i). . . that there are *facts* to which true sentences correspond, (ii). . . that sentences designate *propositions*. . . " (Neale 1995, p. 761 (*cf.*, 2001, p. 9).) "(T)here is a collapsing argument, says Davidson, that precludes the articulation of (a viable theory of facts). The style of argument—used earlier by Church and Quine—is sometimes called the 'Frege Argument'. . . In deference to the minimal machinery and presuppositions of the argument, Barwise and Perry . . . have dubbed it. . . the 'slingshot.' In view of the difficulty involved in attributing the argument to Frege, I shall use this

> label ....(and) reflect carefully upon... an elegant slingshot proof suggested by Gödel" (1995., pp. 763–4).

0.2 AS I SEE IT

Kurt Gödel writes that the assumption that definite descriptions terms together with certain "apparently obvious axioms... (leads) almost inevitably to the conclusion that all true sentences have the same signification (as well as all the false ones)" (Gödel 1944, pp. 128–9). According to Stephen Neale (1995, 1997, and 2001) the argument sketched by Gödel and related arguments due to Church, Davidson, and Quine are of *considerable* philosophical interest. Graham Oppy (1997 and 2004) thinks they are of *minimal* interest. I think these arguments are of *moderate* interest for theories of propositions, showing, as they do, that, if one would have, for sentences to express, more than two propositions, or at least more than two classes of logically equivalent propositions, and, if one counts definite descriptions, or at least certain definite descriptions of classes (a.k.a, 'class–abstractions'), as terms, then

> *one cannot endorse certain propositional–identity (or positional–equivalence) conditions such as that propositions that 'say the same things about the same things' are identical (or logically equivalent).*

0.3 'THE SLINGSHOT' IN GÖDEL'S WORDS

"(W)hat do so-called descriptive phrases... denote or signify* and what is the meaning of sentences in which they occur? The apparently obvious answer that, e.g., 'the author of *Waverley*' signifies Walter Scott, leads to unexpected difficulties. For, if we admit the further apparently obvious axiom, that the signification of a composite expression, containing constituents which have themselves a signification, depends only on the signification of these constituents (not on the manner in which this signification is expressed), then it follows that the sentence 'Scott is the author of *Waverley*' signifies the same thing as 'Scott is Scott;' and this again leads almost inevitably to the conclusion that all true sentences have the same signification (as well as all false ones).** Frege actually drew this conclusion; and he meant it in an almost metaphysical sense, reminding one somewhat of the Eleatic doctrine of the 'One'... 'The True'... being the name he uses for the common signification of all true propositions***... (A)ccording to Russell's terminology and view, true sentences 'indicate' facts... Furthermore, he uses 'denote'... for the relation between things and names... (In his view) different true sentences indicate many different things. Therefore... it is necessary (for him) either to drop the... principle about the signification (i.e., in Russell's terminology the corresponding one about the denotation and indication) of composite expressions or to deny that a descriptive phrase denotes the object described. Russell did the latter*** by taking the viewpoint that a descriptive phrase denotes nothing at all but has meaning only in context... I cannot help feeling that the problem raised by Frege's puzzling conclusion has only been evaded by Russell's Theory of Descriptions and that there is something behind it which is not yet completely understood." (Gödel 1944, pp. 128–30.)

NOTES (GÖDEL'S)

*I use the term 'signify' in the sequel because it corresponds to the German word '*bedeuten*' which Frege, who first treated the question under consideration, used in this connection.

**The only further assumptions one would need in order to obtain a rigorous proof would be: (1) that'$\varphi(a)$' and the proposition 'a is the object which has the property φ and is identical with a' mean the same thing and (2) that every proposition 'speaks about something,' i.e., can be brought to the form $\varphi(a)$. Furthermore one would have to use the fact that for any two objects a, b there exists a true proposition of the form $\varphi(a, b)$ as e.g., $a \neq b$ or $a = a.b = b$.[1]

***Cf. 'Sinn and Bedeutung'... "

0.4 GÖDEL'S SLINGSHOT

By 'Gödel's slingshot' I mean an argument that Gödel sketched mainly in order to state Russell's response to it, and also, one gathers, for its philosophical interest and, to Gödel's mind, deep significance. By 'an argument' I mean a categorical argument, that is, a claim that certain things, premises, are true, and that from them a certain thing, a conclusion, follows. Premises of arguments are not merely assumed or supposed, they are asserted. An argument is sound if and only if its two–part claim is true, that is, if and only if it is valid, and its premises are true. After indicating how Russell could evade what Gödel depicts as essentially Frege's argument in *"Uber Sinn und Bedeutung* (1892), Gödel expressed puzzlement regarding what is 'fundamentally' wrong with this argument.

Coming is a carefully articulated and developed Gödelian slingshot that would be a collapsing argument that established that there are fewer propositions and facts than are dreamt of by some philosophers. This argument, rather than being made, could be merely taken up and studied "to draw out inconsistencies... lurking in a position" (Neale and Dever 1997, p. 150). Indeed, Neale and Josh Dever, in response to objections in (Oppy 1997) to (Neale 1995), write that "Gödel's Slingshot is simply an analytic tool that can reveal when the commitments one has already made lead one to inconsistency" (pp. 150–1, in Section 4, "Methodological Muddles"). It can be taken and studied to such ends, but, when *opposing* it and similar arguments by Church, Quine, and Davidson, Neale casts it and them as "collapsing *argument(s)* designed to SHOW that there are fewer items of a given kind than might be supposed... (in order) to undermine several philosophic theses most notably... that there are *facts* to which true sentences correspond... (and) that sentences designate *propositions*" (1995,

[1]Neither Neale nor I make use of this 'further fact' that Gödel says one would have to use.

p. 761, bold emphasis added; *cf.*, 2001, p. 9).[2] Neale opposes these arguments as arguments: he opposes them at least in part because he opposes their 'Eleatic conclusions' for facts and propositions. I too oppose them for this reason, and study them to expose their errors regarding identities of facts and proposition, and for the interest of their possible description–theory technologies.

0.5 COMING SECTIONS

1. A Gödelian slinghot for a conclusion limited to 'subject–predicate' sentences.
2. Extending this argument to all sentences.
3. *Premise* (iii) of this slingshot, and Neale's assumption G1 for his.
4. Prospects of Gödelian slingshots in three description calculi.
5. Frege's argument.
6. Davidsonian collapsing arguments.
7. Monty and me, and Church's slingshot.
8. Reflections.

1 A GÖDELIAN SLINGSHOT FOR A CONCLUSION LIMITED TO 'SUBJECT–PREDICATE' SENTENCES

The limited conclusion of the argument of this section is that true 'subject–predicate' sentences of a language, or language–segment, L, that meets conditions laid down in coming premises, all express the same proposition or 'stand for' the same fact, and that false ones all express a single other proposition. L can be an interpreted formal language, a natural language such as English, or a segment of a natural language. The coming argument is valid. It wants to embarrass the theory that 'true sentences' express true propositions, that is, facts, and can express *different* true propositions, and that 'false sentences' express false propositions and can express different false propositions. The argument serves this purpose only if its premises are true, if not of natural languages, then at least of formal languages that might recommend themselves as frameworks for 'ideal languages', and of propositions and facts.

Regarding propositions and facts, contrary to Alfred Tarski, as observed in Section 1.3 of (Sobel 2006c), I consider propositions to be the primary bearers

[2] Neale includes under 'collapsing arguments', arguments for other kinds of 'collapses' such as of "modal distinctions... in systems that combine modality and quantification" (*ibid.*). I confine the rubric in what follows to arguments that would 'collapse' *propositions* into just two (or classes of logically equivalent propositions into just two), and *facts* into just one.

of truth and falsity, count a sentence–token as true/false if and only if it expresses a true/false proposition, and count a sentence–type as true if and only if its possible tokens are all true. Facts, plain facts of English, seem to be *identical with* true propositions. For example, the true proposition *that snow is white* is, on the face of it, the fact *that snow is white*. When *that snow is white* is said to be true, and when it is said to be a fact, it is the same thing that is said to be true, and that is a fact, *unless* the tokens of the name 'that snow is white' in the attribution of truth, and in the classification as a fact, denote different things. There is, in my view, no good reason for thinking that they do that. In my view facts *are* true propositions. It may be that "(s)ome philosophers vehemently deny this" (Neale 2001, p. 203), though another possibility is that they mean by facts not 'plain facts of English', but 'truth–makers' as it is sometimes said that 'Russellian facts' would be. Plain facts, since identical with true propositions, cannot be what make true propositions true.[3]

1.1 THE ARGUMENT

PREMISES FIVE

(i) For sentences φ and φ' of L, if a φ' comes from φ by replacement of a term in an 'extensional position' by a term that has the same denotation, or by replacement of a sentence in an 'extensional position' by a sentence that expresses the same proposition, then φ and φ' express the same proposition.

Cf.: "the signification of a composite expression, containing constituents which have themselves a signification, depends only on the signification of these constituents" (Gödel 1944, p. 129). I have assumed that 'constituents' of a composite expression are for Gödel expressions that occupy 'extensional positions'. For example, I assume: that while '*Hesperus*' is a constituent of '*Hesperus* is a planet.', it is not a constituent of 'An ancient name for the third planet from the Sun is '*Hesperus*'.'; and that while 'Helen is home' is a constituent of 'If Helen is home, she will answer the phone.'; it is not a constituent of, 'Henry said, "Helen is home," and nothing more.'

The'significations' in Gödel's sense of terms and sentences of language L are, respectively, their denotations, and the propositions they express. Gödel runs together Russell's *indications* and *denotations*: For Russell, true sentences indicate facts, and names denote objects; to 'signify' in Gödel's sense is to 'indicate or denote' in Russell's. Gödel refers to the just quoted principle in these words: "the above mentioned principle about signification (i.e., in Russell's terminology the corresponding one about DENOTATION AND INDICATION")

[3]"But what *are* propositions and facts such as that the moon is a satellite of the earth?" I'm not sure I understand the question, comparable as it is to questions like, "But what *are* numbers such as three?" and "What *are* colours such as yellow?" As for what propositions are *not*, one cannot do better for starters than "Propositions" in (Cartwright 1987).

of composite expressions” (p. 130, bold emphasis added). This terminological decision of Gödel’s has had unfortunate effects on some of his interpreters. When speaking for myself I use ‘express’ for the relation of sentences to propositions, and for to the relation to facts of sentences that express true propositions (of course, since, as said in note 2, I identify facts and true propositions).

What premise (i) is not. It is not simply the assumption that, as far as its sentences go, L is an *extensional language*, or the *extensional part* of a language, in the sense in which the languages of standard logical systems are extensional, which is that ‘the designations of their complex semantically complete units (terms and formulas) being determined by the designations of their components’ (Kalish, *et. al.*, 1980, p. 379).

To spell out what premise (i) *is not*, let ’$\ll$ext$\gg$ ’ between closed terms or sentences of L be short for ‘has the same extension as’, where the ‘extension’ of a closed term is the object it names, and the ‘extension’ of a sentence is its ‘*truth–value*’.[4]

Premise (i) is *not* simply the assumption that, for any sentences φ and φ', and terms or sentences χ and χ', if

$$\chi \ll \mathrm{ext} \gg \chi',$$

and φ' comes from φ by replacing an occurrence of χ in an extensional position by an occurrence of χ', then

$$\varphi \ll \mathrm{ext} \gg \varphi'.$$

Whether or not L is an extensional language, or an extensional part of a language, in this sense of ‘extensional’ is beside the point of *premise* (i).

What premise (i) is. To spell out what premise (i) *is*, let ‘<expr> ’ between names of closed terms be short for ‘has the same *designatum* as’: ‘<expr> ’ between names means the same as ‘$\ll$ext$\gg$ ’. Let ‘<expr> ’ between sentences

[4]‘Truth–value’? “This phrase is due to Frege.” (Whitehead and Russell 1927, p. 7n.) “Frege’s first use of truth–values is in *Funktion and Begriff* of 1891 and in his paper of 1892"(Church 1958, p. 25n27). “The explicit use of two truth–values appears for the first time in a paper by C. S. Peirce in the *American Journal of Mathematics*, vol. 7 (1885)” (Church, *loc. cit.)*.

“By the truth–value of a sentence I understand the circumstance of its being true or false. There are no other truth–values. For brevity I call the one the True, and the other the False.” (Frege 1960, p. 63.) With these words, writes Göran Sundholm, were ‘truth–values’ introduced into logic: he “argues. . . that the explanations offered (of ‘truth–values’ were) insufficient” (“Truth–Value,” *Logica* 2004 Abstracts, p. 51).

