Paraconsistent Vagueness: Why Not?

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The idea that the phenomenon of vagueness might be modelled by a paraconsistent logic has been little discussed in contemporary work on vagueness, just as the idea that paraconsistent logics might be fruitfully applied to the phenomenon of vagueness has been little discussed in contemporary work on paraconsistency. This is *prima facie* surprising given that the earliest formalisations of paraconsistent logics presented in Jaśkowski (1948) and Halldén (1949) were presented as logics of vagueness.

One possible explanation for this is that, despite initial advocacy by pioneers of paraconsistency, the prospects for a paraconsistent account of vagueness are so poor as to warrant little further consideration. In this paper we look at the reasons that might be offered in defence of this negative claim. As we shall show, they are far from compelling. Paraconsistent accounts of vagueness deserve further attention.

1 What is paraconsistent vagueness?

What is a paraconsistent account of vagueness? In short, it takes up the idea that where vagueness gives rise to indeterminacy, e.g. indeterminacy in the application of predicate to a borderline case, this is to be modelled as a species of overdetermination of truth as opposed to the more common supposition of underdetermination of truth. In this way, then, where a is a borderline case of redness we shall say that the sentence 'a is red' is true and so too its negation 'a is not red'. Thus, where A is false if and only if its negation $\neg A$ is true, the atomic sentence 'a is red' is both true and false, giving rise to truth-value gluts. This contrasts with the more commonly accepted semantic approach to vagueness according to which such a sentence is neither true nor false, giving rise to truth-value gaps.

Moreover, despite some sentences and their negations being true, such an approach should deny that every sentence is true (on pain of triviality). Where validity is defined as preservation of truth then, the non-trivial account of vagueness as overdetermination should reject the validity of the inference from the truth of A and its negation $\neg A$ to the truth of an arbitrary sentence B. The inference, known as *explosion*, is classically valid, of course, and encodes the trivialising effect of contradictory sentences on any theory to which they are jointly admitted. Paraconsistent logics, on the other hand, are non-explosive by definition. An account of vagueness as overdetermination thus should be paraconsistent.

In fact, not only is such an account of vagueness paraconsistent in its pursuit of an inconsistent but non-trivial theory of vagueness, it is *dialethic* since it proposes not simply that there can be contradictory sentences that are true without everything being true, but more strongly that there *are* true contradictory sentences. Such an account therefore will constitute a dialethic paraconsistent account of vagueness.[1] (For more on this distinction see Beall 2004: 6.)

Beyond this central feature, paraconsistent accounts of vagueness vary in much the same way as do their non-trivial truth-value gap counterparts—i.e. paracomplete counterparts (which admit some contradictory pairs of sentences as non-true while denying that every sentence is non-true).

More exactly, some paracomplete accounts go on to retain the necessary truth of excluded middle claims ($A \vee \neg A$) and other classical laws in the face of truth-value gaps (e.g. supervaluationist accounts) with a resulting loss in truth-functionality and consequent non-standard analyses of disjunction and associated logical constants like the existential quantifier. Analogously, some paraconsistent approaches to vagueness might go on to retain the necessary non-truth of contradictions ($A \& \neg A$) and other classical laws in the face of truth-value gluts with a resulting loss of truth-functionality and consequent non-standard analyses of conjunction and associated logical constants like the universal quantifier. For example, the discussive logic of Jaśkowski (1948) or what has also become known as subvaluationism, the dual of supervaluationism (on which see Hyde 1997), are of this kind.

