REVIEW: Eckart Menzler-Trott's *Logic's lost genius: The life of Gerhard Gentzen*

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Menzler-Trott, Eckart *Logic's lost genius*: *The life of Gerhard Gentzen*, translated by Craig Smoryński and Edward Griffor (from *Gentzens Problem*, Birkhäuser Verlag), (History of Mathematics, vol. 33) American Mathematical Society and London Mathematical Society, 2007; 440 pp., USD$89.00.
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REVIEW

Natural deduction now features in virtually every first course in formal logic. In general it does not bear Gentzen's name and this may account for that name not being a household word, indeed being oft forgotten, even in philosophical circles. But natural deduction is Gentzen's very great legacy to philosophy – and to mathematics and computer science too. This style of system does indeed seem natural, echoing the way that we structure arguments in natural language. Yet this formalization took a long time to appear. In the mid-nineteenth century, Boole had introduced a very mathematical style for logical arguments about what are now called 'sets'. He used the same style as Leibniz had done centuries before, though it is not known if Boole knew Leibniz's work. Frege introduced a typographically cumbrous and complicated-looking formalism for predicate calculus in the late nineteenth century. Later the mathematician Hilbert introduced a variant of Russell's system and mathematicians used and still largely use such Hilbert-style systems. But it was Gentzen who did the analysis and invented, or should I say, extracted, the style of natural deduction system that is now so familiar.

Gentzen also invented other variations on natural deduction: the systems of so-called sequent calculus. These are not always as simple and elegant as natural deduction but have various advantages for technical work. Interestingly, equivalences between the various kinds of system even for the same underlying logic, are not always easy to establish, even though they seem obvious.

His exact contemporary was Gödel, though the latter lived much longer than Gentzen. In his early work Gentzen showed that the intuitionistic logic of Brouwer, while looking more restricted than classical logic, was in fact more powerful in that classical logic could be embedded in intuitionist logic, but not vice versa. Gödel also proved this result. What is better known is that Gödel also showed that one could not prove the consistency of arithmetic using only the means of (formal) arithmetic. (This required a further development of the techniques of his famous proof of the incompleteness of arithmetic.) Gödel's result would suggest that trying to prove the consistency of arithmetic was a wild goose chase, but that is not true. The systems that Gentzen developed allowed him to view formal proofs as mathematical objects, subject to operations that converted them into other objects. This meant that he was then in a very powerful position when trying to give a mathematical proof of the consistency of arithmetic. Gödel's results meant that any proof of the consistency of arithmetic, could not be achieved with the limited means available in formal arithmetic. In particular, the most potent axiom, ordinary induction that had been introduced by Peano and formalized by Dedekind, was not sufficient and what was required was transfinite induction using longer orderings than the natural ordering of the natural numbers. Certain of these orderings are easy to describe: thus if one puts all the even numbers 'after' all the odd ones, then one gets an ordering $1, 3, 5, ..., 2, 4, 6, ...$ whose type is called $\omega.2$, and one can make much more complicated ones. Such orderings had been developed by Cantor in the previous century. However, it turns out that one has to use orderings that are 'effective', or as we now say, recursive. The smallest ordinal number required for the consistency of arithmetic is called 'epsilon-nought', where $\varepsilon_0 = \omega^{\omega^{\omega^{\omega^{\cdot^{\cdot^{\cdot}}}}}}$, which looks large but is really quite small in Cantor's paradise of set theory.

However, the efficacy of such transfinite induction depended on being able to manipulate proofs with (relative) ease. Gentzen's system of natural deduction did that. It did much more, though Gentzen was not to know of it. In 1960, Bill Howard, building on work of Haskell B. Curry, established a correspondence between logic in a natural deduction system and Church's lambda calculus. This correspondence is, quite literally, visible when one writes down the corresponding rules of the two kinds of system. So proofs can be viewed, again quite literally, as lambda terms these latter can be used, again quite literally, as computer programs. Such programs have proofs embedded in them, and therefore provide a guarantee of their correctness. This approach has become a major interest of theoretical computer scientists and is heavily deployed in Lawrence Paulson's Isabelle theorem prover based in Cambridge.

Such usefulness was, sadly, not known to Gentzen. Other sadness also permeated Gentzen's life, especially his death, which was a result of Czech, not German, imprisonment. However, his life in Nazi Germany was very troubled. Gentzen was not a Jew; he was baptized an evangelical and joined the Nazi party in 1933, later being discharged from the Wehrmacht in 1942 on the grounds of ill health. His involvement with the Nazis did not seem to interfere with his mathematics, rather his difficulties were in getting a succession of jobs. However the Nazis took far more interest in the nature of mathematics than politicians usually do, and since there were so many excellent Jewish mathematicians, the discipline was particularly vulnerable. (Menzler-Trott gives an exhaustive account of Nazi views on the nature and rôle of mathematics.) In 1945 Gentzen and others were arrested in Prague and put into 'protective custody'. As a German, he was reviled by the Czechs and the unhealthy conditions of his imprisonment led to his death.

The book under review relates all of the above, and much more. The last third of this book is devoted to technical appendices including welcome translations of three of Gentzen's papers, which describe his philosophical motivation and exhibit a genuine concern for mathematical practice. Smoryński presents his view of Hilbert's programme and Hilbert's ultimate triumph over Brouwer, but many of us would question whether Hilbert really was the winner: constructive (or effective) mathematics is flourishing and is essential for computing. I am more inclined to agree with Gentzen's style: on p. 365 (retaining his italics) Gentzen says that to consider the continuum a mathematical fiction or a reality 'is a matter of taste; for mathematical *practice* it has hardly any significance.' The last forty pages of the book are taken up with a technical description of Gentzen's mathematical achievements by Jan von Plato, an appendix which I find difficult to assess. This reviewer is familiar with, and uses, Gentzen's techniques, so finds van Plato's account much simplified; I am not convinced a more casual reader would find it easy to follow.

Gentzen's life is dealt with by presenting a large number of documents (in English translation) that weigh down upon the reader, though having them available will be extremely useful for future scholars. One reads school reports and letters of recommendation but these seem a hindrance rather than a help in providing insights into Gentzen's life and motivations. The translation, mostly by Craig Smoryński, leans more to a word-for-word translation than a literary one and Germanisms sometimes intrude. 'Because one could produce him internationally.' on p. 124 is one of the more startling sentences and 'ordinary' (p. 46) seems an inadequate translation of 'Ordinarius' as applied to a professor. The index is of names only, which seems a pity.

So Eckart Menzler-Trott's book is a book for all libraries; it is not bedtime reading. But Eckart Menzler-Trott is right: Gentzen was a genius, though his abilities are hard to communicate. His wonderfully simple 'natural deduction' is his memorial.

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