Without claiming to interpret or fully agree with Frege, or to explain a notion that will “carry the philosophical burden it has been given ever since” (*ibid.*), here is what I make of truth–values. There are propositions, and amongst their properties are truth and falsity. *Truth* is *the* property that every true proposition, and no false proposition, has. *Falsity* is similarly related to false propositions. So far, plain fact, not stipulation. Now comes a stipulation: truth and falsity are ‘*truth–values*’, and there are no other ‘truth–values’. Truth–value is, by this stipulation, to truth and falsity, as colour is to black and white, except that while there are colours other than black and white, there are no truth–values other than truth and falsity.

be short for 'expresses the same proposition as': '<expr> ' between sentences does *not* mean the same as '≪ext≫ '. *Premise* (i), *spelled out*, is the principle that, for any sentences φ, φ', ψ, and ψ' :

IF φ' comes from φ by replacement of an occurrence of ψ in an extensional position by ψ', and ψ and ψ' are *co-expressive*,

$$\psi <\text{expr}> \psi',$$

or φ' comes from φ by replacement of an occurrence of δ in an extensional position by an occurrence of δ', and δ and δ' are *co-designating* names,

$$\delta <\text{expr}> \delta';$$

THEN

$$\varphi <\text{expr}> \varphi'.$$

(ii) Definite descriptions of L are terms.

(iii) For variable α, closed term A, formula φ_α of L in which α is free in an extensional position, and sentence φ_A of L that comes from φ_α by proper substitution of A for α : if φ_A *is true*; then

$$\mathsf{A} \text{ is identical with the object } \alpha \text{ such that, } \alpha \text{ is identical with } \mathsf{A} \mathbin{\&} \varphi_\alpha,$$

i.e.,

$$\mathsf{A} = \text{the object } \alpha \text{ such that } (\alpha = \mathsf{A} \mathbin{\&} \varphi_\alpha)$$

is a sentence of L, or a translation of a sentence of L,[5]

and this sentence and φ_A *express the same proposition*: in short,

if φ_A is true, *then* $\varphi_\mathsf{A} <\text{expr}> \mathsf{A} = \text{the object } \alpha \text{ such that } (\alpha = \mathsf{A} \mathbin{\&} \varphi_\alpha)$.

Cf.: " 'φ (A)' and the (sentence) 'A is the object which has the property φ and is identical to A' mean the same thing" (Gödel 1944, p. 129n5). I have added, without prejudice, the qualifying clause 'if $\varphi(a)$ is true'. Gödel may have taken this qualification for granted, since he was concerned only parenthetically also

[5]'The Moon is identical with the object that is identical with The Moon and a natural satellite of The Earth' is a sentence of English, and the *literal* translation, under the scheme—A^0: The Moon; F_1 : a is a natural satellite of The Earth—of the sentence of The Description Calculus (Kalish, *et. al.*, 1980, Chapter VI), '$\mathsf{A}_0 = \daleth x(x = \mathsf{A}_0 \wedge \mathsf{F}_1 x)$'. It is *not* a *literal* translation under this scheme of the Russellian sentence of the *R*–calculus (Sobel 2006a, Chapter VIII), '$\{\daleth x(x = \mathsf{A}_0 \wedge \mathsf{F}_1 x\}\mathsf{A}_0 = \daleth x(x = \mathsf{A}_0 \wedge \mathsf{F}_1 x)$': a *literal* translation under that scheme of this symbolic sentence makes *explicit* that *there is exactly one object x* such that x is identical with The Moon and x is a natural satellite of The Earth. The literal translation of this Russellian sentence is, 'There is an object such that it and only it is the Moon and a natural satellite of The Earth, and the Moon is identical with this object': in symbols, $y(x((x = \mathsf{A}_0 \wedge \mathsf{F}_1 x) \leftrightarrow y) \wedge \mathsf{A}_0 = y)$.

with false sentences.[6] He writes not of sentences expressing the same proposition, but of sentences meaning the same thing. He could have written of their *signifying* the same thing. He tells us that, in Russell's terminology, they can be said to *indicate* the same thing. *Premise* (iii) has them *expressing* the same proposition, and, thus, given that this is a true proposition, the same fact.[7]

(iv) *Reflexitivity, symmetry, and transitivity* of the *same–signification relation* (i.e., for terms the same–denotation relation, and for sentences the co-expressive relation). For any terms or formulas of L, ε, ε', and ε'':

$$\text{reflexitivity, } \varepsilon <\text{expr}> \varepsilon\text{; symmetry, } \varepsilon <\text{expr}> \varepsilon' \rightarrow \varepsilon' <\text{expr}> \varepsilon;$$

and

$$\text{transitivity, if } \varepsilon <\text{expr}> \varepsilon' \text{ and } \varepsilon' <\text{expr}> \varepsilon''\text{, then } \varepsilon <\text{expr}> \varepsilon''.$$

In short, $<\text{expr}>$ is an *equivalence relation*.

(v) For any sentence φ of L that expresses a proposition, $\ulcorner{\sim}{\sim}\varphi\urcorner$ is a sentence of L, or a symbolization of a sentence of L, and φ and $\ulcorner{\sim}{\sim}\varphi\urcorner$, or sentences symbolized thereby, express the same proposition.

This premise is not in evidence in (Gödel 1944). I do not know whether in his own (unpublished) reasoning he relied on it. In the coming deduction, the first conjunct of the conclusion concerning truths and facts is reached without aid of *premise* (v), which serves only the deduction of the second conjunct regarding falsehoods.[8]

[6]Neale does not, in his Gödelian slingshot, enter the qualification that '$\varphi(a)$' is true. I come back to this difference between our slingshots in Section 3 below. There is a question whether in his 'reconstruction of Gödel's proof' he at this point "captures Gödel's (intended proof) precisely" (1995, p. 778; *cf.*, 2001, p. 131).

[7]Suppose that it is true *that there are black swans in Australia*. This proposition is *named* by the italicized 'that'–clause, *not* by the sentence 'There are black swans in Australia'. (*Cf.*, King 2002.) Neale writes: "Frege had the idea that a sentence can stand for either Truth or Falsity…On Russell's account, a true sentence STANDS FOR a *fact*." (1995, p. 765, italic emphasis added.) However, in quotations that Neale gives, Russell says not that true sentences stand for, but only that they *express* facts: e.g., "a fact is 'the sort of thing expressed by a whole sentence, not by a single name…We express (what we take to be) a fact…when we say that a certain thing has a certain property…' ((Russell) 1918, pp. 182–3)…" (p. 766). I doubt that Russell ever thought of true sentences as *naming* facts, and believe "that it is (*not*) clear that Russell wanted none of this" where 'this' includes Frege's suggestion that we say that sentences stand for or name The True and The False. Russell's attitude towards this exercise of 'philosophic license' could, for all that mattered to him, have been, "Why not?" (as my friend Sten likes to say).

[8]Neale does not use *premise* (v) or anything like it in his 'reconstruction' of Gödel's argument. The explicit conclusion of his Gödelian slingshot is confined to true sentences. (1995, pp. 778–9, *cf.*, 2001, pp. 130–1.) Similarly for his "Gödel's Proof in Quinean Format" (2001, p. 181). I use *premise* (v) again in the Davidsonian argument of Section 6.1 below that is general for true and false sentences. A fully general Davidsonian argument of Neale's does not use anything like

CONCLUSION

'subject–predicate' sentences of L *that express* TRUE *propositions or facts, express the same true proposition or fact;* AND
'subject–predicate' sentences of L *that express* FALSE *propositions express the same false proposition.*

A 'subject–predicate' sentence is here a sentence in which a term occurs in an'extensional' position: for examples, '*The sun* is a star.', 'No city is larger than *Toronto*.', and 'If *the girl named 'Helen'* is home, she will answer the door.' Neither 'All men are mortal.' nor '*Tiny Tim* was so-called for his size.' are 'subject–predicate' sentences.

The conclusion is that, for example, the true sentences, 'The moon is the natural satellite of the Earth.' and 'The sun is the star around which the Earth orbits.', express the same fact!

1.2 REASONING THAT VALIDATES THIS ARGUMENT

Let 'T' be an operator that, with variables and formulas, makes definite descriptive terms in L, as '℩' does in the language of The Description Calculus of (Kalish, *et. al.*, 1980, Chapter VI). The coming deduction assumes that 'T' and '℩' agree in their logics for proper descriptions in extensional contexts: let this be assumption (*).

FOR THE FIRST CONJUNCT *of the limited conclusion*, namely,

'subject–predicate' sentences of L that express *true* propositions or facts express the same one, we *assume*,

(vi) φ_A is true

(vii) ψ_B is true

for closed terms or 'names' A and B, variable α, formulas φ_α and ψ_α in which α and only α is a variable free in an extensional position, and sentences φ_A and

premise (v) (1995, p. 793; 2001, p. 173). Problems with its way are canvassed in Section 6.2.2 below. Lately, Neale has used something *very* like *premise* (v) in "a proof based on Gödel's slingshot" (2001, p. 183) that would establish *inter alia* that materially equivalent 'subject–predicate' sentences express the same proposition. The first part of this proof, is for *true* 'subject–predicate' sentences: this part is the same as the proof at (1995, pp. 189–90), and uses nothing like *premise* (v). The case for *false* 'subject–predicate' sentences is not taken up in (1995). This deficit is made up in (2001) where the proof has a second part for this case, in which part we meet something like *premise* (v). The proof is addressed to '❽', an arbitrary one place connective. He uses in the second part "the assumption that ❽ is (DNN)" (2001, p. 183), which assumption legitimates inferences in the proof from '❽(Fa)' to '❽(∼∼Fa)', and from '❽(∼∼Gb)' to '❽(Gb)'. The annotations for these inferences feature '❽+DNN', which names a rule of inference, that, unlike the other rules used in his proofs, is not set out before its use, or included in the summary of principles of inference at the end of the book (2001, pp. 251–2).

ψ_B that come respectively from φ_α and ψ_α by replacing all free occurrences of α by occurrences of A and B.

To show: φ_A <expr> ψ_B.

We proceed by cases. In the first case, the referent of A is identical with the referent of B. In the second, not.

For the FIRST CASE, we *assume*,

(viii) $A = B$

From (vi) and *premise* (iii) follows,

(ix) φ_A <expr> $A = (\mathrm{T}\alpha)(\alpha = A \,\&\, \varphi_\alpha)$

and from (vii) and *premise* (iii),

(x) ψ_B <expr> $\psi_B = (\mathrm{T}\alpha)(\alpha = B \,\&\, \psi_\alpha)$.

From (viii), and assumption (*), follows $\ulcorner(\mathrm{T}\alpha)(a = A \,\&\, \alpha = B)\urcorner$ is proper, and both that $\ulcorner(\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B)\urcorner$ <expr> A, and that $\ulcorner(\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B)\urcorner$ <expr> B. (‘$A = B \to \bigvee y \bigwedge x(x = A \wedge x = B \leftrightarrow x = y)$’ and ‘$\bigvee y \bigwedge x(x = A \wedge x = B \leftrightarrow x = y) \to ((\daleth x(x = A \wedge x = B) = A \wedge \daleth x(x = A \wedge x = B) = B)$’ are theorems of The Description Calculus of (Kalish, *et. al.*, 1980, Chapter VI).) From these ‘co-expressions’, it follows, by *premise* (i), that,

(xi) $A = B$ <expr> $A = (\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B)$

and

(xii) $A = B$ <expr> $(\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B) = B$,

From (vi) follows φ_A ‘on a line’, and from this and (viii), and assumption (*), it follows that the descriptions, $\ulcorner\mathrm{T}(\alpha)(\alpha = A \,\&\, \varphi_\alpha)\urcorner$ and $\ulcorner\mathrm{T}(\alpha)(\alpha = A \,\&\, \alpha = B)\urcorner$ are proper for the same thing. (‘$FA \to \bigvee y \bigwedge x(x = A \wedge Fx \leftrightarrow x = y)$’, ‘$A = B \to \bigvee y \bigwedge x(x = A \wedge x = B \leftrightarrow x = y)$’, and thus ‘$FA \wedge A = B \to \daleth x(x = A \wedge Fx) = \daleth x(x = A \wedge x = B)$’ are theorems of The Description Calculus (Kalish, *et. al.*, 1980, Ch. VI.) That is, it follows that,

(xiii) $(\mathrm{T}\alpha)(\alpha = A \,\&\, \varphi_\alpha)$ <expr> $(\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B)$

From (vii) and (viii) it follows similarly that $\ulcorner(\mathrm{T}\alpha)(\alpha = B \,\&\, \varphi_\alpha)\urcorner$ and $\ulcorner(\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B)\urcorner$ are proper for the same thing:

(xiv) $(\mathrm{T}\alpha)(\alpha = B \,\&\, \varphi_\alpha)$ <expr> $(\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B)$

From (xiii), by *premise* (i), it follows that $\ulcorner A = (\mathrm{T}\alpha)(\alpha = A \,\&\, \varphi_\alpha)\urcorner <\mathrm{expr}> \ulcorner A = (\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B)\urcorner$, and from this and (ix), by *premise* (iv) (transitivity), it follows that,

(xv) $\varphi_A <\mathrm{expr}> A = (\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B)$

From (x) and (xiv) it follows similarly that,

(xvi) $\varphi_B <\mathrm{expr}> B = (\mathrm{T}\alpha)(\alpha = A \,\&\, \alpha = B)$

By *premise* (iv) (transitivity, symmetry), from (xv) and (xi) it follows that $\varphi_A <\mathrm{expr}> \ulcorner A = B\urcorner$, from (xvi) and (xii) it follows that $\varphi_B <\mathrm{expr}> \ulcorner A = B\urcorner$, from which 'co-expressions' it follows that,

(xvii) $\varphi_A <\mathrm{expr}> \psi_B$.