Other paracomplete accounts retain the standard semantics for disjunction according to which a disjunction is true if and only if one of its disjuncts is true and consequently reject the necessary truth of some excluded middle claims in the face of truth-value gaps. The logic of Tye (1990) exemplifies such

[1] One could, of course, defend the use of a paraconsistent logic in giving an account of the logic of vagueness as a mere mathematical tool, foregoing the stronger dialethic interpretation. I set this aside in favour of the more obvious reading whereby "truth-value gluts" are construed as real gluts, just as paracomplete "truth-value gap" approaches are generally understood as proposing real gaps, not just invoking gaps as a technical tool.

a truth-functional gap approach, employing Kleene's "strong" three-valued system K3. Other similar paracomplete alternatives include Łukasiewicz's three-valued system L3 with its alternative semantics for the conditional. Analogously, some paraconsistent accounts might retain the standard semantics for conjunction according to which a conjunction is true if and only if both conjuncts are true and consequently reject the necessary non-truth of some contradictions (and hence accept their truth) in the face of truth-value gluts. Priest's paraconsistent logic LP is of this kind. Another closely similar paraconsistent alternative is the paraconsistent logic of Halldén (1949) (though Halldén employs "designated values" where we have spoken of "truth" and defines validity in terms of "preservation of designated value"—on this account some contradictions are "designated").

2 PARACONSISTENCY IS INCOHERENT

An obvious initial reason that has been offered for not pursuing any such paraconsistent approach to vagueness is simply that it is paraconsistent. On such a view, paraconsistency *per se* is objectionable and so any theory that seeks to apply such a logic is doomed from the outset. Keefe (2000: 197), for example, speaking of subvaluationism suggests that:

> many philosophers would soon discount the paraconsistent option (almost) regardless of how successfully it treats vagueness, on the grounds of the unappealing commitments and features of the logical framework as a whole, in particular the absurdity of A and $\neg A$ both being true for many instances of A.

Bearing in mind the foregoing distinction between paraconsistency and dialethic paraconsistency, the objection is properly aimed at the dialethic paraconsistentist given that it appeals to the supposed "absurdity of A and $\neg A$ both being true for many instances of A". Paraconsistency *per se* need make no such commitment in its rejection of explosion. All that is required is the admission of theories that are non-trivial despite their inconsistency. The further supposition that a theory of what is actually true is of this kind, and thus that there actually are contradictory truths, is another matter. However, as already noted, a paraconsistent account of vagueness is dialethic. Thus the objection is properly targeted in this case.

Such an objection is frequently encountered and has been repeatedly dealt with elsewhere (e.g. Priest 1987 and Sainsbury 1995, Ch. 6). While debate at this very general level continues (see, most recently, Priest, Beall and Armour-Garb 2004), we propose to set aside further discussion at this level and focus, instead, on issues that arise with the particular application of paraconsistency to vagueness and some of the problems that this might be thought to engender.

3 KNOWLEDGE AND BORDERLINE CASES

One such problem is that of knowledge of borderline cases. On standard accounts of borderline cases in the sense relevant to vagueness, a borderline case a for a predicate F is characterised as being an object to which the predicate F meaningfully applies, yet which is such that we cannot know of a that it is F nor that it is $\neg F$. Thus, for example, a reddish-yellow apple may be such that, despite being a thing to which 'red' may be meaningfully applied, we cannot know of it that it is red, nor can we know that it is not red. Its colour is such that it falls on the borderline between the two colour categories red and yellow and is neither clearly red nor clearly not red. The existence of borderline cases is then typically taken as necessary for vagueness.

Epistemic theorists go on to explain this irremediable lack of knowledge deemed necessary for vagueness by claiming that there is a fact of the matter, a is F or $\neg F$, but we are precluded from knowing which predicate a satisfies by a barrier to such knowledge arising from a margin-for-error principle governing inexact knowledge. (See Williamson 1994, Ch. 8.) Semantic theorists, on the other hand, typically explain the lack of knowledge by claiming there is simply nothing to be known in respect of which predicate a satisfies. The best explanation of our lack of knowledge, they contend, is the absence of anything to be known. We cannot know that the reddish-yellow apple is red nor that it is not red simply because there is no fact of the matter either way, given the semantic indeterminacy that attends our use of vague predicates like 'red'.

So, for example, paracomplete accounts of vagueness are typified by their claiming that it is neither true nor false that the apple is red and so neither true that it is red nor true that it is not red, and it is this (rather than a barrier to knowledge of what is the case) that underlies the inability to know. There is simply no truth to be known either way.

Paraconsistent accounts, on the other hand, commit to truth both ways (i.e. to truth and falsity), so might be thought to face an apparent difficulty here. It is sometimes suggested that a paraconsistent approach to vagueness is unable to accommodate the existence of borderline cases and so is unable to acknowledge a necessary condition for vagueness. So much for a paraconsistent account of vagueness then, so the argument goes.[2]

More exactly, the concern is as follows. The paraconsistentist will describe the borderline case a for predicate F as satisfying both the predicate and its negation. But if it is true that Fa and true that $\neg Fa$ (since false that Fa) then how can it be that we cannot know of a that it is F nor that it is $\neg F$? We might postulate a barrier to knowledge of truth to explain our ignorance, appealing to the same considerations postulated by epistemic theorists, but then we face the theoretical burden incurred by an epistemic account of vagueness (which semantic accounts seek to avoid) while also postulating a non-classical semantic approach (which epistemic accounts seek to avoid). Theoretical parsimony

[2]The problem was first raised in conversation by Pablo Cobreros.

speaks against such seemingly unnecessary complexity.

Alternatively, we might explain our ignorance by pointing to the fact that, while true, the sentences Fa and $\neg Fa$ are also false. This coupled with the supposition that one cannot know the false would suffice to rule out knowledge. But such a supposition seems difficult to defend. While classically speaking one cannot know the false, this is simply because the false is non-true and one cannot know what is not true. In the situation to hand, truth obtains and the paraconsistentist claims to *know* that it obtains (in addition to the truth of its negation). Fa is true and so too $\neg Fa$, and in accepting a paraconsistent approach to vagueness we take ourselves to come to know this.

Borderline cases then, on the paraconsistent account, are such that we *can* know of them that they satisfy the vague predicate in question and also its negation. The paraconsistentist should take issue with the characterisation of "borderline cases" as described above. It simply begs the question against a paraconsistent approach. What remains the case is that a borderline case a for a vague predicate F is such that we cannot know of it that it is uniquely F (i.e. that a satisfies F and does not satisfy $\neg F$) nor that it is uniquely $\neg F$ (i.e. that a satisfies $\neg F$ and does not satisfy F). Of course, in consistent settings this is equivalent to the simpler characterisation according to which we cannot know of a that it is F nor that it is $\neg F$, but with the admission of a paraconsistent approach to the range of theoretical responses to vagueness this simpler characterisation rules out such an approach by definition and is therefore to be abandoned in favour of the more general characterisation offered.

The situation faced by the paraconsistentist with respect to the characterisation of borderline cases then resembles that encountered in the literature earlier by epistemic theorists. Prior to the 1990s, many definitions of borderline cases characterised them as cases where there was simply no answer to the question whether the predicate in question applied or not. The manifest lack of knowledge was conflated with the lack of anything to know. More recent recognition of the need to avoid begging the question against an epistemic account has shifted discussions towards a more general characterisation that does not prejudge the basis for what all can agree is, at least initially, an epistemic gap. So too now with a paraconsistent account. Room must be made to allow for its possibility so as to not rule out such an account by definition, and the even more general characterisation offered above ought to be accepted.

4 ASSERTION AND DENIAL

We might also wonder as to the assertibility or otherwise of Fa and $\neg Fa$ where a is a borderline case of F. After all, on the paraconsistent account Fa and $\neg Fa$ are both true and, in typical circumstances, this is known to obtain. If we are to assert what we have good evidence for being true, we should then assert Fa and $\neg Fa$. On the other hand, given a standard account of negation according to which A is true if and only if $\neg A$ is false and $\neg A$ is true if and only if A is false, it follows that Fa and $\neg Fa$ are both false, and, again, in typical circumstances, this is known to obtain. Now, if we are to deny what we have good evidence for being false, we should then deny Fa and $\neg Fa$. In such circumstances we seem committed then to both asserting and denying both Fa and its negation. With assertion and denial taken to express the mental states of acceptance and rejection respectively, if we follow the standard assumption that these states are exclusive mental states with respect to any given sentence, then incoherence ensues.