This completes the FIRST CASE: deduced from premises (i) through (iv), assumption (*) that says that logic of the operator 'T' is that of the Fregean description operator '⁊', and assumptions (vi) and (vii) that say that φ_A and ψ_B are true is,

$$\text{if } A = B, \text{ then } \varphi_A <\mathrm{expr}> \psi_B.$$

For the SECOND CASE, we *assume*,

(xviii) $A \neq B$

from which it follows that,

(xix) $\ulcorner A \neq B\urcorner$ is true

It follows from (vi) and *premise* (iii) again that

(xx) $\varphi_A <\mathrm{expr}> A = \mathrm{T}\alpha(\alpha = A \,\&\, \varphi_\alpha)$

From (vi) follows φ_A 'on a line', and from this and (xviii) and assumption (*) that,

(xxi) $\mathrm{T}\alpha(\alpha = A \wedge \varphi_\alpha) <\mathrm{expr}> \mathrm{T}\alpha(\alpha = A \,\&\, \alpha \neq B)$,

as (xiii) follows from (vi), (viii), by assumption (*). It follows from (xix), by *premise* (iii), that

(xxii) $A \neq B <\mathrm{expr}> A = \mathrm{T}\alpha(\alpha = A \,\&\, \alpha \neq B)$.

From (xxi), by *premise* (i), it follows that $\ulcorner A = \mathrm{T}\alpha(\alpha = A \,\&\, \alpha \neq B)\urcorner <\mathrm{expr}> \ulcorner A = \mathrm{T}\alpha(\alpha = A \wedge \varphi_\alpha)\urcorner$, and from this and (xxii), by *premise* (iv) (transitivity), it follows that,

(xxiii) $A \neq B$ <expr> $A = \mathrm{T}\alpha(\alpha = A \,\&\, \varphi_\alpha)$

It follows from (xx) and (xxiii), by *premise* (iv) (symmetry, transitivity), that

(xxiv) φ_A <expr> $A \neq B$

It follows from (vii) and (xviii) similarly that

(xxv) ψ_B <expr> $A \neq B$.

Finally, by *premise* (iv) (symmetry, transitivity), it follows from (xxiv) and (xxv) that

(xxvi) φ_A <expr> ψ_B

That completes the SECOND CASE: deduced from premises (i) through (iv), assumption (*) that says that logic of the operator 'T' is that of the Fregean description operator '⅂', and assumptions (vi) and (vii) that say that φ_A and ψ_B are true is,

if $A \neq B$, then φ_A <expr> ψ_B.

Since φ_A and ψ_B are any true 'subject–predicate' sentences, established by our two cases is that premises (i) through (iv) entail that 'subject–predicate' sentences of L that express *true* propositions express the same one.

FOR THE SECOND CONJUNCT, that

'subject–predicate' sentences of L that express *false* propositions, all express the same one,

we *assume*,

(xxvii) φ_A is false

(xxviii) ψ_B is false

It follows from (xxvii) and (xxviii) that

(xxix) $\ulcorner {\sim}\varphi_A \urcorner$ is true

and

(xxx) $\ulcorner {\sim}\psi_B \urcorner$ is true

Therefore, by adaptations of lines (viii) through (xxvi), it follows that whether or not $A = B$,

(xxx) $\sim\varphi_A <\mathrm{expr}> \sim\psi_B$

We have, by *premise* (iv) (reflexivity), that $\ulcorner\sim\sim\varphi_A\urcorner <\mathrm{expr}> \ulcorner\sim\sim\varphi_A\urcorner$, from which and (xxx) it follows by *premise* (i), that

(xxxi) $\sim\sim\varphi_A <\mathrm{expr}> \sim\sim\psi_B$.

From this it follows, by *premise* (v), that

(xxxii) $\varphi_A <\mathrm{expr}> \psi_B$.

That completes reasoning from premises (i) through (v) for the limited conclusion that,

> *'subject–predicate' sentences of* L *that express true propositions or facts, express the same truth or fact; and sentences of* L *that express false propositions, express the same falsehood.*

2 EXTENDING THE ARGUMENT TO ALL SENTENCES

For the unrestricted conclusion that

> sentences of L that express true propositions or facts express the same truth or fact, and sentences of L that express false propositions express the same falsehood,

Gödel adds something like that,

> FOR EVERY SENTENCE φ OF L THAT EXPRESSES A PROPOSITION, THERE IS A 'SUBJECT–PREDICATE' SENTENCE φ' OF L SUCH THAT φ AND φ' EXPRESS THE SAME PROPOSITION.

He makes what he casts as the 'further assumption' "that every proposition 'speaks about something,' i.e., can be brought to the form $\varphi(a)$" (Gödel 1944, p. 129n).

For example, corresponding in the manner contemplated to sentence '*all men snore*' would be, for one, the sentence '*Clinton is an x such that all men snore*' with subject 'Clinton', and predicate 'is an x such that all men snore' (Neale 1995, p. 778: 'Clinton' is replaced by 'Socrates' in Neale 2001, p. 130.). If such correspondences "are found repugnant one can still follow Gödel's argument through in connection with atomic sentences" (*ibid.*). A problem with some such correspondences, related specifically to this one of Neale's, is that perhaps, if Clinton had not existed, then, while there would still have been the proposition *that all men snore*, there would not have been the proposition *that Clinton is an* x *such that all men snore*. That would be a 'singular proposition' about Clinton, and a case can be made for the existence of this proposition's presupposing the existence of Clinton. (*Cf.*, Cartwright 1998.) If there is this difference between these propositions, then they cannot be one and the same proposition. A response available to Gödel to this problem with his 'further assumption' would be to change it to say that every proposition 'speaks about something that exists necessarily'. I believe that Gödel thought that numbers are necessary existents, and I know that he was inclined to think that God is a necessary existent (*cf.*, Sobel 2004, Chapter IV).

3 PREMISE (III) OF THIS SLINGSHOT, AND NEALE'S ASSUMPTION G1 FOR HIS

Premise (iii) is *weaker* than Neale's G1 (1995, p. 777; 2001, p. 130):

φ_A <expr> $A = \mathrm{T}\alpha(\alpha = A \mathbin{\&} \varphi_\alpha)$.

Premise (iii) licenses entries on lines of a proof of conditionals of the form

φ_A is true $\rightarrow$ φ_A <expr> $A = \mathrm{T}\alpha(\alpha = A \mathbin{\&} \varphi_\alpha)$,

for variable α, closed term A that has a denotation, formula φ_α in which α and only α is free, and sentence φ_A that comes from φ_α by replacing all free occurrences of α in extensional positions by occurrences of A, which conditionals correspond to sentences,

$$\varphi_A \rightarrow (\varphi_A \leftrightarrow A = \mathrm{T}\alpha(\alpha = A \mathbin{\&} \varphi_\alpha)),$$

that are valid in calculi for denoting terms in which T–descriptions are terms. ('$FA \rightarrow (FA \leftrightarrow A = \urcorner x(x = A \wedge Fx))$' is a theorem of The Description Calculus (Kalish, *et. al.*, 1980, Ch. VI.)

Neale's G1, which adds nothing to Gödel's explicit words for his first 'further assumption', licenses entries without further ado of the *consequents* of those conditionals. These consequents correspond to biconditionals that are valid in only *some* calculi for denoting terms in which descriptions are terms. It may be observed that the inferences that Neale bases on G1 to show that "all true sentences stand for the same fact" (2001, p. 178), would be served as well by my *premise* (iii). Similarly for "a proof based on Gödel's slingshot" (2001, p. 183), begun in (1995) on pages 789 and 790, and completed in (2001) on pages 185 and 186.

What, aside from this matter of accommodation in more description calculi, could recommend *premise* (iii) over G1? Nothing, *if* what recommends both G1 and *premise* (iii) is exactly the 'sense' that, for example, φ_A and $\ulcorner A = (\mathrm{T}\alpha)(\alpha = A \mathbin{\&} \varphi_\alpha)\urcorner$ *surely* entail one another, and are '*logically equivalent*' when

'T' means the definite article 'the' of English.[9] Neale writes that "any theory of descriptions must be compatible with this fact (as Russell's is)*" (1995, p. 789; 2001, p. 180), and as The Description Calculus is not.[10] The reference of 'this fact' is precisely to the purported fact that

$$\text{T–INTR} \qquad \varphi_A \quad /\therefore A = T\alpha(\alpha = A \mathbin{\&} \varphi_\alpha)$$

and

$$\text{T–ELIM} \qquad A = T\alpha(\alpha = A \mathbin{\&} \varphi_\alpha) \quad /\therefore \varphi_A$$

"are valid rules of inference in extensional contexts" (1995, p. 789; 2001, p. 180).[11] The sentence

$$\varphi_A \leftrightarrow A = T\alpha(\alpha = A \mathbin{\&} \varphi_\alpha)$$

is a theorem of any calculus in which these rules are primitive or derivable. *The exact truth for Russell's theory, in which descriptions are not terms, is that in it the rules

$$\daleth\text{–INTR} \qquad \varphi_A \quad /\therefore \daleth\alpha(\alpha = A \wedge \varphi_\alpha)A = \daleth\alpha(\alpha = A \wedge \varphi_\alpha)$$

[9] Scare–quotes around 'logical equivalence' since Neale, in (1995) and (2001), uses it in (and presumably only in) a model–theoretic sense that is not applicable to English, for which 'models' are not defined. He writes, "following Tarski, and common practice, let us say that (φ and ψ are logically equivalent) if, and only if, φ and ψ have the same truth–value in every model" (1995, p. 791, *cf.*, 2001, p. 55).

[10] As Neale knows: 1995, p. 798; 2001, p. 194. For example, as said, '$FA \rightarrow (FA \leftrightarrow A = \daleth x(x = A \wedge FA))$' is a theorem of The Description Calculus of (Kalish, *et. al.*, 1980, Chapter VI), but, I now add, '$FA \leftrightarrow A = \daleth x(x = A \wedge FA)$' is not. This sentence is false in the minimal model—Universe: $\{0\}$; Improper Designatum: 0; $F^1 : \{\}$; $A^0 : 0$.

[11] In (1995) and (2001) we find that "any *adequate* theory of descriptions... must be compatible with this fact (the validity of these rules in extensional contexts)" (1995, p. 789; 2001, p. 180), meaning, presumably, any theory of descriptions *that for extensional contexts agrees with the 'logic' of English definite descriptions* must validate these inferences in extensional contexts. These rules are, in Greg Restall's view, valid for "any reasonable theory of descriptions" (Restall 2004, p. 421).

Restall implies, however, that theories such as The Description Calculus of (Kalish, *et. al.*, Chapter VI) satisfy this condition for reasonableness. He should have known, since Neale says so, that this is false. Relatedly, Restall implies that 'FA' and '$A = \daleth x(x = A \wedge Fx)$' are logically equivalent in such theories (p. 425), which is false. Restall also says that these rules are valid for descriptions in Russell's theory, which, as is shortly to be explained in the text, is not*exactly true*.

These mistakes spoil Restall's demonstration without effort—"I simply took (a) pre-existing account... 'off the shelf'." (P. 425)—of the error *of a* Gödelian collapsing argument addressed to an extension of his 'starter' theory of facts that includes a version of this well–known referential theory of descriptions. *The* error, which Restall thinks he identifies by elimination, is that certain rules for substituting identical terms are not unrestrictedly valid in this extension. His demonstration would be better served by a theory such as The Russellian Theory of Descriptions of (Kalish, *et. al.*, 1980, Chapter VIII), which theories are mentioned by Neale for the very virtue that Restall's demonstration–by–elimination needs (Neale 1995, p. 798; 2001, p. 195).

and

$\daleth$–ELIM $\qquad \daleth\alpha(\alpha = A \wedge \varphi_\alpha)A = \daleth\alpha(\alpha = A \wedge \varphi') \qquad \therefore\ \varphi$ are

derivable, and sentences

$$\varphi_A \leftrightarrow \daleth\alpha(\alpha = A \wedge \varphi_\alpha)A = \daleth\alpha(\alpha = A \wedge \varphi_\alpha)$$

are theorems, in which rules and sentences $\ulcorner\daleth\alpha(\alpha = A \wedge \varphi_\alpha)A = \daleth\alpha(\alpha = A \wedge \varphi_\alpha)\urcorner$ is not an identity–sentence. '$\daleth$' is here Russell's description operator that makes formulas. It is used in (Sobel 2006a, Chapter VIII) for an extension of The Description Calculus of (Kalish, *et. al.*, 1980, Chapter VI) in which extension the descriptive term–maker '$\daleth$' and its rules remain.

But, the 'sense' that φ_A and $\ulcorner A = T\alpha(\alpha = A\,\&\,\varphi_\alpha)\urcorner$ should be logically equivalent, along with the associated 'sense' that these rules should be valid, is ***not*** all that ***premise*** (iii) has going for it. To take 'T' in the sense of 'the', let me replace it by 'THE' and shift parentheses. If the English description 'THE$(x)(x = A\,\&\,Fx)$' is proper, which it is if and only if 'FA' is true; then'FA' and '$A =$ THE$(x)(x = A\,\&\,Fx)$' ***can*** express the same proposition, for they will at least express logically equivalent propositions. ***Perhaps***, however, if 'FA' is false, so that 'THE$(x)(x = A\,\&\,Fx)$' is improper, we should say that '$A =$ THE$(x)(x = A\,\&\,Fx)$' does not express a proposition, in which case our two sentences ***cannot*** express the same proposition. An advantage of ***premise*** (iii) over G1 is that it leaves open the somewhat attractive theoretical possibility that, if 'THE$(x)(x = A\,\&\,Fx)$' is improper because 'FA' is false, then though 'FA' expresses a proposition, '$A =$ THE$(x)(x = A\,\&\,Fx)$' does not.[12] Reflection on this advantage can ***detract*** from the sense that in an 'adequate' theory of descriptions φ_A and $\ulcorner A =$ THE$(\alpha)(\alpha = A\,\&\,\varphi_\alpha)\urcorner$ must be logically equivalent, and that THE-INTR and THE-elim must be valid. Related advantages of ***premise*** (iii) over G1 are that it leaves open: (a), that if 'A' abbreviates a non-denoting proper name such as 'Santa Claus', or an improper description, then we should say that neither 'FA' nor '$A =$ THE$(x)(x = A\,\&\,Fx)$' express propositions, so that they cannot express the same proposition; and, (b), that if 'A' abbreviates a non-denoting proper name, then we should say (instead) that though 'FA' does not express a proposition, '$A =$ THE$(x)(x = A\,\&\,Fx)$' expresses a false proposition (for example, a proposition that entails that there is something that is identical with Santa Claus), so that again they do not express the same

[12] *Cf.*: "Mark Sainsbury has pointed out to me that certain forms of free logic must deny that extensional contexts are $+\daleth$–conv (i.e., must deny that both $\daleth$–INTR and $\daleth$–ELIM are valid in extensional contexts)." (Neale 2001, p. 180n4.)

proposition.[13, 14]

4 PROSPECTS OF GÖDELIAN SLINGSHOTS IN THREE DESCRIPTION CALCULI

4.1 RUSSELL'S THEORY OF DESCRIPTIONS

The slingshot of Section 1 is not sound for the language of Russell's Theory of Descriptions. Descriptions in this theory are, contrary to *premise* (ii) of that argument, not 'terms'.[15] Russell's Theory of Descriptions allows him to evade "Frege's puzzling conclusion" (Gödel 1944, p. 130), which Gödel implies is that true sentences all 'indicate' the same fact. "Russell avoids (this) 'Eleatic'

[13]But cannot both G1 and *premise* (iii) be rejected out of hand on the ground that, for example, "'Fa' expresses a monadic (proposition) (the attribution of the property F to a), and '$a = (℩x)(x = a.Fx)$' expresses a dyadic fact (the attribution of identity to the pair of a and a)" (Oppy 1997, p. 125)? Certainly these sentences differ syntactically: the first is a 'subject–predicate', and the second is an 'identity' sentence. It is, however, not clear that we should say that the propositions expressed by these sentences inherit these syntactical differences. Should we say that propositions expressed by 'P', '$(P \wedge P)$', and '$(P \vee P)$' inherit their syntactical differences?

[14]*Premise* (iii) leaves open the possibility that at least some sentences that feature improper definite descriptions in extensional contexts are best said not to express propositions (and so not to be true or false). *Premise* (i) is not similarly tolerant, but it can be modified to be so. For example, another valid Gödelian slingshot comes from mine by, for *one thing*, 'reducing' *premise* (i) to,

> The 'significations' of terms and sentences, that have significations, are, respectively, their denotations and the propositions they express, definite descriptions have denotations if and only if they are proper, and "the signification of a composite expression, containing constituents which have themselves a signification, depends only on the signification of these constituents" (Gödel 1944, p. 129). In particular, if a sentence φ' comes from φ by replacement of a term in an extensional position by a term that has the same denotation, or by replacement of a sentence in an extensional position by a sentence that expresses the same proposition, then, if φ expresses a proposition, φ' express the same proposition.

and, for a *second* thing, adding the premise,

> A sentence is true only if it expresses a proposition, and identity–sentences and their negations are true only if their constituent terms have denotations.

[15]Terms of standard formal languages are distinguished from formulas syntactically, and they are accorded different semantic treatments. The *R*–calculus of (Sobel 2006a, Chapter VIII) features Fregean descriptions such as '$℩xFx$' as well as Russellian descriptions such as '$℩xFx$'. Fregean descriptions occur in exactly the positions that can be occupied by name letters or individual constants. Russellian descriptions occur additionally braced in scope–indicators, for example, '$\{℩xFx\}$'. Interpretations of the language assign to closed Fregean descriptions elements of their domains as denotations. Not so for Russellian descriptions. Interpretations generate extensions not for Russellian descriptions, but for closed Russellian–description formulas such as '$\{℩xFx\}G℩xFx$' to which the truth values of related generalizations: the truth value assigned to '$\{℩xFx\}G℩xFx$' in an interpretations is the truth value assigned to '$\bigvee y(\bigwedge x(Fx \leftrightarrow x - y) \wedge Gy)$'. To the question, but what then does '$℩xFx$' *denote*, the answer is, "It does not denote anything. It does not have an extension. It is, in itself, 'semantically incomplete'."

conclusion because he is a Russellian about definite descriptions" (Neale 1995, p. 779; 2001, p. 133). He avoids 'Eleatic conclusions' that would have sentences that express truths express a *single* truth or fact, and sentences that express falsehoods express a *single* falsehood.

It has been suggested that, since, "when a predicate F applies exactly to one object, the (Russellian) description '$(⁊x)(Fx)$' can be treated for derivational purposes as if it were a singular term" (Rodriguez-Pereya 1998, p. 518), Gödelian slingshots can be modified to make irrelevant that Russellian descriptions are not terms.[16] The *question*, however, is whether when *premises* (iii) and (i) are modified to exploit the term–like behaviour in derivations of proper Russellian descriptions, they lose nothing in plausibility. The plausibility of *premise* (iii) does not suffer when it is so modified, but that of *premise* (i) does.[17] A simple modification to exploit the term–like behaviour of proper Russellian descriptions of the *R*–calculus of Chapter (VIII) in (Sobel 2006a) would, to *premise* (i),

> For sentences φ and φ' of L, if a φ' comes from φ by replacement of a term in an 'extensional position' by a term that has the same denotation, or by replacement of a sentence in an 'extensional position' by a sentence that expresses the same proposition, then φ and φ' express the same proposition.

add,

> If there is exactly one ψ and exactly one ψ', and the object that satisfies ψ is the object that satisfies ψ', in other words, if
>
> $$\{\dot{⁊}\alpha\psi\}\{\dot{⁊}\alpha\psi'\}⁊\alpha\psi = ⁊\alpha\psi',$$
>
> is true, and a sentence φ' comes from sentence φ by replacement of an occurrence of scope–indicator $\ulcorner\{\dot{⁊}\alpha\psi\}\urcorner$ in an extensional context and occurrences of $\ulcorner\{\dot{⁊}\alpha\psi\}\urcorner$ in its scope by an occurrence of $\ulcorner\{\dot{⁊}\alpha\psi\}\urcorner$ and occurrences of $\ulcorner ⁊\alpha\psi'\urcorner$; then φ and φ' express the same proposition. (I use '$\dot{⁊}$' for Russellian descriptions, and '$⁊$' for Fregean descriptions.)

This addition is implausible, and this for reasons that tell with equal force against more complicated modifications that restrict sentence φ to exactly the kinds to which the replacement–license would be addressed in the reasoning of Section 2, if it were adapted to $⁊$–descriptions. Consider that whether or not it is true that though

[16] Neale promises that this is so (2001, pp. 132–3), and with his "Complete Connective Proof" (2001, p. 173) shows that it is so.

[17] Neale criticizes his "Complete Connective Proof" similarly: (2001, pp. 174–5).

$$\{\daleth x Fx\}\{\daleth x Gx\}\daleth x Fx = \daleth x Gx,$$

under the scheme—M : a is the morning star; N : a is the evening star; V : a is Venus—expresses a truth; the propositions expressed by '$\{\daleth x Mx\}V\daleth x Mx$' and '$\{\daleth x Nx\}V\daleth x Nx$' have different implications. The first entails that there is exactly one morning star (i.e., that there is exactly one celestial body is salient in morning skies), and the second entails that there is exactly one evening star (i.e., that there is exactly one celestial body that is salient in evening skies). Since the propositions expressed actually have different *logical* properties, they are certainly distinct propositions. Frege would consider sufficient to the point, the possibility of their having different *epistemological* properties. *Cf.*:

> "(T)he thought in the sentence [18] 'The morning star is a body illuminated by the Sun' differs from that in the sentence 'The evening star is a body illuminated by the Sun'. Anybody who did not know that the evening star is the morning star might hold the one thought to be true, the other false." (Frege 1892 (1960), p. 62.)

His idea may have been that sentences φ and ψ express the same proposition only if the proposition expressed by their equivalence, $\ulcorner(\varphi \leftrightarrow \psi)\urcorner$ is analytically necessary: perhaps Frege would have been happy to add 'even if this person understood these sentences perfectly well'.

4.2 THE DESCRIPTION CALCULUS OF (KALISH, ET. AL., 1980, CHAPTER VI)

Premise (ii) is true of the language of this calculus: $\daleth$–descriptions are 'terms'. There would be trouble for *premise* (iii) for the language of this calculus, *if* it said, *without qualification*, that "'$\varphi(a)$' and the proposition 'a is the object which has the property φ and is identical to a' mean the same thing" (Neale quoting Gödel; 1995, p. 777; 2001, p. 130). As Neale observes, in any calculus for which, "in each model M, some arbitrary element $*_M$ (of its domain)... serves as the referent (in M) of all descriptions that are improper... (, there can be) a model M in which 'Fa' is false and the singular term 'a' refers to $*_M$... In

[18] Let 'a thought *of* a sentence' be not a thought about it, but something like an utterance of it made in thought. Let thoughts$_1$ be thoughts of propositions. These thoughts are 'necessarily in minds'. Let thoughts$_2$ be propositions. These thoughts are not necessarily in minds, ever. Frege's thoughts (*gedanken*) are ***not*** thoughts$_1$: A sentence is said to contain (*enhalten*) a thought (*gedanke*). In his commentary Montgomery Furth writes, "(a) 'thought' being 'not the subjective performance of thinking but its objective content, which is capable of being the common property of many thinkers' ('On Sense and Denotation,' n. 5)" (Gödel 1964, p. xxi, n. 20). Frege's words here quoted suggest that he intended *gedanken* to be propositions, and thus thoughts$_2$. His '*gedanke*' is sometimes translated 'proposition': *cf.*, page 89n5 of Herbert Feigl's translation in (Feigl and Sellars, 1949). However, Frege identifies the *gedanke* of a sentence with its 'sense', and propositions are not senses or meanings of sentences, since there are meaningful sentences that do not express propositions. In my view Frege meant by '*gedanke*' propositions, which he mistakenly identified with senses or meanings of sentences.

M('Fa') is false while ('$a = (\urcorner x)(x = a \cdot Fx)$') is true" (1995, p. 798; 2001, p. 194). The sentence of The Description Calculus,

$$\varphi_A \leftrightarrow A = \urcorner\alpha(\alpha = A \wedge \varphi_\alpha)$$

that corresponds to G1

$$\varphi_A \text{<expr>} A = \urcorner\alpha(\alpha = A \wedge \varphi_\alpha),$$

is not a theorem of this calculus. *Premise* (iii) for The Description Calculus,

$$\varphi_A \text{ is true} \rightarrow \varphi_A \text{<expr>} A = \urcorner\alpha(\alpha = A \wedge \varphi_\alpha),$$

does not have this problem. It corresponds to the theorem

$$\varphi_A \rightarrow (\varphi_A \leftrightarrow A = \urcorner\alpha(\alpha = A \wedge \varphi_\alpha)).$$

Premise (i), on the other hand, *is* problematic for The Description Calculus for reasons elaborated in Section 5.2 for Frege's likely resistance to Gödel's slingshot. Certainly names of the same thing can be freely exchanged without affecting the *truth–value* of an interpreted sentence of The Description Calculus. But *premise* (i) says that such replacements cannot affect the propositions (thoughts) expressed by the 'before and after sentences'. Given that 'names' in The Description Calculus include 'descriptive names', that principle is, to say the least, *prima facie* implausible.