This is, in fact, an issue that is already familiar from the paraconsistency literature.[3] Incoherence is easily avoided in paraconsistent circumstances if we simply reject the above assumption that we should deny what we have good evidence for being false while retaining the idea that we should assert what we have good evidence for being true. After all, in circumstances like the liar paradox where a paraconsistent response has it that we have good evidence for the acceptability of the conclusion (i.e. a sound argument for the conclusion), we ought accept the conclusion despite its being false (as well as true). Its assertion simply expresses this. As Sainsbury (1995: 140) points out, its rejection would require subsequent rejection of some aspect of the reasoning which led to it as conclusion and this would significantly detract from paraconsistency as a response to the liar paradox. Though false, and evidently so, the paraconsistentist ought not deny the conclusion. So too with borderline cases; though Fa may be false it ought not be denied, since it is also true.

On this approach then, we ought only deny that which is evidently false and not true. Of course, from a classical perspective this is simply tantamount to the claim that we ought only deny the evidently false, and this is something the classicist is already committed to. In paraconsistent circumstances, however, the claims come apart and the exclusive nature of assertion and denial (and acceptance and rejection) is restored by advocating the former principle.

The same threat of incoherence can be viewed from a different perspective—one already familiar from discussions of paracomplete accounts of vagueness. If denial of a sentence just is assertion of its negation, as commonly suggested, in denying Fa and its negation—as the paracomplete theorist proposes with respect to borderline case predications—such an account would then also be committed to asserting both the negation of Fa and $\neg Fa$. The paracomplete theorist would then be required to both assert and deny each of Fa and $\neg Fa$. The paracomplete theorist must abandon the idea that the denial of a sentence A is equivalent to the assertion of $\neg A$. While the assertion of $\neg A$ is sufficient for the denial of A, the denial of A is not sufficient for the assertion of $\neg A$.

Analogously for the paraconsistentist: if denial of a sentence just is assertion of its negation, in asserting $\neg Fa$—as proposed on the paraconsistent account with respect to borderline case predications—such an account would

[3] See Priest (1986).

then also be committed to denying Fa. And in also asserting $\neg\neg Fa$ (since equivalent to the assertion of Fa) one would also thereby be committed to the denial of $\neg Fa$. As before, we would then be committed to both asserting and denying both Fa and its negation. As with paracomplete accounts, we should reject the assumption that denial of a sentence is assertion of its negation. While the denial of A is, from a paraconsistent perspective, sufficient for the assertion of $\neg A$, the assertion of $\neg A$ is not sufficient for the denial of A.

Both paracomplete and paraconsistent accounts of vagueness must reject the assumption that denial of a sentence is simply the assertion of its negation. And it should be rejected for good reason: the assumption is not true. This is already a familiar point in respect of paracomplete approaches, we can now appreciate the corresponding point in respect of paraconsistent approaches.

In thus appreciating this in the paraconsistent case, we can now return to our earlier problematic assumption that one ought deny the evidently false. In a paraconsistent context, the following two assumptions are equivalent—the (false) assumption that in asserting $\neg A$ one is thereby committed to the denial of A is *equivalent* to the (false) assumption that one ought deny the evidently false.[4] To see this, suppose that one is in a position to assert $\neg A$. Then one must have evidence for the truth of $\neg A$, and so evidence for the falsity of A.[5] And so, if we assume that one ought deny the evidently false, one ought deny A. Conversely, suppose that A is evidently false. Then we have evidence for the truth of $\neg A$ and so ought be prepared to assert $\neg A$. And so, if we assume that in asserting $\neg A$ one is thereby committed to the denial of A, one ought deny A. Given their equivalence then and the fact that the one assumption has just been seen to be untrue, we can (as, indeed, we did earlier) and must reject the other. Evidence of mere falsity is insufficient for denial.[6]

The paraconsistent account of vagueness faces no special difficulties in re-

[4]See Sainsbury (1995: 141–2).