4.3 'The Russellian Theory' of (Kalish, et. al. 1980, Chapter VIII)

Neale faults theories of description such as that of The Description Calculus a model M for which feature an "element $*_M$ in the domain (over which the variables of quantification range) (which) serves as the referent... of all descriptions that are improper..." The fault is that in "a model M in which 'Fa' is false and...'a' refers to $*_M$...('Fa') is false while ('$A = (\iota x x)(x = a.Fx)$') is true" (Neale 1995, p. 798; 2001, p. 195.) This at once frustrates Neale's Gödelian slingshot, which uses G1 and requires that sentences

$$\varphi_A \leftrightarrow A = \iota\alpha(\alpha = A \,\&\, \varphi_\alpha)$$

should be theorems, and, in Neale's opinion "demonstrates... just how bad a treatment (of descriptions The Description Calculus) is" (*ibid.*)

Neale writes that "(t)he problem just raised could be eradicated, of course, by a special stipulation to the effect that only those singular terms that are also descriptions can be assigned $*_M$ as their reference in M." (*Ibid.*) The 'Russellian Theory' of (Kalish, *et. al.*, 1980, Chapter VIII) does that and more. In it, descriptions are still terms, and descriptive names are said all to *designate* in a model whether or not they are proper, but if improper they designate a thing *that is not in the universe of the model that is the range of quantifiers*, they designate a thing that *does not exist* in the model. In this case, though they 'designate', they are said not *denote*, that being a relation that terms have only to *existing* things, that is, to elements of the universe of the model (Kalish, *et. al.*, 1980, p. 396). In this 'Russelian theory', every sentence,

$$\varphi_A \leftrightarrow A = 7\alpha(\alpha = A \wedge \varphi_\alpha),$$

is a theorem.

I use '7' for the descriptive terms of the calculus of Chapter VIII, and not '⌐' which is used for the descriptive terms of both that chapter and the descriptive terms of Chapter VI. Not every instance of a theorem of 'The Russellian Theory' is a a theorem. However, there is below a derivation for the sentence,

$$FA \leftrightarrow A = 7x(x = A \wedge Fx)$$

of a kind that ensures that every instance of this sentence is a theorem of 'The Russellian Theory'. The primitive rules of this theory come from those of The Description Calculus by restricting the rule of Identity to terms other than 7–descriptions, elevating Symmetry and derived forms of Leibniz's Law to the status of primitive rules, restricting UI and EG to terms other than 7–terms, * deleting the rules Proper Descriptions and Improper Descriptions for ⌐–descriptions, and adding, for '7', the rules:

RD(7) For distinct variables α and β and formula φ

$$/\therefore \quad \bigwedge \beta[\beta = 7\alpha\varphi] \leftrightarrow \bigwedge \alpha(\varphi \leftrightarrow \alpha = \beta);$$

DP–1 For k–place predicate letter π and terms $\tau_1, \ldots, \tau_k$,

$$\pi\tau_1 \ldots \tau_k/\therefore \quad \bigvee \alpha\alpha = \tau_1 \wedge \ldots \wedge \alpha\alpha = \tau_k;$$

and

DP–2 For terms γ and δ,

$$\gamma = \delta/\therefore \quad \alpha\alpha = \gamma.$$

These rules are said to "embody the initial Russellian intuition that subject–predicate sentences are false if the subject term lacks a denotation" (p. 400). A remarkable feature of this theory is that not every instance of a theorem is itself a theorem. (pp. 398–9 and 404.) However all instances of theorems derivable without recourse to DP–1 of DP–2 are themselves theorems (p. 404).. Now comes in Figure 1 such a derivation of '$FA \leftrightarrow A = 7x(x = A \wedge Fx)$'.

The Russellian Theory 'delivers' G1, if one assumes that logically equivalent sentences express the same proposition. In any case it is not inconsistent with

1.	~~SHOW~~ $FA \leftrightarrow A$	(24, DD)
2.	~~SHOW~~ $FA \rightarrow= 7x(A \wedge Fx)$	(14, CD)
3.	FA	(CD)
4.	~~SHOW~~ $x(x = A \wedge Fx \leftrightarrow x = A)$	(5, UD)
5.	~~SHOW~~ $x = A \wedge Fx \rightarrow x = A$	(7, CD)
6.	$x = A \wedge Fx$	(CD)
7.	$x = A$	7, S
8.	~~SHOW~~ $x = A \rightarrow x = A \wedge Fx$	(11, CD)
9.	$x = A$	(CD)
10.	Fx	3, 9, LL
11.	$x = A \wedge Fx$	9, 10, Adj
12.	$x = A \wedge Fx \leftrightarrow x = A$	5, 8, CB
13.	$y(y = 7x(x = A \wedge Fx) \leftrightarrow$	
	$x(x = A \wedge Fx \leftrightarrow x = y))$	RD(7)
14.	$A = 7x(x = A \wedge Fx)$	13, UI(A): p. 410,
		BC, 4, MP
15	~~SHOW~~ $A = 7x(x = A \wedge Fx) \rightarrow FA$	(23, CD)
16.	$A = 7x(x = A \wedge Fx)$	(CD)
17.	$7x(x = A \wedge Fx) = A$	16, Sym
18.	$yy = 7x(x = A \wedge Fx)$	
19.	$y(y = 7x(x = A \wedge Fx)$	
	$\leftrightarrow x(x = A \wedge Fx \leftrightarrow x = y))$	RD(7)
20.	$x(x = A \wedge Fx \leftrightarrow x = A)$	19, UI(A),[a] BC,
		16, MP
21.	$7x(x = A \wedge Fx) = A \wedge F7x(x = A \wedge Fx) \leftrightarrow$	18, 19, UID(7x
	$7x(x = A \wedge Fx) = A$	$(x = A \wedge Fx))$[b]
22.	$F7x(x = A \wedge Fx)$	21, BC, 17, MP, S
23.	FA	16, 22, LL
24.	$FA \leftrightarrow A = 7x(x = A \wedge Fx)$	2, 15, CB

[a] "Inference rules (include)... UI... restricted to variables and name letters" (Kalish, *et. al.*, 1980, p. 410).

[b] UID is 'Universal Instantiation for 7–descriptions'. It is a 'derivable rule'. "UID (is the inference license)...

$$\bigvee \gamma\gamma = 7\alpha\varphi. \bigwedge \beta\psi \therefore \psi'$$

...α, β, and γ are variables, φ is a symbolic formula in which γ is not free, and ψ′ is a symbolic formula that comes from...ψ by proper substitution of ⌜7αφ⌝ for β...(This rule) need not be adopted as primitive..." (Kalish, *et. al.*, 1980, pp. 399–400, with some notational adjustments.)

Figure 1

G1, as is any theory that says that sentences φ_A and $\ulcorner A = \mathrm{T}\alpha(\alpha = A \,\&\, \varphi_\alpha)\urcorner$ are not necessarily equivalent. And this theory agrees with *premise* (ii). Its treatment of descriptions is a bit closer to the ways of English descriptions than is that of The Description Calculus of Chapter VI, though it too 'goes weird' for improper descriptions, and lacks resources, as every Fregean treatment of descriptions must, for perspicuous representations of multiple interpretations of English sentences in which *scopes* of definite descriptions are indeterminate.[19] This theory has, incidentally, the dubious distinction (recently hinted) of having theorems not all instances of which are themselves theorems.[20] And, to wrap up its relation to our slingshot, this broadly Fregean theory is no better placed than The Description Calculus when it comes to *premise* (i) and the idea that exchanging descriptive names of the same thing in extensional contexts cannot affect what proposition is expressed.

5 FREGE'S ARGUMENT

5.1 FREGE'S ARGUMENT, IN PARTICULAR, ITS CONCLUSION IS NOT AS GÖDEL WOULD HAVE IT BE

> "(A)ccording to Russell's terminology and view, true sentences 'indicate' facts... But (on Russell's view) different true sentences may indicate many different (facts)." (Gödel 1944, p. 129.) "Frege's puzzling conclusion has only been evaded by Russell's theory of descriptions ... " (P. 130.)

But what, for Gödel, *was* 'Frege's puzzling conclusion'? It would be that all true sentences signify the same fact or true proposition. That, however, is not the conclusion that Frege drew in the passage that Gödel 'taps'. Frege's own conclusion was neither that true sentences all have the same meaning, nor that they all express the same thought or proposition. Nor was it that true sentences all have the same truth–value, for which conclusion no argument is called for. Frege's conclusion was that, casting sentences as names, this truth–value of true sentences can be said to be what they name. Frege took for

[19]This is the most important shortcoming of Fregean treatments of descriptions. "Confusion of primary and secondary occurrences (of descriptions) is a ready source of fallacies where descriptions are concerned." (Russell, 1919, p. 179.) Far from serving resolutions of these confusions, formal analyses in Fregean theories can enhance them. This is illustrated for a 'liar paradox' in (Sobel 2006b).

[20]"Within our Russellian theory (as in Russell's theory itself), every formula that contains descriptive terms is equivalent to a formula in which no descriptive term occurs, and T512

$$G\gamma x Fx \leftrightarrow \bigvee y(\bigwedge x(Fx \leftrightarrow x = y) \wedge Gy)$$

is the guide to be employed in finding such a formula." (Kalish, *et. al.*, 1980, p. 405.) It is not, however, the simple a guide one might guess it to be, since not every instance of T512 is a theorem (pp. 398–9 and 404).

granted that true sentences can differ in 'sense', and in the 'thoughts' or propositions they 'have' or that they express (though he does not insist on a distinction between meanings or senses of sentences, and thoughts they express). It is only in what he likes to think of as their 'extensions' or 'denotations' that he says the true ones are all alike in having for their extension or denotations The True, and that the false ones are all alike in having for this The False.

> *Cf.*: "Frege takes sentences as names (which is consistent with their also being 'vehicles for thoughts'), and adopts the compositionality principle, according to which when a constituent name is replaced by another having the same denotation, the denotation of the entire name is not changed. Relying on these views, Frege claims that all true sentences have the same denotation, THE TRUE, and all false sentences also have the same denotation, THE FALSE." (Lee 2002, p. 543.)

If sentences are cast as names, as it may be convenient to do for theories of artificial languages, and 'extensional' names are 'compositional' which seems necessary, then it seems that sentences must name their truth–values. *Cf.*: " '(w)hat feature except the truth–value can be found that belongs to... sentences quite generally and remains unchanged by substitutions (of co-referential parts)?' (Frege 1960, pp. 64–5)."

5.2 FREGE WOULD, I THINK, POINT THE FINGER NOT AT PREMISE (II), BUT AT PREMISE (I)

Gödel's slingshot purports to show that true sentences not only, *qua* names, name their shared truth–value, but that they *qua* indicators all indicate or express the same truth or fact.[21] That is *contrary* to Frege's view of sentences and their 'thoughts'. How might he meet the contrary challenge of Gödel's slingshot? My guess is that he would not give up the idea that 'closed' definite descriptions are names, and that he would agree that Russell's Theory of Descriptions merely evades what he, Frege, might think of as the momentarily puzzling conclusion of '*Gödel's Fregean argument*' (!) according to which there is exactly one true proposition or fact. To prevent the collapse of all true propositions into a single one, Frege would, I think, rest with a *rejection* of the implication of *premise* (i) that the thought or proposition expressed by a true sentence, in so far as this thought or proposition depends on constituents of this sentence that have denotations, "depends only on (their denotations)... (not on the manner in which this (denotation) is expressed)" (Gödel 1944, p. 128). Rejecting that would allow him to say that the thought or proposition,

[21]Gödel does not, in his own slingshot, identify 'facts' with 'true propositions'. He, for the most part (an exception is in p. 129n7), leaves propositions out of his discussion, and confines himself to 'facts' which he casts as the correlates of the 'senses', for Frege the thoughts or propositions, of true sentences.

that the University of Toronto is the largest university in Toronto,

is different from the thought or proposition,

that the University of Toronto is the oldest university in Toronto,

while accepting that 'the largest university in Toronto' and 'the oldest university in Toronto' are names that denote the same thing. Frege could add positively that thoughts of sentences, in so far as they depend on constituents that have denotations, depend on the *senses* of these constituents, and the manner in which they present their denotations.

5.3 GÖDEL'S ERROR

The error, if I may say so, of *Gödel's* slingshot, which was *not* 'deep'. It was facilitated by his decision to run together the relation of names to objects they 'denote', and that of true sentences to the facts they, in Russell's term, 'indicate'. He did this because he judged "that 'denote' and 'indicate' (in Russell's sense) together correspond to Frege's '*bedeuten*'" (Gödel 1944, p. 129), which, Gödel tells us corresponds to 'signify' (p. 128n). But the relation of true sentences to facts they 'indicate', and more generally of sentences to propositions they 'indicate', is *different* from that of names to objects they 'denote' or name. We *have* 'names' in English for propositions, for example, "[1] 'Logicism', (2) 'the proposition that mathematics reduces to logic', and [3] 'that mathematics reduces to logic'" (King 2002, p. 341) are names of a single proposition. But *sentences* are *not* names of propositions: for example, 'Mathematics reduces to logic.' is a not a *name* of the proposition that mathematics reduces to logic, but a sentence with which this proposition can be expressed. 'Mathematics reduces to logic *is true*' is not 'good English'; "Mathematics reduces to logic' is true' is 'good philosophic English' only in contexts for which predication of truth to sentences has been explained.

I doubt that Russell ever thought of sentences as names. Frege knew that in doing so he was exercising a kind of 'license'. He wrote, "Let us for the moment assume that the sentence has a nominatum!" (Frege 1892 (1949), pp. 89–90, exclamation mark original. The 1960 translation of (Frege 1892) does not end this sentence with an exclamation mark.) Frege's own argument was *not* a 'collapsing argument' with an 'almost metaphysical, Eleatic conclusion', but a way of settling, for a theory that *lets* sentences have *nominata*, what the *nominata* of sentences must be, and then stipulating names for these *nominata*. The *nominata* of sentences cannot be their thoughts or propositions. They must be their conditions of truth and falsity. "For brevity I call... one the True, the other the False" (Frege 1892 (1960), p. 63). Why not? No cause for alarm, or excitement.[22]

[22] *Cf.*: "(W)ith Frege... we DECLARE all true sentences to denote... truth, and all false sentences to denote... falsehood." (Church 1958, p. 25.) Church writes that he *postulates* truth

Gödel wrote, "I cannot help feeling that the problem raised by Frege's puzzling conclusion has only been evaded by Russell's theory of descriptions and that there is something behind it which is not yet completely understood." (Gödel 1944, p. 130.) He was right about that in more ways than he suspected, the first one of which is that what Gödel considered to be 'Frege's puzzling conclusion' was not a conclusion drawn by Frege, or one to which Frege was committed. One may wonder, incidentally, why Gödel considered the conclusion of his own argument to tease Russell, that there is just one Truth or Fact, and just one Falsehood, puzzling. There is evidence that at least in 1956 he was himself committed to the view that true propositions are all ***necessarily equivalent***: in a note toward an 'ontological proof of the necessary existence of God' Gödel wrote that, given that a thing's 'essence' is its 'complete individual property', "$\varphi(x) \supset N\varphi(x)$" (Gödel 1995, p. 435).[23] This together with his view "that every proposition 'speaks about something,' i.e., can be brought to the form $\varphi(a)$" (Gödel 1944, p. 129n), entails that, for any true sentences φ and ψ, $\ulcorner\Box(\varphi \leftrightarrow \psi)\urcorner$ is a true sentence, so that φ and ψ express necessarily equivalent propositions. That is near enough to 'the puzzling conclusion' that true sentences express the same proposition as to make hardly any difference.

6 DAVIDSONIAN COLLAPSING ARGUMENTS

> "The proof implicit in Gödel's (1944) paper is certain to call to mind a better known proof that appears explicitly in Church's (1943) review of Carnap's (1942) book *Introduction to Semantics*[24] Ver-

and falsehood, while noting that "Frege, as a thoroughgoing Platonic realist... would... say that... *there are two such things*" (p. 25). I am on Frege's side here.

[23]This may be compared to the threat to free will that Leibniz perhaps suspected was made by his theory of 'complete individual concepts' for names of individuals that all truths about individuals are 'analytic' and necessary. (*Cf.*, Sobel 2004, Chapter VI, Appendix A).

[24]Church does not argue in (1943) *that* true sentences all designate the same proposition, and he says nothing of facts in (1943). He does not run a 'categorical collapsing argument' of any kind. He offers only a 'hypothetical collapsing argument' against Carnap's assumption that "the designata of sentences are propositions" (Church 1943, p. 299). He writes that "it is possible to prove that the designata of sentences... must be truth–values rather than propositions" (p. 299). Church explains how, by Carnap's own principles for 'synonymity' in the sense of 'co-designating'—the principles that "synonymous (co-designating) expressions are interchangeable," and that "L–equivalent sentences are synonymous (co-designating)"—*if* sentences designated propositions, any true sentences φ and ψ would designate the same proposition, even if "one sentence is L–true and the other not," whereas at least in that case propositions corresponding to φ and ψ "are certainly not the same for any ordinary meaning of the word 'proposition'" (p. 300). With Frege, Church says that sentences do not designate, but *express*, propositions.

Church implies that Carnap made trouble for himself by "allowing (in his semantical theory) only one kind of meaning" (p. 302) and not, as Frege does, distinguishing the *sense* of a sentence that determines the proposition it expresses, from the *designatum* of a sentence which, assuming "common properties" for a language (p. 299), must be its truth–value. Gödel has this end note to his paper: "I wish to express my thanks to Professor Alonzo Church... who helped me to find the correct English expressions in a number of places." (Gödel 1944, p. 153.) It is remarkable sions of Church's proof have been deployed by Quine and Davidson to various philosophical ends, and discussed widely in the literature. By contrast, the literature contains relatively few discussions of Gödel's proof, which is interesting because Gödel's premises are weaker... The basic difference is that Church, Quine, and Davidson draw upon purported *logical equivalences*, such as... between.... $\varphi \ldots a = (\iota x)(x = a \cdot \varphi) \ldots (\iota x)(x = a) = (\iota x)(x = a \cdot \varphi)$. The net effect of this is that the Church–Quine–Davidson slingshot makes use of a more contentious substitution principle." [25] (Neale 1995, p. 791; *cf.*, 2001, pp. 166–7.)

6.1 AN ARGUMENT 'AFTER' DAVIDSON

The argument of this section is addressed to 'The Russellian Theory of Descriptions' of (Kalish, *et. al.*, 1980, Chapter VIII) that "considers false any subject–predicate sentence whose subject term is an improper definite description" (p. 395). In this theory definite descriptions are terms, and in interpretations improper descriptions all 'designate' some one thing that is "*not* in the universe of the model" (p. 395). Let a 'true sentence' of The Russellian Theory under an interpretation for the language of this calculus, be a sentence that under that interpretation expresses a true proposition. Understand 'false sentence' of this calculus under an interpretation similarly. The coming Davidsonian argument for the conclusion that all true sentences of The Russellian Theory under an interpretation express the same true proposition, and that all 'false sentences' express the same false proposition, uses, in place of ***premise***

that Gödel was not persuaded by Church not to use 'signify' as he does to cover both 'denote' or 'designate', and 'express' or 'indicate'.

[25] It is, for several reasons, at best misleading to say that Gödelian slingshots use weaker premises. First, though "Church, Quine, and Davidson draw upon purported *logical equivalences*" (*ibid.*), they need not have. The Davidsonian argument coming 'draws from logical equivalences' only that, in The Russellian Theory of (Kalish, *et. al.*, 1980, Chapter VIII), for any sentence φ and term A, $\ulcorner 7\alpha(\alpha = A) = 7\alpha(\alpha = A \wedge \varphi)\urcorner$ express the same proposition (please see lines (g) and (h) of the argument). The argument could have taken as premises these same–proposition claims. The argument does ***not*** draw on the logical equivalence in The Russellian Theory of φ and $\ulcorner A = 7\alpha(\alpha = A \wedge \varphi)\urcorner$.

Second, Neale's reconstruction of Gödel's argument uses

$$\text{G1}\ \varphi_A <\text{expr}> A = \mathsf{T}\alpha(\alpha = A \,\&\, \varphi_\alpha).$$

My Gödelian slingshot uses *Premise* (iii)

$$\varphi_A \text{ is true} \rightarrow \varphi_A <\text{expr}> A = \mathsf{T}\alpha(\alpha = A \,\&\, \varphi_\alpha).$$

Premise (iii) is not a special case of "The *Principle of Substituvity for Logical Equivalences* (PSLE) " (Neale 1995, p. 792, italics original). G1 is a special case of PSLE for some interpretations of 'T' (e.g., 'the', 'ד', and '7'), but not for all (it is not for 'ד'). *Cf.*: "Gödel produces a Slingshot whose premises are quite different from—and PERHAPS in some sense weaker than—those used by Quine, Church, and Davidson." (Oppy 1997, p. 127, small caps emphasis added.)

(iii) addressed to The Russellian Theory, the principle that logically equivalent RT–sentences express, under an interpretation, the same proposition—let this principle be LgEquv/SameProp. The argument does not need a premise to the affect that, for every sentence, there is a 'subject–predicate' sentence that expresses the same proposition. Reasoning to the first conjunct of the conclusion can be as follows. For RT–sentences φ and ψ, and RT–name A that is not a ℩–description, we assume that,

(a) φ is true

(b) ψ is true

There are the following forms of theorems in RT,

(c) $\varphi \leftrightarrow ℩\alpha(\alpha = A) = ℩\alpha(\alpha = A \wedge \varphi)$, and

(d) $\psi \leftrightarrow ℩\alpha(\alpha = A) = ℩\alpha(\alpha = A \wedge \psi)$.

(See proof in Figure 2.) It follows from (c) and (d) being forms of theorems that

(e) φ is logically equivalent to $\ulcorner ℩\alpha(\alpha = A) = ℩\alpha(\alpha = A \wedge \varphi)\urcorner$, and

(f) ψ is logically equivalent to $\ulcorner ℩\alpha(\alpha = A) = ℩\alpha(\alpha = A \wedge \psi)\urcorner$

From these logical equivalences by LgEquv/SameProp, it follows that

(g) φ <expr> $℩\alpha(\alpha = A) = ℩\alpha(\alpha = A \wedge \varphi)$[26]

(h) ψ <expr> $℩\alpha(\alpha = A) = ℩\alpha(\alpha = A \wedge \psi)$

It follows from (a) and (b), and (c) and (d) that $\ulcorner ℩\alpha(\alpha = A \wedge \varphi)\urcorner$ and $\ulcorner ℩\alpha(\alpha = A \wedge \psi)\urcorner$ are proper for the same thing,

(j) $℩\alpha(\alpha = A \wedge \varphi)$ <expr> $℩\alpha(\alpha = A \wedge \psi)$

It follows from (j), *by premise* (i) of my Gödelian slingshot, that

(k) $℩\alpha(\alpha = A) = ℩\alpha(\alpha = A \wedge \varphi)$ <expr> $℩\alpha(\alpha = A) = ℩\alpha(\alpha = A \wedge \psi)$

[26] *Cf.*, Neale's G1:

φ_A <expr> $A = T\alpha(\alpha = A \,\&\, \varphi_\alpha)$,

α a variable *that is free in* φ_α, A a term, and φ_A sentence that comes from φ_α by proper substitution of A for α. The italicized difference explains how this Davidsonian argument is, without aid of Gödel's assumption that every sentence 'means the same' as some subject–predicate sentence, general for all sentences.

Adapting a line from (Neale 1995, p. 793; *cf.*, 2001, p. 171): "It will not do to object... that ($\ulcorner ℩\alpha(\alpha = A \wedge \varphi)\urcorner$ in line (g)) is not well–formed or not interpretable unless φ contains an occurrence of α that ($\ulcorner ℩\alpha\urcorner$) can bind." There is no such restriction on ℩–terms in The Russellian Theory of (Kalish, *et. al.*, 1980, Chapter VIII).

All sentences of the forms (c) and (d) are instances of '$(P \leftrightarrow \iota x(x = A) = \iota x(x = A \wedge P))$', which theorem has the following derivation in which neither DP–1 nor DP–2 are used. As said in Section 4.3 above, all instances of theorems of this 'Russellian calculus' that have such derivations are themselves theorems.

1.	~~SHOW~~ $P \leftrightarrow \iota x(x = A) = \iota x(x = A \wedge P)$	$(31,$ DD$)$
2.	~~SHOW~~ $P \rightarrow \iota x(x = A) = \iota x(x = A \wedge P)$	$(19,$ CD$)$
3.	P	(CD)
4.	$A = \iota x(x = A) \leftrightarrow x(x = A \leftrightarrow x = A)$	RD*(7): Logic, 400,* UI
5.	~~SHOW~~ $x(x = A \leftrightarrow x = A)$	$(7,$ UD$)$
6.		
7.	$x = A \leftrightarrow x = A$	T91
8.	$A = \iota x(x = A)$	4, BC, 5, MP
9.	$A = \iota x(x = A \wedge P) \leftrightarrow x(x = A \wedge P \leftrightarrow x = A)$	RD*(7): Logic, 400,* UI
10.	~~SHOW~~ $x(x = A \wedge P \leftrightarrow x = A)$	$(17,$ UD$)$
11.	~~SHOW~~ $x = A \wedge P \rightarrow x = A$	$(13,$ CD$)$
12.	$x = A \wedge P$	(CD)
13.	$x = A$	12, S
14.	~~SHOW~~ $x = A \rightarrow x = A \wedge P$	$(16,$ CD$)$
15.	$x = A$	(CD)
16.	$x = A \wedge P$	3, 15, Adj
17.	$x = A \wedge P \leftrightarrow x = A$	11, 14, CB
18.	$A = \iota x(x = A \wedge P)$	9, BC, 10, MP
19.	$\iota x(x = A) = \iota x(x = A \wedge P)$	8, 18, T
20.	~~SHOW~~ $\iota x(x = A) = \iota x(x = A \wedge P) \rightarrow P$	$(30,$ CD$)$
21.	$\iota x(x = A) = \iota x(x = A \wedge P)$	(CD)
22.	~~SHOW~~ $x(x = A \leftrightarrow x = A)$	$(23,$ UD$)$
23.	$x = A \leftrightarrow x = A$	T91
24.	$\bigvee y \bigwedge x(x = A \leftrightarrow x = y)$	22, EG
25.	$yy = \iota x(x = A) \leftrightarrow \bigvee y \bigwedge x(x = A \leftrightarrow x = y)$ p. 401; derived without aid of DP-1 or DP-2	*T500 : Logic*,
26.	$yy = \iota x(x = A)$	25, BC, 24, MP
27.	$yy = \iota x(x = A \wedge P)$	26, 21, EGD*: Logic*, p. 399
28.	$\bigvee y \bigwedge x(x = A \wedge P \leftrightarrow x = y)$	T500, BC, 27, MP
29.	$x(x = A \wedge P \leftrightarrow x = a)$	28, EI
30.	P	29, UI, BC, Id, MP
31.	$P \leftrightarrow \iota x(x = A) = \iota x(x = A \wedge P)$	2, 20, BC

Figure 2

From (g), (h), and (k), it follows by *premise* (iii) (that <expr> is an equivalence relation), that

(l) φ <expr> ψ.