[5]We are assuming that if a body of evidence E is evidence for B then it is evidence for any claim equivalent to B.

[6]Given the revised conditions for rejection mooted above, one might wonder here whether the paraconsistentist is yet entitled, by their own lights, to reject the problematic assumptions under consideration concerning the relation between truth, falsity, negation, assertion and denial. Rejection requires non-truth after all, and so the paraconsistentist must be careful to establish more than the mere falsity of the assumptions in question. The query is a justified one but we need have no concern here. The arguments above can be taken to establish non-truth, as claimed. To be sure, the arguments against the problematic assumptions were *reductio* arguments and at least some forms of *reductio* arguments are, by paraconsistent lights, insufficient to establish the non-truth and subsequent *rejection* of an argument's assumption. However, even by paraconsistent lights *reductio ad absurdum*, as opposed to *reductio ad contradictionem*, offers a valid reason to reject the assumption giving rise to the absurdity. After all, no-one wants to accept an absurdity—where an absurdity is something unacceptable by your own lights. In the current setting, however, contradictions are not always absurdities (in the required sense). In the arguments in the main text, we have only relied on the paraconsistently-kosher *reductio ad absurdum* to licence the rejection of the assumption in question. This, it seems, we are licensed to do. (For more on this see Priest 1989.) We thank an anonymous referee for this line of defence of our strategy.

spect of assertion and denial.

5 METAPHYSICAL ISSUES

A weak commitment to paraconsistency involves a commitment to our being able to reason non-trivially in the context of inconsistent theories. So, for example, medieval philosophers reasoning hypothetically from the assumption that God does not exist would, against a background where God's existence was taken to be necessary, be reasoning from an assumption that contradicted this truth. We can, it seems, reason non-trivially in these and other similarly inconsistent circumstances. The weak paraconsistentist advocates paraconsistency to model such circumstances but there is no commitment to the truth of the hypothetically considered situation.

A stronger commitment to paraconsistency involves, as we discussed earlier, a commitment to the *truth* of some contradictions from which we might non-trivially reason. In thus committing themselves to such a strong dialethic account of vagueness one might wonder whether the strong paraconsistentist thus commits to the inconsistency of the world thus described. Since the world is manifestly not inconsistent, were the paraconsistentist thus committed, they would be refuted by the falsity of this seemingly untoward consequence. Or, at least, so it might be argued. What is the paraconsistent vagueness theorist to say here?

Let us begin with the assumption that "the world is manifestly not inconsistent". Is this so obvious? Supposing we grant that the world is not manifestly inconsistent, that objects that are borderline cases of 'red' (for example) are not seen to be both red and not red. To infer from this that such objects are not both red and not red would be a *non sequitur* on a par with supposing that simply because we do not see the world as composed of atoms it follows that it is not so composed. We may not see that the table in front of us is made up of atoms (as postulated by physical theory) simply because we do not have the physical theory to hand which gives an account of what we are in fact seeing. Seeing the world for what it is—seeing it as composed of atoms, as governed by particular physical laws, etc.—is a consequence of the theory-ladeness of such "observations" coupled with the theories brought to the act of seeing. So too with metaphysical theorising. We might see the borderline case of 'red' and it might also be the case that such a borderline case is both red and not red, without it thereby following that we see the borderline case as red and not red. Metaphysical theory can affect how we see the world as much as any other theory, and much of the world's nature can remain opaque without theory to guide us (Beall and Colyvan 2001b).