For the second conjunct of the conclusion, we assume,

(m) φ is false

(n) ψ is false

It follows that

(o) $\ulcorner{\sim}\varphi\urcorner$ is true

(p) $\ulcorner{\sim}\psi\urcorner$ is true

It then follows as above that

(q) $\sim\varphi$<expr>$\sim\psi$

from which it follows from (q), by *premise* (iii) and *premise* (i), that

(r) $\sim\sim\varphi$<expr>$\sim\sim\psi$,

and from (r), by *premise* (v), that

(s) φ <expr> ψ.

The argument cannot be addressed to The Descriptions Calculus (Kalish, *et. al.*, 1980, Chapter VIII), since ℩–analogues of the schemata on (c)) and (d) are not all theorems. (The sentence '$P \leftrightarrow \text{℩}x(x = A) = \text{℩}x(x = A \land P)$' is false in the model, U :$\{0, 1\}$; Improper designatum: 1; P : false; A : 1.) The argument *can* be addressed to theories for denoting terms that feature an operator for class–abstraction terms: λ–analogues of (c)) and (d) are theorems of such theories.[27]

6.2 ANOTHER DAVIDSONIAN ARGUMENT

Neale presents an argument (1995, p. 793)[28] from which can be gathered a Davidsonian argument to show, without using anything like *premise* (v), that sentences that have the same truth–value express the same proposition.

[27] Consider the sentence '$P \leftrightarrow \lambda x(x = A) = \lambda x(x = A \land P)$'. Suppose '$P$' is true. Then, necessarily, the terms '$\lambda x(x = A)$' and '$\lambda x(x = A \land P)$' both denote the singleton $\{A\}$, so that '$\lambda x(x = A) = \lambda x(x = A \land P)$' is also true, and the biconditional is true. Suppose 'P' is false, then '$\lambda x(x = A) = \lambda x(x = A \land P)$' is also false, so that in this case too the biconditional is true. The identity is false in this case since in it, while '$\lambda x(x = A)$' still denotes $\{A\}$, '$\lambda x(x = A \land P)$' denotes the null–class $\{\}$.

[28] I believe it is the argument *intended* on (2001, p. 172). The argument actually produced on that page is the *same* as the argument on the next page that is supposed to solve a problem for it.

6.2.1 THE ARGUMENT

The following Davidsonian argument that I draw from (1995, p. 793) uses, in addition to premises that can be gathered from annotations, the premise,

for any sentence χ that has a truth–value and term A that has a denotation

χ is logically equivalent to $\ulcorner T\alpha(\alpha = A) = T\alpha(\alpha = A \,\&\, \chi)\urcorner$

Let that be *premise* X. We assume, for argument, that

(1) φ and ψ have the same truth–value.

Now to show that φ and ψ express the same proposition.

(2) φ <expr> $T\alpha(\alpha = A) = T\alpha(\alpha = A \,\&\, \varphi)$.

premise X, LgEquv/SameProp

(3) $T\alpha(\alpha = A \,\&\, \varphi)$ <expr> $T\alpha(\alpha = A \,\&\, \psi)$. from (1)

(4) $T\alpha(\alpha = A) = T\alpha(\alpha = A \,\&\, \varphi)$ *premise*, 3, *premise* (i)

<expr> $T\alpha(\alpha = A) = T\alpha(\alpha = A \,\&\, \psi)$.

(5) ψ <expr> $T\alpha(\alpha = A) = T\alpha(\alpha = A \,\&\, \psi)$.

premise X, LgEquv/SameProp

(6) φ <expr> ψ. 3, 4, 5, *premise* (iii) (transitivity)

A somewhat *leaner* argument uses, instead of LgEquv/SameProp and *premise* X, the premise—let it be *premise* (iii)*—that, for any sentence χ that has a truth–value, and any term A that has a denotation,

χ <expr>$\ulcorner T\alpha(\alpha = A) = T\alpha(\alpha = A \,\&\, \chi)\urcorner$.

A *less ambitious* argument could proceed without LgEquv/SameProp from *premise* X to the still startling conclusion that sentences that are materially equivalent are all logically equivalent. Inferences, in this argument, from *premise* X would be to 'demotions' of (2) and (5) from statements of propositional identities, to statements of logical equivalences. The inference from (3) would be to a similar demotion of (4).

6.2.2 RELATIONS OF SEVERAL FREGEAN THEORIES OF DESCRIPTIONS TO THIS ARGUMENT

While the specification of *premise* X to 7–descriptions of the 'Russellian Theory' is true, the inference from (1) to (3) is not valid for these descriptions (*cf.*: Kalish, *et. al.*, 1980; T504 on p. 402). It is the other way for the ℩–descriptions of the Descriptions Calculus of (Kalish, *et. al.*, 1980). While the inference from (1) to (3) is valid for the ℩–descriptions (*cf.*, Kalish, *et. al.*, 1980; T404 on p. 322), the specification of *premise* X to ℩–descriptions is false. "Frege (1893 (*Die Grundgesetze der Arithmetik*)) suggests an alternative treatment according to which an improper description refers to *the class of entities satisfying its matrix*." (1995, p. 797; 2001, p. 193): on this treatment, '$\mathsf{T}xx = 1$' refers to 1, '$\mathsf{T}x(x$ is 1 or $2)$' refers to $\{1, 2\}$, and '$\mathsf{T}x(x$ is 1 and $2)$' refers to $\{\}$. The argument is valid for this treatment: *Premise* X, specified for this treatment, is true. On this treatment, if X is false, then, '$\mathsf{T}xx = A$' refers to A, '$\mathsf{T}x(x = A \,\&\, X)$' refers to the null–class $\{\}$, and the identity sentence '$\mathsf{T}xx = A = \mathsf{T}x(x = A \,\&\, X)$' is also false; if X is true, then it is also true. And the inference from (1) to (3) is valid on this treatment.

Neale says of the argument that I have mined for this Davidsonian argument, that it is valid if descriptions in it are given a non–term Russellian analysis (1995, p. 794). However, while an adaptation of *premise* X to ℩–descriptions (complete with scope–indicators) is true, the inference from (1) to an adaptation of (3) to ℩–descriptions is not valid: for example, if 'P' and 'Q' are both false, then (1) is true of them; but the specification of (3) to these sentences and a term 'A', '$\{\text{℩}x(x = A \wedge P)\}\{\text{℩}x(x = A \wedge Q)\}\text{℩}x(x = A \wedge P) = \text{℩}x(x = A \wedge Q)$, is (doubly) false. This seems to be noticed and remedied in (2001) by a 'cumbersome method' due to Quine that results in an argument that is valid for ℩–descriptions (2001,p. 173). My current Davidsonian collapsing argument, similarly encumbered, would have in place of '$\mathsf{T}\alpha(\alpha = A \,\& \cdot \chi)$' the necessarily proper description '$\mathsf{T}\alpha((\alpha = A \,\&\, \chi) \vee (\alpha = B \,\&\, \sim\chi))$. Encumbered it would be ('dressed out' appropriately with scope–indicators) valid for ℩–descriptions. It would also valid for the ℩–descriptions of (Kalish, *et. al.*, 1980, Chapter VI), and for those of the 'alternative treatment of (Frege 1893)'; 'for what these Fregean theories are worth', given that they (as ***all*** Fregean theories of descriptions as terms that must have referents)[29] are non-starters as accounts of the "treatment of descriptions in natural language" (2001, p. 194).[30]

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[29]Strawsonian theories treat definite descriptions as terms that, when improper, lack references. A good one would be worth something. Though, if it 'had no use for scope–indicators', it would not be a fully adequate account of 'descriptions in natural language'. *Some* of Neale's 'counter–examples' to Strawsonian theories as "account(s) of descriptions in natural language" might be parried by a Strawsonian theory that could differentiate possible scopes of definite descriptions, and say that whether an improper descriptions "renders the proposition false. . . (or) prevents a proposition from being expressed. . . (*did*) turn on structural or logical facts about the sentence used" (1995, pp. 802–3; 2001, pp. 199-201).

[30]According to this 'alternative treatment', the author of (Kalish, *et. al.*, 1980) is the class

The inference from (1) to (3) in my unencumbered Davidsonian argument is, as it happens, valid for the class–abstractions of Church's argument: for example, if 'P' and 'Q' are true, then '$\lambda x(x = A \wedge P)$' and '$\lambda x(x = A \wedge Q)$' both denote $\{A\}$; and if 'P' and 'Q' are false, these λ–terms denote $\{\}$. Furthermore, ***premise*** X is true for λ–terms. These terms—these 'specialized definite descriptions'[31]—have, for this Davidsonian collapsing argument, one advantage or another over the descriptive terms of The Description Calculus and The Russellian Theory of Descriptions of (Kalish, *et. al.*, 1980), and an advantage over the non–term ℩–descriptions of Russell's Theory of Descriptions.

6.3 DAVIDSON'S OWN ARGUMENT

Davidson does not argue in (1969) for a conclusion concerning sentences and propositions. He argues against the utility of facts for a theory that would explain truth. He argues that we are unlikely to "find (a) way to pick out facts" that, (i), does not "distinguish facts as finely as statements," and, (ii), does not, for any sentences φ and ψ, identify the fact denoted by $\ulcorner$the fact that $\varphi\urcorner$ with the fact denoted by $\ulcorner$the fact that $\psi\urcorner$ when either φ and ψ are logically equivalent, or φ and ψ differ from one another only "in that a singular term (in φ) has been replaced by a coextensive singular term," for if this distjunctive identity–condition holds for facts, then there are not several facts, but only a single Great Fact. (Davidson 1969, pp. 752–3.) Davidson says that "we cannot hope to explain truth by appeal to (facts)" (p. 753), ***unless*** we can find a way between these extremes of a one–to–one correspondence of true statements and facts, and a many–one relation. He thinks there is no middle way, and that "(t)alk about facts reduces to predication of truth" (*ibid.*). His argument against facts of use in a 'theory of truth' is 'guardedly categorical'.

Davidson's 'statements' are, I think, my 'propositions'. Since I identify facts with true propositions, I distinguish them exactly as finely as propositions. Furthermore I agree that "we cannot hope to explain truth (of propositions) by appeal to (facts)" (*ibid.*),[32] and I do 'reduce' classifications of propositions as facts, and predications of truth to propositions, one to the other.

{Donald Kalish, Richard Montague, Gary Mar}, and the author of (Kalish and Montague, 1964) is a proper sub–class of it.

[31] A literal translation of '$\lambda x Fx$' can be 'the class such that an object x is a member of this class if and only if Fx'. In a first order theory for classes that features logical predicates 'κ' and 'ε' for 'A is a class' and 'A is an element of *b*', the λ–operator could be defined thus: for variables α and β and formula φ,

$$\lambda\alpha\varphi = \text{℩}\alpha(\kappa\alpha \wedge \beta(\beta\varepsilon\alpha \leftrightarrow \varphi)).$$

[32] I agree with this, though not because I distinguish facts exactly as finely as true propositions, but because I *identify* them with true propositions. Actual 'states of things', named not by 'that'–clauses but by gerundives such as 'snow's being white', provided they ***are*** distinguished exactly as finely as are true propositions, and correspond to them one–to–one, are I think 'what in the world make propositions true'. I do not understand why Davidson thought that an explanation of truth in terms of 'facts' would need ***not*** to distinguish facts exactly as finely as true

Davidson does not suggest that the plurality of true statements is threatened, or that there is a single Great Truth. He says that "there are very strong reasons, as Frege pointed out, for supposing that if sentences, when standing alone or in truth–functional contexts, name anything, then all true sentences name the same thing" (1969, p. 750). He does not say that there any reasons, let alone strong ones, for supposing that sentences name statements, that in particular, a sentence φ not only *expresses* the proposition named by $\ulcorner$the statement that $\varphi\urcorner$, but *names* it.

7 MONTY AND ME, AND CHURCH'S λ–SLINGSHOT

7.1 BRINGING CHURCH AGAINST A PRINCIPLE OF IDENTITY FOR SUBJECT–PREDICATE PROPOSITIONS

The Davidsonian 'categorical' collapsing argument' of Section 6.1 for the conclusion that sentences of 'The Russellian Theory of Descriptions' of (Kalish, *et. al.*, 1980, Chapter VIII) that are true under an interpretation all express under it the same true proposition, and that 'false sentences' express a single false proposition, uses *premise* (i). These arguments suffer thereby for a reason developed in Section 5.2: *premise* (i) is untenable for languages in which definite descriptions are terms (and indeed, Section 4.1, not only for languages in which they are not terms). Related 'non-categorical' arguments of some philosophic interest do not have this problem, for they would be *reductios* of 'theories' of propositions that include the objectionable principles of *premise* (i).