And this brings us to our second point. Is the paraconsistentist, in accepting a theory according to which our borderline case of 'red' is both red and not red, really committed to the world being inconsistent (and their consequently seeing it as such in light of their theoretical commitment)? The answer is clearly 'no'. A commitment to a paraconsistent account of vagueness no more requires a commitment to the world's being inconsistent than a commitment to a paracomplete account of vagueness requires a commitment to the world's being incomplete. Supervaluationism is a case in point. Most supervaluationists are committed to our treating the indeterminacy due to vagueness as a species of underdetermination or incompleteness of *reference*. This postulated indeterminacy of reference, there simply being no fact of the matter as to whether or not 'red' refers in such a way as to include our borderline case in its extension or anti-extension, is deemed sufficient to explain the vagueness of language. Paracomplete vagueness, on this view, amounts to nothing more than semantic indeterminacy according to which what is represented by a given vague term, e.g. 'red', is left open. There are many candidate referents no one of which is determined. As McGee and McLaughlin put it, we learn "a disturbing philosophical lesson" that the semantic approach to vagueness forces us to accept "the inscrutability of reference" (2000: 130).

Similarly, the paraconsistentist can explain the indeterminacy due to vagueness as a species of overdetermination or inconsistency of reference. Paraconsistent vagueness, on this view, amounts to nothing more than semantic indeterminacy according to which what is represented by a given vague term, e.g. 'red', is not left open but is, rather, determined in a number of mutually inconsistent ways. There are many candidate referents no one of which is uniquely determined. There is no requirement for a paraconsistent account of vagueness to be committed to the idea that inconsistency in respect of the satisfaction of a vague predicate by its borderline case is evidence of the inconsistency of the instantiation of a vague property by that borderline case. Our semantics of vagueness might admit of inconsistency without the world being thereby taken as inconsistent.[7]

The general point here is a familiar one. Not all features of models represent features of that which is being modelled. As opposed to any *particular* account of the vagueness of natural language, paracomplete or paraconsistent, consider vagueness more generally. As Russell was keen to remind us, the mere vagueness of natural language or any other system of representation is no sure sign of the vagueness of that which is represented. In the case of natural language, to overlook this is to commit what Russell (1923: 84–5) described as the "fallacy of verbalism":

> There is a certain tendency in those who have realized that words are vague to infer that things also are vague... This seems to me precisely a case of the fallacy of verbalism—the fallacy that consists in mistaking the properties of words for the properties of things.

[7]See Mares (2004) for more on the distinction between what he calls semantic dialetheism and metaphysical dialetheism. Using Mares' way of carving things up, the paraconsistentist need only commit to semantic dialetheism and not to metaphysical dialetheism.

Russell was in fact committed to the stronger claim that not only was such an inference fallacious, there could be no sound argument for its conclusion since its conclusion was false.

> Vagueness and precision alike are characteristics which can only belong to a representation, of which language is an example. They have to do with the relation between a representation and that which it represents. Apart from representation, whether cognitive or mechanical, there can be no such thing as vagueness or precision; things are what they are, and there is an end of it.

Whether he was right about this or not is another matter.[8] The point here is simply the weaker one that one is not *required* to read the property of the representation into that which is represented.

Of course, the general question of when it is legitimate to read features of a model into that which is being modelled is a difficult one. We must simply look to the feature in question and the arguments for and against its being a feature of that being modelled. In the case of the paracomplete account of vagueness as merely semantic, McGee and McLaughlin recognise what they take to be a disturbing feature—"the inscrutability of reference". Analogously on the paraconsistent account that views vagueness as merely semantic, there is a corresponding lesson that semantic vagueness forces us to accept what we might term "the fecundity of reference". If such a consequence is disturbing enough, one might reject attempts to contain either the incompleteness or inconsistency to the model alone, and decide instead that it is best seen as a feature of the world. A merely semantic view of vagueness might be abandoned in favour of a view that sees the phenomenon as ultimately having a metaphysical foundation. But such a commitment is not mandatory without further argument.[9]

Moreover, as we have seen, even if such a commitment is made and the incompleteness or inconsistency manifested by vague language is indeed taken as evidence of the incompleteness or inconsistency of that which vague language is taken to represent, e.g. the world, the initial concerns deriving from the opacity of this postulated feature of the world is itself no barrier to such a paraconsistent account of vagueness. For all that has been said so far, a paraconsistent account of vagueness encounters no particular metaphysical difficulties.

What might further argument for an inconsistent world look like? It's worth pausing for a moment to answer this question, if for no other reason but to get clear about how the more-modest, semantic account differs from an account according to which the world itself is inconsistent. One very general

[8] See, for example, Colyvan (2001B) and Hyde (2008).

[9] See Beall and Colyvan (2001B) for a tentative stab at such an argument.

way to argue that features of a model should be treated as faithfully representing the corresponding bits of the world, is via an indispensability argument: if the model in question needs to posit some entity a with properties F and G and no other theory forgoing reference to a, F and G does as well, then we say that a, F and G are indispensable. Moreover, we ought to believe in the existence of as and believe that they have the properties F and G attributed to them in the relevant theory.[10] In the case of vagueness, what we would require would be that our best scientific theories required vague predicates (which, in part, means that there's no better theory that does not need to resort to such predicates) and that our best theory of vagueness be a paraconsistent one.[11] If we have all this, then there will be some object a with inconsistent property instantiations—a will have both the vague property F and $\neg F$. We are not here committing to this or any other arguments for metaphysical dialetheism; we have claimed that further argument is required and we draw attention to one way that further argument might proceed. But there are several ways a semantic dialetheist can resist this further argument (e.g. by showing that the inconsistent properties are not indispensable and can be explained by means of fecundity of reference to consistent properties).

6 BITING BULLETS

Finally, we should be clear as to the kind of response paraconsistency presents to vagueness and associated paradox, the sorites paradox. Paraconsistency is frequently presented as a "bullet biting" response to the particular problem at issue, a resigned acceptance of the seemingly unacceptable. So, for example, when discussing paradoxes such as the liar, the paraconsistent response proposes that one simply accept the contradictory conclusion engendered by the paradox, and in this sense amounts to biting the bullet. It should be clear that the paraconsistent approaches to vagueness under discussion here are not such responses.

The signature paradox associated with vagueness, the sorites paradox, presents us with arguments to the effect that, for a sorites-prone predicate F, everything within F's range of significance satisfies F if anything does and nothing satisfies F if anything in its range does not. The paradox thus presents a *prima facie* case for what Williamson has termed an "all-or-nothing" view of sorites-prone predicates. All objects within F's range satisfy F or none do. Some theorists do indeed advocate biting this bullet. Unger (1979), for example, takes typical examples of the paradox involving a negated atomic predicate $\neg F$ to be sound thus establishing that everything satisfies $\neg F$. And, assuming consis-

[10] See Colyvan (2001A) for a general defence of such indispensability arguments, Colyvan (2001B) for an indispensability argument for metaphysical vagueness, Hyde (2008) for another account of metaphysical vagueness, and Beall and Colyvan (2001B) and Colyvan (2008) for indispensability argumens for belief in inconsistent objects.

[11] And also that the indispensability argument can be suitably defended.

tency, he consequently maintains that any sorites which begins from the assumption that some object *does* satisfy F (thus by sorites reasoning apparently establishing that all do) is unsound by virtue of an untrue premise. Atomic predicates like F have no satisfiers. The disjunctive bullet is thus sometimes chewed one way, and other times the other way. The sorites paradox is taken to show yet another cost of adhering to classical logic and semantics.

Paraconsistentists, on the other hand, seek to avoid the untoward conclusions of sorites reasoning rather than learning to accept them. Recall that paraconsistency is characterised by its ability to accept inconsistency in a theory (e.g. a theory of vagueness according to which, for some a and vague predicate F, both Fa and $\neg Fa$ are accepted in the theory) without triviality, i.e. without everything thereby being accepted in the theory. What we might term the "strong triviality" of a theory, everything being true in the theory, is able to be avoided despite the acceptance of contradiction. Quite obviously, a weaker species of triviality, the truth in the theory of every sentence predicating a *negated atomic vague predicate* of something, is also able to be avoided. Paraconsistent logic with its attendant rejection of explosion as a rule of inference no more requires weak triviality of inconsistent theories than it does strong triviality. And in its application to vagueness, it is advocated precisely to avoid any such commitment to weak triviality of the kind endorsed, for example, by Unger.