Church runs against Carnap's theory of propositions as *designata* of sentences a non-categorical argument that takes ideas from Frege's identification of truth–values as the denotations of sentences. Monty Furth pummelled me with something like this argument at U.C.L.A. one day in 1964 when I confessed affection for propositions. He pressed the issue of when propositions are the same, and when they are different; and I suggested, for openers, that propositions expressed by 'subject–predicate' English sentences that 'say the same thing about the same thing' are identical. That condition is, in 'material mode', part of *premise* (i) of the slingshot in Section 1. He 'called time' to get notes from his office that he had taken at recent lectures given by Church. With these in hand, Monty made an argument to show that the theory I had floated would have true 'subject–predicate' sentences expressing the same proposition, assuming that logically equivalent proposition express the same proposition (otherwise they express logically equivalent propositions), and similarly for false 'subject–predicate' proposition. The argument of Section 6.1 above can be adapted to make his point. For this adaptation, assume

statements, *unless*, after all, his statements were sentences nor propositions. Propositions, the primary bearers of truth–values, are presumably not distinguished exactly as finely as sentences. They are not. if it is sometimes possible, when we say something, to say not merely something logically equivalent, but the very same thing, in other words.

that 'φ' and 'ψ' symbolize 'subject–predicate' sentences. The inference to (k) by *premise* (i), is in this adaptation by the propositional–identity condition that I floated that day. The identity on line (j) entails that the propositions expressed by the sentences on line (k) 'say the same thing about the same thing' albeit under different names for it: each says of the same thing, $℩\alpha(a = A \wedge \varphi)$ or $℩\alpha(\alpha = A \wedge \varphi)$, that it is $℩\alpha(\alpha = A)$, the thing that is A.

7.2 BACK TO CHURCH AND CLASS–ABSTRACTIONS

Monty's argument actually ran in terms not of generic definite descriptions but specifically of the class–abstractions Church had used in his criticism of Carnap's theory of propositions as designata of sentences. Neale says it is "(a) superficial difference between the arguments of Church and Gödel... that Church uses the abstraction operator '(λx)'—where '$(\lambda x)\varphi$' is read as 'the class of all x such that φ'—while Gödel (implicitly) uses the definite description operator '(℩x)'." (1995, pp. 791–2n30; 2001, p. 166.) English 'class–abstractions' are evidently definite descriptions of a particular form, and they share with definite descriptions of all forms the *possibility* of impropriety. Though the general run of English class–abstractions are necessarily proper descriptions, thanks to the availability of the null–class as the class that is uniquely described;[33] that Mars has a moon. And it is necessary that the proposition that Mars has a moon is either true or false. there are a few 'anomalous' ('aberrant', 'paradoxical') class–abstractions in currency that are improper: for example, 'the class of classes that are not members of themselves' is a necessarily improper 'class–abstraction'; it is demonstrable that there is no such class.[34] Still, while the differences between the arguments of Church and Gödel are ***in a limited sense*** superficial, they are, contrary to Neale, not without dialectical signifi-

[33] Assume that 'A' abbreviates '1' and 'P' abbreviates 'Mars has a moon'. Then '$\lambda x(x = A \wedge P)$' is necessarily proper. It is well–formed, and by the rule for evaluating λ–terms, it is proper for the unit–class $\{1\}$ if it is true that Mars has a moon, and for the null–class $\{\}$ if it is false.

[34] While improper definite descriptions are ubiquitous in ordinary discourse, and it is often ***not*** 'analytically necessary' that they are improper (consider 'the author of *Principia Mathematica*'), 'improper class–abstractions' are rare in ordinary discourse, and it seems that, of improper class–abstractions, it is always 'analytically necessary' that they are improper (that is, 'that there is no such class'). As a consequence, class–abstractions in ordinary discourse almost always 'behave logically as terms', as do all proper definite descriptions, and so can be safely treated as terms. Related, one assumes, to this ***non***–superficial difference between definite descriptions in general, and class–abstractions in particular, is that while the best known, and most often adopted, treatment of definite descriptions in general is Russell's Theory of Descriptions in which they are ***pseudo*** terms, there are few formal treatments of class–abstractions as ***other*** than terms in languages in which would–be paradoxical class–abstractions are not well–formed ("(t)his or some other departure from the informal notion (being) necessary... —in the presence of... assumptions it is difficult to avoid—(on pain otherwise of)... antinomies" (Church 1958, p. 29)). Neale cites only Quine and Smullyan for treatments of class–abstractions that make them not terms, but ***pseudo*** terms defined by, or in a similar manner to, Russellian descriptions: "In Smullyan's (1948)... '$((\lambda x)Fx)$' is a scope marker just like Whitehead and Russell's '$((℩x)Fx)$'" (Neale 1995, p. 792n32; 2001, p. 167).

cance.[35] Contrary to Neale, it *makes* a difference whether collapsing arguments are run in terms of definite descriptions or class–abstractions. For one thing, Church's argument for Gödel's 'nearly Eleatic' conclusions regarding all true, and all false sentences, does not "to be of any interest whatsoever (need to) be supplemented with a precise semantics for (class–abstractions)" (Neale 1995, p. 794). There is not a multiplicity of alternative semantics for treatments of class–abstractions as terms in currency, some of which serve all purposes of this argument, and some of which do not. For other things: Church's class–abstraction argument is *simpler* than Gödel's definite–description argument; Church's argument does not address first only 'subject–predicate' sentences, and from conclusions for them proceed to all sentences. It deals at once with all true sentences and false ones. And Church's class–abstraction–argument is in several ways '*leaner*' than Gödel's definite–description argument.

As I construct Gödel's argument in Sections 1 and 2 above, and would construct Church's argument along lines drawn in Section 6.2.1 above, their arguments are in two ways alike, and in one way different without either being 'leaner' for the difference. They are *alike* in what they draw from *premise* (i) of Section 1 above. Each needs only that substitutions of terms for identicals in extensional positions in identity–sentences proceed not only *salva veritate* (which they do by definition of 'extensional positions'), but also *salva pronuntiatio*. Also, this is *premise* (iv), each depends on, the relation signified by '<expr>', the relation of having the same denotation or expressing the same proposition, being an equivalence relation. These arguments *differ* when it comes to *premise* (iii) of Gödel's argument,

if φ_A is true, then

$$\varphi_A < \text{expr} > A = \text{℩}\alpha(\alpha = A \,\&\, \varphi_\alpha)$$

is true,

A a term in which no variable is free, α a variable that is free in formula φ_α, and φ_A a sentence that comes from φ_α by proper substitution of A for α. A related principle, sufficient for my construction of Church's argument, is *premise* (iii)**,

$$\varphi < \text{expr} > \lambda\alpha(\alpha = A) = \lambda\alpha(\alpha = A \wedge \varphi)$$

φ a sentence that express a proposition, A a closed term that has a denotation. However, these different premises are logically independent: Neither argument is 'leaner' for this difference between them.[36]

[35]Neale writes that "Church is fully aware that it makes no difference whether descriptions or class abstracts are used in setting up the slingshot." (Neale 1995, p. 792n; 2001, p. 167.)

[36]However, while *premise* (iii) is not, when specified for it, a theorem of every theorem of descriptions; *premise* (iii)**, when specified for it, is a theorem of every theory of class–abstractions—it is a theorem of theories that casts them as terms, and, 'dressed out' appropriately with scope–indicators, of theories that cast them as specialized Russellian descriptions.

Now come three ways in which Church's argument is 'leaner' for its differences from Gödel's. First, while Gödel's argument assumes, this is *premise* (ii), that *all* definite descriptions are terms, Church's argument assumes only that definite descriptions that are *class–abstractions* are terms. Second, Church's argument, in contrast with my construction of Gödel's, does not use, for the conclusion that all *false* sentences express the same proposition, anything like *premise* (v) according to which, for any sentence φ, φ <expr>$\ulcorner\sim\sim\varphi\urcorner$: it deals at once both with true, and with false sentences. Third, Church's argument, to reach Gödel's 'nearly Eleatic' conclusions not only for 'subject–predicate' sentences, but for *all* sentences, uses nothing like Gödel's 'further assumption (2)' that every sentence 'means the same thing as some subject–predicate sentence': with *premise* (iii)** Church's argument deals at once with all sentences. This is the most important way in which my construction Church's argument would be not only simpler, but 'leaner', than my construction of Gödel's argument.

8 REFLECTIONS

Probably collapsing arguments that work with descriptive terms have recommended themselves to Neale as objects to study, because of his general interest in definite descriptions, and his opposition to Fregean theories that cast them as terms that must refer, which interest and opposition I share. He comments very briefly on the possibility of 'abstraction slingshots' in order to say why he will examine only 'description slingshots' (Neale 1995, p. 791–2n; 2001, pp. 166–7).[37] But if one is concerned to oppose collapsing arguments, as best one can, then paying at least equal attention to arguments such as Church's that run in terms of class–abstractions is recommended, for this attention can facilitate identifying the bad *compound* idea that is *common* to economical collapsing arguments of both descriptive–term and class–abstraction–terms technologies. It is the tripartite idea that: *for one thing*, if not all definite descriptions, then at least those that are class–abstractions, are terms; *for a second thing*, sentences that differ only in co-extensive terms express the same propositions; and, *for a third thing*, something like *premise* (iii)** above is true.

'The culprit' is this *combination* of ideas. However, 'shame of place' should, I think, go to the second of them. Assuming the abbreviations—A: the number one; M: Mars has at most two moons; and S: Saturn has at least three moons—the as it happens true identity,

$$\lambda x(x = A \wedge M) = \lambda x(x = A \wedge S),$$

[37] In (1995) Neale explains his concentration on 'description slingshots' thus: "it makes no difference whether descriptions or class abstracts are used in setting up the slingshot. . . For the sake of continuity, I will stick to statements that contain descriptions." (p. 792n30) In (2001) he writes: "it makes no difference whether descriptions or class abstracts are used in setting up the basic slingshot. For epistemological reasons I have a preference for (first order definable) description over abstraction, so I will examine only versions that make use of descriptions." (p. 167)

without a doubt entails that the propositions expressed by '$\lambda x(x = A) = \lambda x(x = A \wedge M)$' and '$\lambda x(x = A) = \lambda x(x = A \wedge S)$' have the same truth–value and are materially equivalent. But the suggestion that the displayed identity entails that these propositions are not only materially, but *logically*, equivalent, *let alone* the suggestion that they are *identical*, is, if I may say so, *ridiculous*.[38] There is *nothing* going for this part of *premise* (i) for anyone who makes use in his philosophy of 'proposition'.

Another example to this point can be reached from discussion in (Lee 2002) of a case involving descriptions such as '$\iota x(x =$ Socrates $\wedge x$ is an Athenian)' and '$\lambda x(x =$ Socrates $\wedge$ Socrates is an Athenian not a Spartan)'. Switching to class–abstractions, and 'losing' the third occurrences of 'x', we have that the identity–proposition that

$$\lambda x(x = \text{Socrates} \wedge \text{Socrates is mortal}) = \lambda x(x = \text{Socrates} \wedge \text{Socrates is an Athenian not a Spartan}),$$

entails that the propositions that

$$\lambda x(x = \text{Socrates}) = \lambda x(x = \text{Socrates} \wedge \text{Socrates is mortal}),$$

and that

$$\lambda x(x = \text{Socrates}) = \lambda x(x = \text{Socrates} \wedge \text{Socrates is an Athenian not a Spartan})$$

are *materially equivalent*. It is given that Socrates is mortal, and that he is an Athenian, not a Spartan, so these propositions are both true, as is the identity–premise from which their material equivalence follows. However, "***in a world in which Socrates is* (*mortal*) *and yet a Spartan* (*not an Athenian*)**" (p. 547, italics added), though the proposition that

$$\lambda x(x = \text{Socrates}) = \lambda x(x = \text{Socrates} \wedge \text{Socrates is mortal})$$

is true, since {Socrates} = {Socrates}, the proposition that

$$\lambda x(x = \text{Socrates}) = \lambda x(x = \text{Socrates} \wedge \text{Socrates is an Athenian not a Spartan})$$

is false, since {Socrates} $\neq$ {}. In such a world these propositions are not materially equivalent. They are thus *not logically equivalent*, let alone *identical*.

[38] Oppy sees Gödel's *particular* slingshot, addressed as it was to Russell's philosophical/logical situation, as founded on a compound error. *Cf.*: "I think that it is pretty obvious that anyone who treats facts as structured entities (as Russell did) will resist the suggestion that 'Fa' and '$a =$ the $x : x = a \,\&\, Fx$' express the same fact (so that Russell's escape from the slingshot argument is overdetermined: it isn't just his analysis of descriptions that sets him free)." (e-mail, 27 January 2004.) To complicate we should recall that Russell's *first* resistance to that suggestion would go to the *incompleteness* of '$A = \iota x(x = A \,\&\, Fx)$' which he would see as short for '$\{\iota x(x = A \,\&\, Fx)\}A = \iota x(x = A \,\&\, Fx)$'. Also while he would resist the *identity* of the propositions expressed by this sentence and 'FA', he would see that they are logically equivalent, and that therefore it is not a sufficient answer on his part to Gödel's slingshot that it claims identity for these propositions. The logical equivalence of all truths, and of all falsehoods, would be collapse enough.

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