The sorites paradox presents a *prima facie* case for the seemingly unacceptable weak triviality of theories involving vague predicates and it is by limited acceptance of contradiction that the paraconsistentist hopes to avoid such triviality while also avoiding the stronger triviality that results from the acceptance of contradiction *per se*. Finding the bullet of weak triviality too abhorrent to bite they invite limited contradiction instead.

Although our purpose in this paper is to clear the way for paraconsistent accounts in general, without entering the fray on which particular paraconsistent approach is the best, it might be worth saying just a few words about how a particular paraconsistent account of vagueness would look and why it need not be seen as bullet biting. As we have already made clear, what all paraconsistent approaches have in common is that they treat the borderline region of a vague predicate as glutty, with both the predicate and its negation applying to objects in the borderline region. One way of fleshing out this basic suggestion is a subvaluational semantics developing Jaśkowski (1948). (See Hyde 1997).[12] According to this approach, we consider all the permissible precisifications of the vague predicate in question and quantify over them in determining the truth, falsity or otherwise of any sentence, atomic or complex—as with the familiar supervaluationist approach—but unlike the supervaluationist approach we define truth in terms of true on *at least one* permissible precisification, rather than

[12] We have already mentioned another paraconsistent approach of Halldén (1949). More recently, David Ripley (2008) has advocated LP as a logic of vagueness.

in terms of *all* such precisifications.[13] It is important to note that subvaluationism is the formal dual of supervaluationism, so shares all the formal virtues as well as vices of that approach. Any reason for choosing supervaluationism over subvaluationism will need to appeal to non-formal (philosophical) features of the accounts in question. It's fair to say that thus far no such features have been identified (Beall and Colyvan 2001; Hyde 2001).

The important point, for present purposes, is that subvaluationism is no more a bullet-biting response than is supervaluationism. Indeed, there is something arbitrary about insisting—as supervaluationism does—on truth in terms of *all* permissible precisifications. While truth in terms of *at least one* permissible precisification is (for all that has been said so far) also arbitrary, it is not some last resort one is pushed to for want of a better response. On the contrary, it is a very natural position and a natural response to the arbitrariness of the dominant supervaluationist approach. And properly understood, it really does seem on a par with supervaluationism, including the much touted conservativism with respect to classical logic (Hyde 2008, Ch. 4).[14]

7 CONCLUSION

Paraconsistent accounts of vagueness have been present in the literature for as long as modern formal treatments of paraconsistency themselves and yet they have attracted very little attention in the more recent explosion of literature on the subject of vagueness. Suspecting as-yet unarticulated concerns that prohibit their acceptance, we have looked at a range of problems that might be variously mooted as points of concern in respect of a paraconsistent account. Having looked at these problems the paraconsistentist might face when applying themselves to vagueness—problems to do with knowledge and borderline cases, assertion and denial, concerns about incurring commitments to an inconsistent world, or concerns arising from the kind of response offered to the associated sorites paradox—we conclude that, at least on the grounds considered here, the paraconsistentist is free to continue making their case.[15]

[13] An LP response, on the other hand, proceeds similarly with respect to atomic sentences but evaluates complex sentences by means of the familiar LP truth-functions.

[14] The conservativism with respect to classical logic is typically cited by advocates of supervaluationism as one of the strengths of the latter. But from where we sit, preserving classical logic in the context of vagueness—where all the data is pointing away from classical logic—hardly seems virtuous. Moreover, the paraconsistent deviation incurred by subvaluationism is no greater than that incurred by the paracomplete supervaluationist response.

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